Aghabozorgi et al., *Cogent Mathematics & Statistics* (2018), 5: 1510357
https://doi.org/10.1080/25742558.2018.1510357

cogent mathematics & statistics

PURE MATHEMATICS | RESEARCH ARTICLE

On some classes of hypergroups

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Received: 19 April 2018
Accepted: 07 August 2018
First Published: 15 August 2018

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Reviewing editor:
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Abstract: In this paper, we first introduce the notion of power(ful) hypergroups, proving that the classes of cancelative semigroups and complete hypergroups are powerful hypergroups. We investigate some properties of this class of (semi) hypergroups.

Subjects: Group Theory; Combinatorics; Pure Mathematics

Keywords: (semi)hypergroup; powerful hypergroup; complete hypergroup

1. Introduction

Algebraic hyperstructures represent a natural extension of classical algebraic structures. In contemporary algebraic hyperstructures, one studies a series of algebraic objects, defined with the aid of one or several hyperoperations in a set of elements of this or that kind. One obtains one or another algebraic hyperstructure theory, depending upon the collection of these hyperoperations, their properties, and the nature of the set. One of these theories is the theory of semihypergroups. Definition of semihypergroups in general goes back at least to the 1934 by Marty (1934). Indeed, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. In a semigroup, the composition of two elements is an element, while in a semihypergroup, the composition of two elements is a non-empty set. Semihypergroups have many applications in automata, probability, geometry, lattices, binary relations, graphs, hypergraphs, and other branches of science such as biology, chemistry and physics. Semihypergroups have attracted the attention of numerous researchers, both for their various properties and nowadays for their generalizations to Γ-semihypergroups (Hila, Davvaz, & Dine, 2012), LA-semihypergroups (Hila & Dine, 2011), or for their connections with groups (Aghabozorgi, Davvaz, & Jafarpour, 2013a), polygroups (Aghabozorgi, Davvaz, & Jafarpour, 2013b), or complete parts (Jafarpour, Leoreanu-Fotea, & Zolfaghari, 2013).

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PUBLIC INTEREST STATEMENT

Algebraic hypertructures represent a field of algebra of major attraction and productive of many significant results in algebra. There are a lot of topics about hyperstructures, which can be in depth analyzed and there also are open problems and new connections to other fields that can be explored more in the future. Marty defined hypergroups as a generalization of groups. There are applications of hypergroups to the following subjects: geometry, hypergraphs, binary relations, combinatorics, codes, cryptography, probability and etc.

There are applications of hypergroups to the following subjects: geometry, hypergraphs, binary relations, combinatorics, codes, cryptography, probability and etc. In this paper, a new class of hypergroups is introduced.

In this paper, we first introduce the notion of power(ful) hypergroups, proving that the classes of cancelative semigroups and complete hypergroups are powerful hypergroups, we show that a subsemihypergroup of a powerful semihypergroup is not necessary a subsemihypergroup of H_n^*. Finally, we prove that if H_n is a powerful hypergroup, then $\frac{H_n^*}{\beta^*}$ is the trivial group.

2. Preliminaries

We recall here some basic notions of hypergroup theory and we fix the notations used in this note. We referee the readers to the following fundamental books Corsini (1993), Corsini & Leoreanu (2003) and Vougiouklis (1994).

Let H be a non-empty set and $P^*(H)$ denote the set of all non-empty subsets of H. Let $\circ$ be a *hyperoperation* (or *join operation*) on H, that is, a function from the Chartesian product $H \times H$ into $P^*(H)$. The image of the pair $(a,b) \in H \times H$ under the hyperoperation $\circ$ in $P^*(H)$ is denoted by $a \circ b$. The join operation can be extended in a natural way to subsets of H as follows: for non-empty subsets A, B of H, define $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$. The notation $a \circ A$ is used for $\{a\} \circ A$ and $A \circ a$ for $A \circ \{a\}$. Generally, the singleton $\{a\}$ is identified with its element a. The hyperstructure $(H, \circ)$ is called a *hypergroupoid* and it is called commutative if $a \circ b = b \circ a$, for all $a, b \in H$. The hypergroupoid $(H, \circ)$ is called a *semihypergroup* if $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in H$, which means that

$$\bigcup_{u \in a \circ b} u \circ c = \bigcup_{v \in b \circ c} a \circ v.$$

A semihypergroup $(H, \circ)$ is called *complete* if, for all natural numbers $n, m \geq 2$ and all tuples $(x_1, x_2, \ldots, x_n) \in H^n$ and $(y_1, y_2, \ldots, y_m) \in H^m$, we have the following implication:

$$\prod_{i=1}^{n} x_i \bigcap \prod_{j=1}^{m} y_j \neq \emptyset \Rightarrow \prod_{i=1}^{n} x_i = \prod_{j=1}^{m} y_j.$$

where $\prod_{i=1}^{n} x_i = x_1 \circ x_2 \circ \cdots \circ x_n$ and $\prod_{j=1}^{m} y_j = y_1 \circ y_2 \circ \cdots \circ y_m$.

A semihypergroup $(H, \circ)$ is called a *hypergroup* if the reproduction law holds: $a \circ H = H \circ a = H$, for all $a \in H$. A non-empty subset K of a hypergroup $(H, \circ)$ is called a *subhypergroup* if it is a hypergroup, too. An element e of H is called an *identity* if, for all $x \in H$, $x \in x \circ e \cap e \circ x$ and $a' \in H$ is called an *inverse* of a in H if $e \in a \circ a' \cap a' \circ a$. Suppose that $(H, \circ)$ and $(H', *)$ are two semihypergroups. A function $f : H \to H'$ is called a *homomorphism* if $f(a \circ b) \subseteq f(a) * f(b)$, for all a and b in H. We say that f is a *good homomorphism* if, for all a and b in H, $f(a \circ b) = f(a) * f(b)$. If $(H, \circ)$ is a hypergroup and $\rho \subseteq H \times H$ is an equivalence relation, then, for all non-empty subsets A, B of H, we set

$$A \overline{\overline{\rho}} B \Leftrightarrow a \rho b, \text{ for all } a \in A,\ b \in B.$$

The relation ρ is called *strongly regular on the left* (respectively, *on the right*) if $x \rho y$ implies $a \circ x \overline{\overline{\rho}} a \circ y$ (respectively, $x \rho y$ implies $x \circ a \overline{\overline{\rho}} y \circ a$), for all $(x, y, a) \in H^3$. Moreover, ρ is called *strongly regular* if it is strongly regular on the right and on the left. The significance of the strongly regular relation is given by the following result.

Theorem 2.1. (Corsini, 1993) *If $(H, \circ)$ is a semihypergroup (respectively, a hypergroup) and ρ is a strongly regular relation on H, then the quotient H/ρ is a semigroup (respectively, a group) under the operation defined by*

$$\rho(x) \otimes \rho(y) = \rho(z), \textit{ for all } z \in x \circ y.$$

Denote the class $\rho(x)$ of the element x by $\bar{x}$ and write just $\bar{x}\bar{y}$ instead of $\bar{x} \otimes \bar{y}$. For all natural numbers $n > 1$, define the relation β_n on a semihypergroup H, as follows:

$$a \beta_n b \Leftrightarrow \exists (x_1, \ldots, x_n) \in H^n \text{ such that } \{a, b\} \subseteq \prod_{i=1}^{n} x_i,$$

and take $\beta = \bigcup_{i=1}^{n} \beta_n$, where $\beta_1 = \{(x,x) | x \in H\}$ is the diagonal relation on H. Denote by β^* the transitive closure of β. The relation β^* is a strongly regular relation (Corsini, 1993). This relation was introduced by Koskas (Koskas, 1970) and studied mainly by Freni (1991), proving the following basic result: If H is hypergroup, then $\beta = \beta^*$. Note that, in general, for a semihypergroup we may have $\beta \neq \beta^*$. Moreover, the relation β^* is the smallest equivalence relation on a hypergroup H, such that the quotient H/β^* is a group. The *heart* ω_H of a hypergroup H is defined like the set of all elements x of H, for which the equivalence class $\beta^*(x)$ is the identity of the quotient group H/β^*.

3. Power semihypergroup

In this section, we first introduce the notion of power(ful) hypergroup, proving that the classes of cancelative semigroups and complete hypergroups are powerful hypergroup. By an example we show that a subsemihypergroup of a powerful semihypergroup is not necessary a subsemihypergroup of H_n^*.

For every $n \in \mathbb{N}$, we denote $H_n = \{1, 2, ..., n\}$. Moreover, if $\circ$ is a hyperoperation on H_n we define $e_{ij} = (i,j)$ and $D_k = \{e_{ij} \mid k \in i \circ j\}$, for every $(i,j,k) \in H_n^3$. Notice that

$$e_{ij} = e_{rs} \Leftrightarrow (i,j) = (r,s), \forall (i,j,r,s) \in H_n^4.$$

Proposition 3.1. *Let $(H_n, \circ)$ be a hypergroupoid. The $(H_n^*, \circ_D)$ is a hypergroupoid, where $H_n^* = \{e_{ij} \mid (i,j) \in H_n^2\}$ and for every $(i,j,s,t) \in H_n^4$,*

$$e_{ij} \circ_D e_{st} = \bigcup_{k \in \{i,s\} \circ \{j,t\}} D_k.$$

Proof. It is easy to see that $\{e_{ij}, e_{st}\} \subseteq e_{ij} \circ_D e_{st}$, for every $(i,j,s,t) \in H_n^4$. □

From now on we call $\circ_D$ the power hyperoperation of $\circ$. Moreover, $(H_n^*, \circ_D)$ is called the power hypergroupoid of $(H_n, \circ)$. If $(H_n, \circ)$ is a (semi) hypergroup and the power hypergroupoid is a (semi) hypergroup, then we call $(H_n, \circ)$ a powerful (semi)hypergroup and $(H_n^*, \circ_D)$ is the power (semi) hypergroup of $(H_n, \circ)$.

Example 3.2. *Let $(H_2, +)$ be the cyclic group of order 2. Then, the power hypergroup of $(H_2, +)$ is as below*:

$\circ_D$	e_{00}	e_{01}	e_{10}	e_{11}
e_{00}	$\{e_{00}, e_{11}\}$	H_2^*	H_2^*	H_2^*
e_{01}	H_2^*	$\{e_{01}, e_{10}\}$	H_2^*	H_2^*
e_{10}	H_2^*	H_2^*	$\{e_{01}, e_{10}\}$	H_2^*
e_{11}	H_2^*	H_2^*	H_2^*	$\{e_{00}, e_{11}\}$

In this case $(H_2^*, \circ_D)$ is a hypergoup.

Proposition 3.3. *The power hypergroupoid $(H_n^*, \circ_D)$ of $(H_n, \circ)$ is commutative.*

Proof. The proof is straightforward. □

Proposition 3.4. *Let $(H_n, \circ)$ be a groupoid.*

(*i*) If at $(H_n, \circ)$ left cancellation property holds then $|D_k| = n$, for all $k \in H_n$.
(*ii*) If at $(H_n, \circ)$ right cancellation property holds then $|D_k| = n$, for all $k \in H_n$.
(*iii*) If $(H_n, \circ)$ is a group that $|H_n| \geq 5$, then for all $(u,v,a,b) \in H_n^4$ we have $e_{vs} \circ_D e_{ab} \neq H_n^*$.

Proof. (i) Let $(k, i) \in H_n^2$. Because the left cancellation property holds we conclude that there exists a unique $j \in H_n$ such that $k = i \circ j$. Notice that if $k \neq i \circ j$ for every $j \in H_n$ then there exists $(s, t) \in H_n^2$ such that $s \neq t$ and $e_{is} = e_{it}$, which is a contradiction. Thus $|D_k| = n$. The proof of (ii) is similar to the part (i).

(iii) Let $(u, v, a, b) \in H_n^4$. According to parts (i) or (ii) and the definition of $\circ_D$ we have

$$|e_{vs} \circ_D e_{ab}| = |D_{v\circ s} \cup D_{v\circ b} \cup D_{a\circ s} \cup D_{a\circ b}| \leq 4n \langle n^2 = |H_n^*|.$$

Hence, for $|H_n| \geq 5$ we get $e_{vs} \circ_D e_{ab} \neq H_n^*$. □

Theorem 3.5. *If $(H_n, \circ)$ is a group, then $(H_n^*, \circ_D)$ is a hypergroup.*

Proof. First we prove that $D_{u\circ v} \circ_D e_{ab} = H_n^* = e_{ab} \circ_D D_{u\circ v}$, for all $(u, v, a, b) \in H_n^4$.

$$\begin{aligned} D_{u\circ v} \circ_D e_{ab} &= \{e_{xy} | u \circ v = x \circ y\} \circ_D e_{ab} \\ &= \bigcup_{u\circ v = x\circ y} e_{xy} \circ_D e_{ab} \\ &\supseteq \bigcup_{u\circ v = x\circ y} D_{x\circ b} \\ &= \bigcup_{g\in H_n} D_g = H_n^*. \end{aligned}$$

Similarly, we have $e_{ab} \circ_D D_{u\circ v} = H_n^*$. Let $(u, t, v, s, a, b) \in H_n^6$. Then, we obtain

$$(e_{ut} \circ_D e_{vs}) \circ_D e_{ab} \supseteq D_{u\circ t} \circ_D e_{ab} = H_n^*.$$

On the other hand,

$$e_{ut} \circ_D (e_{vs} \circ_D e_{ab}) \supseteq e_{ut} \circ_D D_{v\circ s} = H_n^*.$$

Therefore, $(e_{ut} \circ_D e_{vs}) \circ_D e_{ab} = e_{ut} \circ_D (e_{vs} \circ_D e_{ab})$

Remark 3.6. The following example shows that $(H_n^*, \circ_D)$ is not necessarily a hypergroup.

Example 3.7. Suppose that $H_2 = \{1, 2\}$. Consider the groupoid $(H, \circ)$ endowed with the operation $\circ$ defined as follows:

$\circ$	1	2
1	1	1
2	1	2

$\circ_D$	e_{11}	e_{12}	e_{21}	e_{22}
e_{11}	D_1	D_1	D_1	H_2^*
e_{12}	D_1	D_1	H_2^*	H_2^*
e_{21}	D_1	H_2^*	D_1	H_2^*
e_{22}	H_2^*	H_2^*	H_2^*	D_2

In this case, $(H_2, \circ)$ is a semihypergroup but $(H_2^*, \circ_D)$ is not a semihypergroup, because $D_1 = \{e_{11}, e_{12}, e_{21}\}$ and $(e_{11} \circ_D e_{11}) \circ_D e_{12} = D_1 \circ_D e_{12} = H_2^*$, while $e_{11} \circ_D (e_{11} \circ_D e_{12}) = e_{11} \circ_D D_1 = D_1$. Hence, $(e_{11} \circ_D e_{11}) \circ_D e_{12} \neq e_{11} \circ_D (e_{11} \circ_D e_{12})$, which implies that $(H_2, \circ)$ is not a powerful semihypergroup.

Example 3.8. Suppose that $H_2 = \{1, 2\}$. Consider the hypergroupoid $(H, \circ)$ endowed with the hyperoperation $\circ$ defined as follows:

$\circ$	1	2
1	1	1, 2
2	2	1

$\circ_D$	e_{11}	e_{12}	e_{21}	e_{22}
e_{11}	D_1	H_2^*	H_2^*	H_2^*
e_{12}	H_2^*	H_2^*	H_2^*	H_2^*
e_{21}	H_2^*	H_2^*	D_2	H_2^*
e_{22}	H_2^*	H_2^*	H_2^*	D_1

In this case, $(H_2, \circ)$ is not a semihypergroup, because $(1 \circ 2) \circ 2 = \{1, 2\} \neq \{1\} = 1 \circ (2 \circ 2)$, but $(H_2^*, \circ_D)$ is a hypergroup.

If $(H_n, \circ)$ and $(H_n, \circ')$ are semihypergroups, for every map

$f : (H_n, \circ) \to (H_n, \circ')$ we define the map f^* as follows:

$f^* : (H_n^*, \circ_D) \to (H_n^*, \circ'_D)$

$f^*(e_{ij}) = e_{f(i)f(j)}$.

Moreover, if K is a non-empty subset of H_n, then we denote the set $K^* = \{e_{ij} \mid (i, j) \in K^2\}$. In the following example, we show that if $(H_n, \circ)$ is a powerful semihypergroup and K is a subsemihypergroup of $(H_n, \circ)$, then K^* is not necessarily a subsemihypergroup of $(H_n^*, \circ_D)$.

Example 3.9. *Let* $(H_2, \circ)$ *and* $(H_2^*, \circ_D)$ *be as follow*:

H_2	1	2
1	1	2
2	2	1

H_2^*	e_{11}	e_{12}	e_{21}	e_{22}
e_{11}	D_1	H_2^*	H_2^*	H_2^*
e_{12}	H_2^*	D_2	H_2^*	H_2^*
e_{21}	H_2^*	H_2^*	D_2	H_2^*
e_{22}	H_2^*	H_2^*	H_2^*	D_1

Then $K = \{1\}$ is a subgroup of $(H_2, \circ)$ but $K^* = \{e_{11}\}$ is not subhypergroup of $(H_2^*, \circ_D)$, because $D_1 = \{e_{11}, e_{22}\}$ and $e_{11} \circ_D e_{11} = D_1 \not\subseteq K^*$.

Theorem 3.10. *Let* $(H_n, \circ)$ *and* $(H_n, \circ')$ *be a powerful semihypergroups and* $f : (H_n, \circ) \to (H_n, \circ')$ *be a map.*

(1) If f is a homomorphism, then f^* is a homomorphism.

(2) If f is a isomorphism, then f^* is a isomorphism.

Proof. (1) Let $(e_{ij}, e_{st}) \in H_n^{*^2}$. Then,

$$\begin{aligned} f^*(e_{ij} \circ_D e_{st}) &= f^*\Big(\bigcup_{k \in \{i,s\} \circ \{j,t\}} D_k\Big) \\ &= \bigcup_{k \in \{i,s\} \circ \{j,t\}} f^*(D_k) \\ &= \bigcup_{k \in \{i,s\} \circ \{j,t\}} \{f^*(e_{ij}) \mid k \in i \circ j\} \\ &\subseteq \bigcup_{f(k) \in \{f(i), f(s)\} \circ' \{f(j), f(t)\}} \{e_{f(i)f(j)} \mid f(k) \in f(i) \circ' f(j)\} \\ &= \bigcup_{f(k) \in \{f(i), f(s)\} \circ' \{f(j), f(t)\}} D_{f(k)} \\ &= e_{f(i)f(j)} \circ'_D e_{f(s)f(t)} \\ &= f^*(e_{ij}) \circ'_D f^*(e_{st}). \end{aligned}$$

The proof of (2) is straightforward.

Notice that if f is a good homomorphism in Theorem 3.10 then f^* is not necessarily a good homomorphism.

Proposition 3.11. *Let $(H_n, \circ)$, $(H_n, \circ')$, and $(H_n, \circ'')$ be powerful semihypergroups, and $f : (H_n, \circ) \to (H_n, \circ')$ and $g : (H_n, \circ') \to (H_n, \circ'')$ be maps. Then $(gf)^* = g^* f^*$.*

Proof. Let $e_{ij} \in H_n^*$. Then

$$\begin{aligned}(gf)^*(e_{ij}) &= e_{(gf)(i)(gf)(j)}\\ &= e_{g(f(i))g(f(j))}\\ &= g^*(e_{f(i)(f(j)})\\ &= g^* f^*(e_{ij}).\end{aligned}$$

The proof of (2) is straightforward.

Notice that if f is a good homomorphism in Theorem 3.11 then f^* is not necessarily a good homomorphism.

Example 3.12. *Consider the Cayley tables of Example 3.9 and let $f : (H_2, \circ) \to (H_2, \circ)$ be a good homomorphism satisfying $f(1) = f(2) = 1$. In this case, we have $f^*(e_{11} \circ_D e_{11}) = \{f(e_{11})\}$, while $f^*(e_{11}) \circ_D f^*(e_{11}) = e_{11} \circ_D e_{11} = \{e_{11}, e_{22}\}$. Hence, $f^*(e_{11} \circ_D e_{11}) \neq f^*(e_{11}) \circ_D f^*(e_{11})$. So, f^* is not a good homomorphism.*

Theorem 3.13. *Every complete hypergroup is a powerful hypergroup.*

Proof. Suppose that $(H_n, \circ)$ is a complete hypergroup. We have

$$\begin{aligned}(e_{us} \circ_D e_{vt}) \circ_D e_{ab} &\supseteq \bigcup_{k \in u \circ s} D_k \circ_D e_{ab}\\ &\supseteq \bigcup_{k \in u \circ s, k \in x \circ y} e_{xy} \circ_D e_{ab}\\ &= \bigcup_{u \circ s = x \circ y} e_{xy} \circ_D e_{ab} \; (H_n \text{ is complete})\\ &\supseteq \bigcup_{t \in x \circ b, u \circ s = x \circ y} D_t = \bigcup_{t \in H_n} D_t = H_n^*.\end{aligned}$$

Similarly, we have $e_{us} \circ_D (e_{vt} \circ_D e_{ab}) = H_n^*$. □

Proposition 3.14. *Let H_n be a complete hypergroup. Then $\frac{H_n^*}{\beta^*}$ is a trivial group.*

Proof. For every $(i, j, s, t, u, v) \in H_n^6$, the equality $(e_{ij} \circ_D e_{st}) \circ_D e_{uv} = H_n^*$ shows that $\beta_3 = \beta^*$, so $\beta_3(x) = \beta^*(x) = H_n^*$, for all $x \in H_n^*$. Hence, $\frac{H_n^*}{\beta^*}$ is the trivial group.

4. Complement of H_n

Definition 4.1. (Aghabozorgi, Cristea, & Jafarpour, 2016) Let $(H, \circ)$ be a semihypergroup such that $x \circ y \neq H$, for all $x, y \in H$. We call the complement of $(H, \circ)$ the hypergroupoid $(H, \circ^c)$ endowed with the complement hyperoperation: $x \circ^c y = H - x \circ y$. We say that the semihypergroup $(H, \circ)$ is complementable if its complement $(H, \circ^c)$ is a semihypergroup too, and in this case $(H, \circ^c)$ is called the complement semihypergroup of $(H, \circ)$.

Proposition 4.2. (Aghabozorgi et al., 2016) *Every non-trivial group is complementable.*

Lemma 4.3. *If $(H_n, \circ)$ is a group and $(H_n, \circ^c)$ is the complement of $(H_n, \circ)$, then for every $(i, j, s, t) \in H_n^4$,*

(1) *if* $i = s$ *and* $j = t$, *then* $e_{ij} \circ_D^c e_{st} = H_n^* - D_{i\circ j}$;
(2) *if* $i \neq s$ *or* $j \neq t$, *then* $e_{ij} \circ_D^c e_{st} = H_n^*$.

Proof. (1) Let $i = s$ and $j = t$. Then

$$\begin{aligned} e_{ij} \circ_D^c e_{st} = e_{ij} \circ_D^c e_{ij} &= \bigcup_{k \in i \circ^c j} D_k \\ &= \bigcup_{k \in H_n - \{i \circ j\}} D_k \\ &= H_n^* - D_{i\circ j}. \end{aligned}$$

(2) Let $i \neq s$ or $j \neq t$. Then either $i \circ j \neq i \circ t$ or $s \circ j \neq s \circ t$ and so we have

$$\begin{aligned} e_{ij} \circ_D^c e_{st} &= \bigcup_{k \in \{i,s\} \circ^c \{j,t\}} D_k \\ &= \left(\bigcup_{k \in H_n - \{i \circ j\}} D_k \right) \cup \left(\bigcup_{k \in H_n - \{i \circ t\}} D_k \right) \cup \left(\bigcup_{k \in H_n - \{s \circ j\}} D_k \right) \cup \left(\bigcup_{k \in H_n - \{s \circ t\}} D_k \right) \\ &= \bigcup_{k \in H_n} D_k \\ &= H_n^*. \end{aligned}$$

Remark 4.4. In Lemma 4.3, $\circ_D^c$ is not the complement of the hyperoperation $\circ_D$. Indeed, in Example 3.2 $e_{00} \circ_D^c e_{00} = \{e_{01}, e_{10}\}$. But $e_{00} (\circ_D)^c e_{00} = \{e_{00}, e_{11}\}$. Thus $e_{00} \circ_D^c e_{00} \neq e_{00} (\circ_D)^c e_{00}$.

Theorem 4.5. *If* $(H_n, \circ)$ *is a group, then* $(H_n, \circ^c)$ *is a powerful hypergroup.*

Proof. Using Lemma 4.3, we have $(e_{ij} \circ_D^c e_{st}) \circ_D^c e_{uv} = H_n^* = e_{ij} \circ_D^c (e_{st} \circ_D^c e_{uv})$ for every $(i, j, s, t, u, v) \in H_n^6$. □

5. Conclusion

In this paper, we introduced and analyzed the notion of power hyperoperation. A new class of semihypergroups called powerful semihypergroups are investigated. We showed that cancelative semigroups and complete hypergroups are powerful hypergroups. This construction of semihypergroups can be used for classification of the class of semihypergroups.

Acknowledgements
We thank the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions.

Funding
This work was supported by the Vali-e-Asr University of Rafsanjan [Aghabozorgi].

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Citation information
Cite this article as: On some classes of hypergroups, H. Aghabozorgi, M. Jafarpour & B. Davvaz, *Cogent Mathematics & Statistics* (2018), 5: 1510357.

References

Aghabozorgi, H., Cristea, I., & Jafarpour, M. (2016). On complementable semihypergroups. *Communications in Algebra*, 44(4), 1740–1753.
Aghabozorgi, H., Davvaz, B., & Jafarpour, M. (2013a). Nilpotent groups derived from hypergroups. *Journal Algebra*, *382*, 177–184. doi:10.1016/j.jalgebra.2013.02.011
Aghabozorgi, H., Davvaz, B., & Jafarpour, M. (2013b). Solvable polygroups and derived subpolygroups. *Communications in Algebra*, 4(18), 3098–3107.
Corsini, P. (1993). *Prolegomena of hypergroup theory*. Tricesimo: Aviani Editore.
Corsini, P., & Leoreanu, V. (2003). *Applications of hyperstructure theory*. Dordrecht: Kluwer Academical Publications.
Freni, D. (1991). Une note sur le cour d'un hypergroupe et sur la clôture transitive ß*de . (French) [A note on the core of a hypergroup and the transitive closure ß*of ß]. *Riv Mat Pura Applications*, *8*, 153–156.
Hila, K., Davvaz, B., & Dine, J. (2012). *Study on the structure of Γ-semihypergroups. Communications in Algebra*, *40*, 2932–2948. doi:10.1080/00927872.2011.587855
Hila, K., & Dine, J. (2011). On hyperideals in left almost semihypergroups. *ISRN Algebra, Art. ID 953124*, 8.
Jafarpour, M., Leoreanu-Fotea, V., & Zolfaghari, A. (2013). *Weak complete parts in semihypergroups,*

Iran. *Journal Mathematical Sciences Informatics*, *8*, 101–109.

Koskas, M. (1970). Groupoides, demi-hypergroupes et hypergroupes. *Journal Mathematical Pures Applications*, *49*, 155–192.

Marty, F. (1934). *Sur une generalization de la Notion de Groupe, 8th congress mathematics* (pp. 45–49). Scandenaves, Stockholm, Sweden.

Vougiouklis, T. (1994). *Hyperstructures and their representations*. Palm Harbor, FL: Hadronic Press.