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Functions and wv -Lindelöf with respect to a hereditary class

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Abstract: A collection of a nonempty subsets of is called hereditary class if it is closed under hereditary property. In this work, we define and study the notion of some generalizations of ν -Lindelöf generalized topological spaces with respect to a hereditary class, namely; ν -Lindelöf hereditary generalized topological spaces. Moreover, investigate basic properties of the concepts, its relation to known concepts and its preservation by functions properties.

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1. Introduction and preliminaries

A lot of attention has been made to study properties of covering in topological spaces, which include open and different kinds of generalized open sets. Furthermore, several authors have introduced the generalization of Lindelöf space separately for many reasons and according to the sets that they are interested in. In this work, we use the notions of generalized topology and hereditary classes introduced by (Császár, 2002), (Császár, 2005) and (Császár, 2007), respectively. In order to define some of generalizations of $\nu\mathcal{H}$ -Lindelöf (Qahis, AlJarrah, & Noiri, 2016), namely; $w\nu\mathcal{H}$ -Lindelöf hereditary- generalized topological spaces. In literature, (Sarsak, 2012) introduced and studied ν -Lindelöf sets in generalized topological spaces. Recently, (Abuage and Kiliçman, 2017) introduced $w\nu$ -Lindelöf generalized topological spaces. The notion ν -Lindelöfness in term of a hereditary classes was studied by (Qahis et al., 2016).

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PUBLIC INTEREST STATEMENT

Lindelöfness and its generalizations are important and interesting concepts in topology. Furthermore, Lindelöf and its generalizations have been done to generalized topological spaces, the earlier generalizations to generalized covering properties is ν -Lindelöf. Recently, the concept of ν -Lindelöfness with respect to hereditary class has been introduced. This paper will introduce and define one of generalizations of ν -Lindelöf; namely, wv -Lindelöf with respect to hereditary class. Some properties and counterexamples are showed. Functions properties are investigated, and we proved that the image of a $w\nu\mathcal{H}$ -Lindelöf under an almost ν -continuous function is $w\nu\mathcal{H}$ -Lindelöf.

The strategy of using generalized topologies and hereditary classes to extend classical topological concepts have been used by many authors such as (e.g. Császár, 2007; Kim & Min, 2012; Ramasamy, Rajamani, & Inthumathi, 2012; Zahran, El-Saady, & Ghareeb, 2012), among others. Realization of generalized continuous function was introduced by (Császár, 2002), interesting types of functions in generalized topological spaces have been introduced by many mathematicians, such as Al-Omari and Noiri (2012), Al-Omari and Noiri (2013), Min (2009), Min (2010a), and Min (2010b). The purpose of this paper is to study the effect of functions on $w\nu\mathcal{H}$ -Lindelöf generalized topological spaces. We also show that some functions preserve this property. The main result is that the image of a $w\nu\mathcal{H}$ -Lindelöf under an almost (ν, μ) -continuous function is $w\mu\mathcal{H}$ -Lindelöf.

Suppose a non-empty set X , $P(X)$ denotes the power set of X and ν be a non-empty family of $P(X)$. The symbol ν implies a generalized topology (briefly. \mathcal{GT}) on X (Császár, 2002) if the empty set $\emptyset \in \nu$ and $U_\gamma \in \nu$ where $\gamma \in \Omega$ implies $\bigcup_{\gamma \in \Omega} U_\gamma \in \nu$. The pair (X, ν) is called generalized topological space (briefly. \mathcal{GTS}) and we always denote it by $\mathcal{GTS}(X, \nu)$ or X_ν . Each element of \mathcal{GT} ν is said to be ν -open set and the complement of ν -open set is called ν -closed set. Let A be a subset of a $\mathcal{GTS}(X, \nu)$, then $i_\nu(A)$ (resp. $c_\nu(A)$) denotes the union of all ν -open sets contained in A (resp. denotes the intersection of all ν -closed sets containing A), and $X_\nu \setminus A$ denotes the complement of A , $c_\nu(X_\nu \setminus A) = X_\nu \setminus (i_\nu A)$. Moreover, A is said to be ν -regular open (resp. ν -regular closed) iff $A = i_\nu c_\nu(A)$ (resp. $A = c_\nu i_\nu(A)$) (Császár, 2008). If a set $X_\nu \in \nu$, then a $\mathcal{GTS}(X, \nu)$ is called ν -space (Noiri, 2006), and will be denoted by a ν -space (X, ν) or a ν -space X_ν . X_ν is said to be quasi-topological space (Császár, 2006), if the finite intersection of ν -open sets of ν belongs to ν and denoted by $\mathcal{QTS}(X, \nu)$. If $\beta \subseteq P(X)$ and $\emptyset \in \beta$. Then β is called a ν -base (Császár, 2004) for ν if $\{\bigcup \beta' : \beta' \subseteq \beta\} = \nu$, and we say that ν is generated by β . A $\mathcal{GTS}(X, \nu)$ is said to be ν -extremally disconnected (Császár, 2004) if the ν -closure of every ν -open set is ν -open. Moreover, a subset A of a $\mathcal{GTS}(X, \nu)$ is called ν -clopen if it is both ν -open and ν -closed.

Let (X, ν) be a \mathcal{GTS} , a cover U of a subsets of X is called ν -open cover if the elements of U are ν -open subsets of X (Thomas & John, 2012). A $\mathcal{GTS}(X, \nu)$ is said to be ν -Lindelöf (Sarsak, 2012) (resp. $w\nu$ -Lindelöf (Abuage & Kiliçman, 2017)) if for each ν -open cover $U = \{U_\gamma : \gamma \in \Omega\}$ of X_ν admits a countable sub-collection $\{U_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu = \bigcup_{n \in \mathbb{N}} U_{\gamma_n} \text{ (resp. } \Lambda_\nu = c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n})),$$

where Λ_ν is the union of all ν -open set in X_ν .

A non-empty family \mathcal{H} of subsets of X_ν is called a hereditary class (Császár, 2007) if $A \in \mathcal{H}$ and $B \subset A$ imply that $B \in \mathcal{H}$ (Kuratowski., 1933). Given a generalized topological space (X, ν) with a hereditary class \mathcal{H} , for a subset A of X_ν , the generalized local function of A with respect to \mathcal{H} and ν (Császár, 2007) is defined as follows: $\mathcal{A}^* = \{x \in X_\nu : U \cap A \notin \mathcal{H} \text{ for all } U \in \nu_x\}$, where $\nu_x = \{U : x \in U \text{ and } U \in \nu\}$; and the following are defined: $c_\nu^* = \mathcal{A} \cup \mathcal{A}^*$ and the family $\nu^* = \{A \subset X_\nu : X_\nu \setminus A = c_\nu^*(X_\nu \setminus A)\}$ is a \mathcal{GT} on X_ν . The elements of ν^* are called ν^* -open and the complement of a ν^* -open set is called ν^* -closed set. It is clear that a subset A is ν^* -closed if and only if $\mathcal{A}^* \subset A$. If the hereditary class \mathcal{H} satisfies the additional condition: if $A, B \in \mathcal{H}$ implies $A \cup B \in \mathcal{H}$, then \mathcal{H} is called an ideal on X_ν (Kuratowski., 1933). We call (X, ν, \mathcal{H}) a hereditary generalized topological space and denoted by $\mathcal{HGT}(X, \nu, \mathcal{H})$ or simply $X_{\nu, \mathcal{H}}$. Let a $\mathcal{GTS}(X, \nu)$, we denoted by \mathcal{H}_c the hereditary class of countable subsets of X_ν .

Definition 3.1 (Sarsak, 2012) Let (X, ν) and $A \subseteq X_\nu$. Then a collection $\{U \cap A : U \in \nu\}$ is said to be generalized topology on A , and denote by $\nu(A)$. A \mathcal{GT} $\nu(A)$ on A forms a generalized topological subspace of X_ν , denoted by $(A, \nu(A))$.

Let $(\mathcal{X}_G, \nu, \mathcal{H})$ be a $\mathcal{H}GTS$ and $\mathcal{A} \subseteq \mathcal{X}_G$, $\mathcal{A} \neq \emptyset$. We denote by $\mathcal{H}_{\mathcal{A}}$ the collection $\{H \cap (\mathcal{A} \cap \Lambda_\nu) : H \in \mathcal{H}\}$ and by $(\mathcal{A}, \nu(\mathcal{A}))$ the subspace of (\mathcal{X}_G, ν) on \mathcal{A} .

Definition 3.2 (Qahis et al., 2016) Let (\mathcal{X}_G, ν) be a GTS and \mathcal{H} be a hereditary class on \mathcal{X}_G . A $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is called $\nu\mathcal{H}$ -Lindelöf or ν -Lindelöf respect to a hereditary class on \mathcal{X}_G if each ν -open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of Λ_ν has a countable subcollection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that $\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n} \in \mathcal{H}$.

Lemma 3.3 (Csa'sz'ar, 2008)

(a) If \mathcal{F} is ν -closed set then $i_\nu(\mathcal{F})$ is ν -regular open.

(b) If \mathcal{U} is ν -open set then $c_\nu(\mathcal{U})$ is ν -regular closed.

Theorem 3.4 (Császár, 2007) Let (\mathcal{X}_G, ν) be a GTS and \mathcal{H} be a hereditary class on \mathcal{X}_G

(i) A $GT \nu^*$ finer than ν ,

(ii) If \mathcal{A} be a subset of \mathcal{X}_G , then $\mathcal{A}^* \subseteq c_\nu(\mathcal{A})$.

Theorem 3.5 (Császár, 2007) Let (\mathcal{X}_G, ν) be a GTS and \mathcal{H} be a hereditary class on \mathcal{X}_G and \mathcal{U} be a subset of \mathcal{X}_G . If \mathcal{U} is ν^* -open, then for each $x \in \mathcal{U}^*$ there is $\mathcal{U} \in \nu_x$ and $H \in \mathcal{H}$ such that $x \in \mathcal{U} \setminus H \subset \mathcal{U}^*$.

Lemma 3.6 (Carpintero, Rosas, Salas-Brown, & Sanabria, 2016) Let a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$. If \mathcal{H} is a hereditary class on \mathcal{X}_G , then $g(\mathcal{H}) = \{g(H) : H \in \mathcal{H}\}$ is a hereditary class on \mathcal{Y}_G .

2. $w\nu$ -Lindelöf with respect to a hereditary class \mathcal{H}

The following concepts give a characterization of $w\nu\mathcal{H}$ -Lindelöf.

Definition 4.1 Let (\mathcal{X}_G, ν) be a GTS and \mathcal{H} be a hereditary class on \mathcal{X}_G . A $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is said to be $w\nu\mathcal{H}$ -Lindelöf or $w\nu$ -Lindelöf respect to a hereditary class on \mathcal{X}_G if each ν -open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of Λ_ν has a countable subcollection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})) \in \mathcal{H}.$$

Proposition 4.1 A $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf if and only if every collection $\{\mathcal{F}_\gamma : \gamma \in \Omega\}$ of ν -closed sets of \mathcal{X}_G such that $(\bigcap_{\gamma \in \Omega} \mathcal{F}_\gamma) \cap \Lambda_\nu = \emptyset$ admits a countable sub-collection $\{\mathcal{F}_{\gamma_n} : n \in \mathbb{N}\}$ such that $i_\nu(\bigcap_{n \in \mathbb{N}} \mathcal{F}_{\gamma_n}) \cap \Lambda_\nu \in \mathcal{H}$.

Proof. Necessity. Let $\{\mathcal{F}_\gamma : \gamma \in \Omega\}$ be a collection of ν -closed sets of \mathcal{X}_G such that $(\bigcap_{\gamma \in \Omega} \mathcal{F}_\gamma) \cap \Lambda_\nu = \emptyset$. Then $\Lambda_\nu = \mathcal{X}_G \setminus (\bigcap_{\gamma \in \Omega} \mathcal{F}_\gamma) = \bigcup_{\gamma \in \Omega} (\mathcal{X}_G \setminus \mathcal{F}_\gamma)$, i.e., the collection $\{\mathcal{X}_G \setminus \mathcal{F}_\gamma : \gamma \in \Omega\}$ is a ν -open cover of Λ_ν .

Since \mathcal{X}_G is $w\nu\mathcal{H}$ -Lindelöf, there is a countable sub-collection $\{\mathcal{X}_G \setminus \mathcal{F}_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{F}_{\gamma_n}))) \in \mathcal{H}.$$

Thus

$$\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{F}_{\gamma_n}))) = \Lambda_\nu \setminus (c_\nu(\mathcal{X}_G \setminus (\bigcap_{n \in \mathbb{N}} \mathcal{F}_{\gamma_n}))) = \Lambda_\nu \setminus (\mathcal{X}_G \setminus i_\nu(\bigcap_{n \in \mathbb{N}} \mathcal{F}_{\gamma_n})) \in \mathcal{H}.$$

It is obviously to show that;

$$\Lambda_\nu \cap i_\nu(\bigcap_{n \in \mathbb{N}} \mathcal{F}_{\gamma_n}) = \Lambda_\nu \setminus (\mathcal{X}_G \setminus i_\nu(\bigcap_{n \in \mathbb{N}} \mathcal{F}_{\gamma_n})) \in \mathcal{H}.$$

Sufficiency, Suppose $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν , then $\Lambda_\nu = \bigcup \mathcal{U}_\gamma$ and $\{\mathcal{X}_G \setminus \mathcal{U}_\gamma : \gamma \in \Omega\}$ is a collection of ν -closed sets of \mathcal{X}_G . Thus $(\mathcal{X}_G \setminus \bigcup_{\gamma \in \Omega} \mathcal{U}_\gamma) \cap \Lambda_\nu = \emptyset$, i.e., $\bigcap_{\gamma \in \Omega} (\mathcal{X}_G \setminus \mathcal{U}_\gamma) \cap \Lambda_\nu = \emptyset$. By hypothesis, there is a countable sub-collection $\{\mathcal{X}_G \setminus \mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that $i_\nu(\bigcap_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{U}_{\gamma_n})) \cap \Lambda_\nu \in \mathcal{H}$. Since,

$$i_\nu(\bigcap_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{U}_{\gamma_n})) \cap \Lambda_\nu = \Lambda_\nu \setminus (\mathcal{X}_G \setminus (i_\nu(\bigcap_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{U}_{\gamma_n}))))$$

Then,

$$\Lambda_\nu \setminus (\mathcal{X}_G \setminus (i_\nu(\bigcap_{n \in \mathbb{N}} (\mathcal{X}_G \setminus \mathcal{U}_{\gamma_n})))) = \Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n}))) \in \mathcal{H}.$$

Which implies that a $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is a $w\nu\mathcal{H}$ -Lindelöf.

Proposition 4.2 Let (\mathcal{X}_G, ν) be a GTS with a hereditary class \mathcal{H} , then (\mathcal{X}_G, ν) is $w\nu$ -Lindelöf if and only if $(\mathcal{X}_G, \nu, \mathcal{H}_c)$ is $w\nu\mathcal{H}_c$ -Lindelöf $\mathcal{H}GTS$.

Proof. The necessity is obvious. Sufficiency, suppose $(\mathcal{X}_G, \nu, \mathcal{H}_c)$ is $w\nu\mathcal{H}_c$ -Lindelöf $\mathcal{H}GTS$. Let $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν . Then by hypothesis, there is a countable sub-collection $\{\mathcal{U}_n : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_n)) \in \mathcal{H}_c.$$

Assume, $\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_n)) = \{x_i : i \in \mathbb{N}\}$, pick out \mathcal{U}_{γ_i} such that $x_i \in \mathcal{U}_{\gamma_i}$ for each $i \in \mathbb{N}$. Thus,

$$\Lambda_\nu = (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_n)) \cup (c_\nu(\bigcup_{i \in \mathbb{N}} \mathcal{U}_{\gamma_i})).$$

This implies that a \mathcal{X}_G is $w\nu$ -Lindelöf GTS .

By proposition above, it is clear that a $GTS (\mathcal{X}_G, \nu)$ is $w\nu$ -Lindelöf if and only if $(\mathcal{X}_G, \nu, \{\emptyset\})$ is $w\nu\{\emptyset\}$ -Lindelöf $\mathcal{H}GTS$.

Proposition 4.3 A $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is $\nu\mathcal{H}$ -Lindelöf then it is $w\nu\mathcal{H}$ -Lindelöf $\mathcal{H}GTS$.

Proof. Let $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν . Since a $\mathcal{H}GTS (\mathcal{X}_G, \nu, \mathcal{H})$ is $\nu\mathcal{H}$ -Lindelöf then there is a countable sub-collection $\{\mathcal{U}_n : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} (\mathcal{U}_n) \in \mathcal{H}.$$

But, $\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_n)) \subseteq \Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} (\mathcal{U}_n)$. So, $\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_n)) \in \mathcal{H}$, and the proof is completed.

The converse of above proposition is not true as the following example shows:

Example 4.1 Let \mathbb{R} be the real set, choose $a \in \mathbb{R}$, $\beta = \{\{a, x\} : x \in \mathbb{R}, a \neq x\}$ and a hereditary class $\mathcal{H} = \{\emptyset, \mathbb{R}\}$. If the $GT \nu(\beta)$ generated on \mathbb{R} by the ν -base β . Then $(\mathbb{R}, \nu(\beta), \mathcal{H})$ is a $\mathcal{H}GTS$, and for each non-empty ν -open set \mathcal{U} of \mathbb{R} , we have $c_\nu \mathcal{U} = \mathbb{R}$. So, each ν -open cover $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ of \mathbb{R} , there is a countable sub-collection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\mathbb{R} \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})) \in \mathcal{H}.$$

Thus $\mathcal{H}GTS (\mathbb{R}, \nu(\beta), \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf. Now, $U = \{\{0, x\} : x \in \mathbb{R}\}$ is a ν -open cover of \mathbb{R} and let $\{\{0, x_n\} : n \in \mathbb{N}\}$ be a countable sub-collection of U , it follows that $\mathbb{R} \setminus (\bigcup_{n \in \mathbb{N}} \{0, x_n\}) \notin \mathcal{H}$. Therefore, a $\mathcal{H}GTS (\mathbb{R}, \nu(\beta), \mathcal{H})$ is not $\nu\mathcal{H}$ -Lindelöf.

In the following proposition, we will show that a ν -Lindelöf with respect to hereditary classes is special case of $w\nu$ -Lindelöf \mathcal{GTS} .

Proposition 4.4 Let a $\mathcal{GTS} (\mathcal{X}_G, \nu)$.

- (i) (\mathcal{X}_G, ν) is $w\nu$ -Lindelöf if and only if $(\mathcal{X}_G, \nu, \mathcal{H}_n)$ is $\nu\mathcal{H}_n$ -Lindelöf.
- (ii) (\mathcal{X}_G, ν) is $w\nu$ -Lindelöf if and only if $(\mathcal{X}_G, \nu, \mathcal{H})$ is $\nu\mathcal{H}$ -Lindelöf with a ν -codense hereditary class \mathcal{H} .

Proof. (i) (\Rightarrow) Let (\mathcal{X}_G, ν) is $w\nu$ -Lindelöf and $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν . Thus there is a countable sub-collection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that $\Lambda_\nu = c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})$. Which implies that,

$$\Lambda_\nu \setminus c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}) = \emptyset.$$

So, $i_\nu(\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}) = \emptyset$, and hence $\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n} \in \mathcal{H}_n$. Which proves that a $\mathcal{HGTS} (\mathcal{X}_G, \nu, \mathcal{H}_n)$ is $\nu\mathcal{H}_n$ -Lindelöf.

(\Leftarrow) Suppose $(\mathcal{X}_G, \nu, \mathcal{H}_n)$ is $\nu\mathcal{H}_n$ -Lindelöf and let $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν . Thus there is a countable sub-collection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n}) \in \mathcal{H}_n.$$

This implies that $i_\nu(\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}) = \emptyset$, then $\Lambda_\nu \setminus c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}) = \emptyset$. Thus the proof is completed.

(ii) (\Rightarrow) From (i) \mathcal{H} is a ν -codense hereditary class.

(\Leftarrow) Let $(\mathcal{X}_G, \nu, \mathcal{H})$ is $\nu\mathcal{H}$ -Lindelöf and $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a ν -open cover of Λ_ν . Thus there is a countable sub-collection $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\}$ such that

$$\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n}) \in \mathcal{H}.$$

Since a hereditary class \mathcal{H} is ν -codense on \mathcal{X}_G , $\Lambda_\nu \setminus \bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})$ has empty ν -interior, then $\Lambda_\nu = c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})$. Which implies that (\mathcal{X}_G, ν) is $w\nu$ -Lindelöf \mathcal{GTS} .

Proposition 4.5 Let $(\mathcal{X}_G, \nu, \mathcal{H})$ be a $w\nu\mathcal{H}$ -Lindelöf \mathcal{HGTS} and \mathcal{A} be a ν -clopen subset of \mathcal{X}_G . Then $(\mathcal{A}, \nu(\mathcal{A}), \mathcal{H}_\mathcal{A})$ is $w\nu(\mathcal{A})\mathcal{H}_\mathcal{A}$ -Lindelöf.

Proof. Let \mathcal{A} be a ν -clopen subset of \mathcal{X}_G . If $\{\mathcal{O}_\gamma = \mathcal{U}_\gamma \cap \mathcal{A} : \mathcal{U}_\gamma \in \nu \text{ for each } \gamma \in \Omega\}$ be a $\nu(\mathcal{A})$ -open cover of $\mathcal{A} \cap \Lambda_\nu = \mathcal{A}$. Hence the family $\{\mathcal{U}_\gamma : \gamma \in \Omega\} \cup (\mathcal{X}_G \setminus \mathcal{A})$ forms a ν -open cover of Λ_ν . Since \mathcal{X}_G is an $w\nu\mathcal{H}$ -Lindelöf space, then there is a countable subfamily $\{\mathcal{U}_{\gamma_n} : n \in \mathbb{N}\} \cup (\mathcal{X}_G \setminus \mathcal{A})$ such that

$$\Lambda_\nu \setminus [c_\nu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})) \cup (\mathcal{X}_G \setminus \mathcal{A})] = H \in \mathcal{H}.$$

Now,

$$\begin{aligned} \mathcal{A} \cap H &= \mathcal{A} \cap (\Lambda_\nu \setminus [c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}) \cup (\mathcal{X}_G \setminus \mathcal{A})]) \\ &= \mathcal{A} \cap (\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})) \cap (\Lambda_\nu \setminus (\mathcal{X}_G \setminus \mathcal{A}))) \\ &= \mathcal{A} \cap (\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})) \cap (\Lambda_\nu \cap \mathcal{A})) \\ &= \mathcal{A} \cap (\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n})) \cap \mathcal{A}) = \mathcal{A} \cap (\Lambda_\nu \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} \mathcal{U}_{\gamma_n}))) \end{aligned}$$

$$\begin{aligned}
 &= \mathcal{A} \cap (\Lambda_\nu \cap (\mathcal{X}_G \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}))) = (\mathcal{A} \cap \Lambda_\nu) \cap (\mathcal{X}_G \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}))) \\
 &= \mathcal{A} \cap (\mathcal{X}_G \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}))) = \mathcal{A} \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n})) = \mathcal{A} \setminus (\mathcal{A} \cap (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n})))
 \end{aligned}$$

However, $c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}) \cap \mathcal{A} = c_{\nu(\mathcal{A})}(\bigcup_{n \in \mathbb{N}} \mathcal{V}_{\gamma_n})$. Therefore, we have $\mathcal{A} \cap H = \mathcal{A} \setminus (c_{\nu(\mathcal{A})}(\bigcup_{n \in \mathbb{N}} \mathcal{V}_{\gamma_n})) \in \mathcal{H}_A$.

This proves that a subset \mathcal{A} is an $w\nu(\mathcal{A})\mathcal{H}_A$ -Lindelöf.

Proposition 4.6 Let (\mathcal{X}_G, ν) be a ν -space and \mathcal{H} be a hereditary class on \mathcal{X}_G . If $(\mathcal{X}_G, \nu^*, \mathcal{H})$ is $w\nu^*\mathcal{H}$ -Lindelöf then $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf $\mathcal{H}GTS$.

Proof. The proof follows from Theorem 3.4 and Proposition 4.5, since every ν -closed (ν -open) set is ν^* -closed (ν^* -open) set. Thus every ν -closed set is ν^* -closed set.

In the following example, we show that the converse of Proposition 4.6 is not true.

Example 4.2 Let \mathbb{R} be the set of real numbers and $\nu = \{U \subset \mathbb{R} : U \text{ is uncountable}\} \cup \{\emptyset\}$ be a $\mathcal{G}T$ on \mathbb{R} . Suppose $\mathcal{H} = \{\mathbb{R} \setminus U : U \in \nu\}$ be a hereditary class on \mathbb{R} , observe that \mathcal{H} is not closed under countable union. A ν -space \mathbb{R} is $\nu\mathcal{H}$ -Lindelöf (see Qahis et al., 2016) so it is $w\nu\mathcal{H}$ -Lindelöf. Now, for each $x \in \mathbb{R}$, $\{x\}$ is ν^* -open. Further, $\{x\}$ is ν -closed set so it is ν^* -closed, and hence $c_{\nu^*}(\{x\}) = \{x\}$. Furthermore, $\{\{x\} : x \in \mathbb{R}\}$ is a ν^* -open cover of a ν^* -space \mathbb{R} . Assume that there is a countable collection $\{\{x_i\} : i \in \mathbb{N}\}$ such that $\mathbb{R} \setminus (c_{\nu^*}(\bigcup_{i \in \mathbb{N}} \{x_i\})) \in \mathcal{H}$. And this is not possible. Therefore, a ν^* -space \mathbb{R} is not $w\nu^*\mathcal{H}$ -Lindelöf.

The converse of Proposition 4.6 will be hold if a hereditary class \mathcal{H} is closed under countable union as the following:

Proposition 4.7 Let (\mathcal{X}_G, ν) be a ν -space and a hereditary class \mathcal{H} on \mathcal{X}_G is closed under countable union, then $(\mathcal{X}_G, \nu^*, \mathcal{H})$ is $w\nu^*\mathcal{H}$ -Lindelöf if and only if $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf $\mathcal{H}GTS$.

Proof. The necessity is obviously by Proposition 4.6. For sufficiency, suppose a $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf and \mathcal{H} is closed under countable union. Given $\{U_\gamma^* : \gamma \in \Omega\}$ a ν^* -open cover of \mathcal{X}_G , then for each $x \in \mathcal{X}_G$, $x \in U_{\gamma_x}^*$ for some $\gamma_x \in \Omega$. By Theorem 3.5, there is $U_{\gamma_x} \in \nu_x$ and $H_{\gamma_x} \in \mathcal{H}$ such that $x \in U_{\gamma_x} \setminus H_{\gamma_x} \subset U_{\gamma_x}^*$. Since the collection $\{U_{\gamma_x} : \gamma \in \Omega\}$ is a ν -open cover of \mathcal{X}_G , then there exists a countable sub-collection $\{U_{\gamma_{x_n}} : n \in \mathbb{N}\}$ such that

$$\mathcal{X}_G \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} U_{\gamma_{x_n}})) = H \in \mathcal{H}.$$

Since \mathcal{H} is closed under countable union, then $\bigcup \{H_{\gamma_{x_n}} : n \in \mathbb{N}\} \in \mathcal{H}$. Then, $H \cup [\bigcup \{H_{\gamma_{x_n}} : n \in \mathbb{N}\}] \in \mathcal{H}$. Note that $\mathcal{X}_G \setminus (c_{\nu^*}(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}^*)) \subset H \cup [\bigcup \{H_{\gamma_{x_n}} : n \in \mathbb{N}\}] \in \mathcal{H}$. So,

$$\mathcal{X}_G \setminus (c_{\nu^*}(\bigcup_{n \in \mathbb{N}} U_{\gamma_n}^*)) \in \mathcal{H}.$$

Therefore, $(\mathcal{X}_G, \nu^*, \mathcal{H})$ is $w\nu^*\mathcal{H}$ -Lindelöf $\mathcal{H}GTS$.

2.1. Function properties on $w\nu$ -Lindelöf with respect to a hereditary class \mathcal{H}

attention has been made to study properties of covering in topological

spaces was introduced by (Császár, 2002). Let ν and μ be generalized topologies on \mathcal{X}_G and \mathcal{Y}_G , respectively. Then a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) is called (ν, μ) -continuous iff $U \in \mu$ implies that $g^{-1}(U) \in \nu$.

Definition 4.2 Let \mathcal{A} be a subset of $\mathcal{GTS}(\mathcal{X}_G, \nu)$, then \mathcal{A} is called ν -preopen (resp. $\nu - \beta$ -open) (Csa 'sz'ar, 2005) if $\mathcal{A} \subseteq i_\nu c_\nu(\mathcal{A})$ (resp. $\mathcal{A} \subseteq c_\nu i_\nu c_\nu(\mathcal{A})$)

The complement of ν -preopen (resp. $\nu - \beta$ -open) is said to be ν -preclosed (resp. $\nu - \beta$ -closed), we denote by π the class of all ν -preopen sets in \mathcal{X}_G , by β the class of all $\nu - \beta$ -open sets in \mathcal{X}_G .

Definition 4.3 A function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ is called:

(1) almost (ν, μ) continuous (Min, 2009), if for each $t \in \mathcal{X}_G$ and each μ -open set \mathcal{U} containing $g(t)$, there is a ν -open set \mathcal{V} with $t \in \mathcal{V}$ such that $g(\mathcal{V}) \subseteq i_\mu c_\mu(\mathcal{U})$.

(2) almost (π, μ) continuous (resp. almost (β, μ) continuous) (Abuage, Kiliman, & Sarsak, 2017) if for each $t \in \mathcal{X}_G$ and each μ -regular open set \mathcal{U} in \mathcal{Y}_G containing $g(t)$, there is a ν -preopen (resp. $\nu - \beta$ -open) set \mathcal{V} containing t such that $g(\mathcal{V}) \subseteq \mathcal{U}$.

Remark 4.1 Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be a function between \mathcal{GTS} 's (\mathcal{X}_G, ν) and (\mathcal{Y}_G, μ) . Then we have the following implications but the reverse relations may not be true in general:

almost (ν, μ) continuous \Rightarrow almost (π, μ) continuous \Rightarrow almost (β, μ) continuous

Example 4.3 Let $\mathcal{X}_G = \{a, b, c\}$ and $\nu = \{\emptyset, \{a, b\}\}$ be a \mathcal{GT} on \mathcal{X}_G . Then $\pi = \nu \cup \{\{a\}, \{b\}\}$. Define a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{X}_G, \nu)$ as follows: $g(a) = a$, $g(b) = g(c) = c$. Then g is almost (π, μ) continuous function but not almost (ν, μ) continuous.

Example 4.4 Let $\mathcal{X}_G = \{a, b, c\}$ and $\nu = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$ be a \mathcal{GT} on \mathcal{X}_G . Then $\pi = \nu$ and $\beta = \nu \cup \{\{a, b\}, \{a, c\}, \mathcal{X}_G\}$. Consider a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{X}_G, \nu)$ defined by $g(a) = g(b) = b$, $g(c) = a$. Then g is almost (β, μ) continuous function without begin almost (π, μ) continuous.

Proposition 4.8 Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) continuous surjection from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) , and \mathcal{H} be a hereditary class on \mathcal{X}_G . If \mathcal{X}_G is $w\nu\mathcal{H}$ -Lindelöf then \mathcal{Y}_G so is.

Proof. Let $\{\mathcal{U}_\gamma : \gamma \in \Omega\}$ be a μ -open cover of \mathcal{Y}_G . Since g is almost (ν, μ) -continuous, that means $g^{-1}(i_\mu c_\mu(\mathcal{U}_\gamma))$ is a ν -open in \mathcal{X}_G . Thus $\{g^{-1}(i_\mu c_\mu(\mathcal{U}_\gamma)) : \gamma \in \Omega\}$ is a ν -open cover of \mathcal{X}_G , then there is a countable sub-collection $\{g^{-1}(i_\mu c_\mu(\mathcal{U}_{\gamma_n})) : n \in \mathbb{N}\}$ such that

$$\mathcal{X}_G \setminus c_\nu(\bigcup_{n \in \mathbb{N}} g^{-1}(i_\mu c_\mu(\mathcal{U}_{\gamma_n}))) \in \mathcal{H}.$$

Now;

$$\mathcal{X}_G \setminus (c_\nu(g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})))))) \subseteq \mathcal{X}_G \setminus (c_\nu(g^{-1}(\bigcup_{n \in \mathbb{N}} (c_\mu \mathcal{U}_{\gamma_n}))))$$

$$= \mathcal{X}_G \setminus (c_\nu(\bigcup_{n \in \mathbb{N}} g^{-1}(c_\mu(\mathcal{U}_{\gamma_n}))))$$

$$\subseteq \mathcal{X}_G \setminus c_\nu(\bigcup_{n \in \mathbb{N}} g^{-1}(i_\mu c_\mu(\mathcal{U}_{\gamma_n}))) \in \mathcal{H}.$$

By Lemma 3.3 $c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n}))$ is μ -regular closed in \mathcal{Y}_G and g is an almost (ν, μ) continuous, we have $g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})))$ is ν -closed in \mathcal{X}_G . Thus

$$\mathcal{X}_G \setminus (g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})))) = \mathcal{X}_G \setminus (c_\nu(g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{\gamma_n})))))) \in \mathcal{H}.$$

By Lemma 3.6,

$$\begin{aligned} g(\mathcal{X}_G \setminus (g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{r_n})))))) &= g(\mathcal{X}_G \setminus (g(g^{-1}(c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{r_n})))))) \\ &= \mathcal{Y}_G \setminus (c_\mu(\bigcup_{n \in \mathbb{N}} (\mathcal{U}_{r_n}))) \in g(\mathcal{H}). \end{aligned}$$

Which proves that a $\mathcal{H}GTS \mathcal{Y}_G$ is $w_\mu g(\mathcal{H})$ -Lindelöf.

If $\mathcal{H} = \{\emptyset\}$ in the above Proposition, then we have the following result:

Corollary 4.1 (Abuage & Kiliçman, 2017) *Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -continuous surjection from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) , if a ν -space \mathcal{X}_G is w_ν -Lindelöf then a μ -space \mathcal{Y}_G so is.*

Theorem 4.4 (Ekici, 2012) *Let (\mathcal{X}_G, ν) be a GTS where $c_\nu(\emptyset) = \emptyset$. Then (\mathcal{X}_G, ν) is a submaximal and extremally disconnected GTS if any subset of (\mathcal{X}_G, ν) is $\nu - \beta$ -open if and only if it is ν -open.*

Obviously, if $\mathcal{X}_G \in \nu$ in $GTS(\mathcal{X}_G, \nu)$ then $c_\nu(\emptyset) = \emptyset$, so the following proposition proves immediately by Theorem 4.4.

Proposition 4.9 *Let (\mathcal{X}_G, ν) be a submaximal and ν -extremally disconnected ν -space. Then a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ is an almost (ν, μ) -continuous if and only if it is almost (β, μ) -continuous.*

Corollary 4.2 *Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (β, μ) -continuous surjection, and \mathcal{H} be a hereditary class on \mathcal{X}_G . If \mathcal{X}_G is submaximal, ν -extremally disconnected and $w_\nu \mathcal{H}$ -Lindelöf ν -space. Then \mathcal{Y}_G is $w_\mu g(\mathcal{H})$ -Lindelöf.*

Proof. The proof follows directly from Proposition 4.8 and Proposition 4.9.

Lemma 4.5 *Let a (\mathcal{X}_G, ν) be a submaximal QTS then every ν -preopen set is ν -open.*

Proof. Assume, a subset \mathcal{V} is a ν -preopen, then by Proposition 3.11 (Sarsak, 2013) $\mathcal{V} = \mathcal{U} \cap \mathcal{A}$ for some ν -regular open set \mathcal{U} and ν -dense set \mathcal{A} of \mathcal{X}_G . Since (\mathcal{X}_G, ν) is submaximal QTS , so \mathcal{A} is ν -open set of \mathcal{X}_G and thus \mathcal{V} is ν -open set of \mathcal{X}_G .

Next proposition proves directly, by Lemma 4.5, so the proof omitted.

Proposition 4.10 *Let (\mathcal{X}_G, ν) be a submaximal QTS then a function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ is an almost (ν, μ) -continuous if and only if it is almost (π, μ) -continuous.*

By Propositions 4.8 and Proposition 4.10 the following corollary concluded:

Corollary 4.3 *Let $g : (\mathcal{X}_G, \tau) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (π, μ) -continuous surjection, and \mathcal{H} be a hereditary class on \mathcal{X}_G . If a space \mathcal{X}_G is submaximal and weakly Lindelöf then \mathcal{Y}_G is $w_\mu g(\mathcal{H})$ -Lindelöf.*

Definition 4.6 (Al-Omari & Noiri, 2012) *A function $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ is said to be*

(a) *almost (ν, μ) -open if $g(\mathcal{V}) \subseteq i_\mu c_\mu(g(\mathcal{V}))$ for each ν -open set \mathcal{V} in \mathcal{X}_G ,*

(b) *contra (ν, μ) -continuous if $g^{-1}(\mathcal{U})$ is ν -closed in \mathcal{X}_G for every μ -open set \mathcal{U} in \mathcal{Y}_G .*

(Al-Omari and Noiri, 2012), showed that if a function g from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) is an almost (ν, μ) -open and contra (ν, μ) continuous, then g is almost (ν, μ) continuous

Moreover, if g is a contra (ν, μ) continuous and a μ -space \mathcal{Y}_G is μ -extremely disconnected, then g is almost (ν, μ) continuous Proposition 4.8, we conclude the following corollaries:

Corollary 4.4 Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -open and contra (ν, μ) -continuous surjection from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) , with a hereditary class \mathcal{H} on \mathcal{X}_G . If \mathcal{X}_G is $w\nu\mathcal{H}$ -Lindelöf then \mathcal{Y}_G so is.

Corollary 4.5 Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be a contra (ν, μ) -continuous and a μ -space \mathcal{Y}_G is μ -extremely disconnected from a ν -space (\mathcal{X}_G, ν) into a μ -space (\mathcal{Y}_G, μ) , with a hereditary class on \mathcal{X}_G . If \mathcal{X}_G is $w\nu\mathcal{H}$ -Lindelöf then \mathcal{Y}_G so is.

Proposition 4.11 Let $g : (\mathcal{X}_G, \nu, \mathcal{H}) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -continuous surjection, if $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf and μ -space \mathcal{Y}_G is countable, then \mathcal{Y}_G is $w\mu$ -Lindelöf.

Proof. Suppose $(\mathcal{X}_G, \nu, \mathcal{H})$ be a $w\nu\mathcal{H}$ -Lindelöf and g be an almost (ν, μ) -continuous surjection, by Proposition 4.8 \mathcal{Y}_G is $n\mu g(\mathcal{H})$ -Lindelöf. Since \mathcal{Y}_G is countable so a hereditary class \mathcal{H} is countable, by applying Proposition 4.2 the proof is completed.

Proposition 4.12 Let $g : (\mathcal{X}_G, \nu, \mathcal{H}) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -continuous surjection, if $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf and μ -space \mathcal{Y}_G is countable, then $(\mathcal{Y}_G, \mu, g(\mathcal{H}_n))$ is $\mu g(\mathcal{H}_n)$ -Lindelöf.

Proof. The proof follows immediately by Propositions 4.8, Proposition 4.2 and Proposition 4.4 (i).

Proposition 4.13 Let $g : (\mathcal{X}_G, \nu, \mathcal{H}) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -continuous surjection, if $(\mathcal{X}_G, \nu, \mathcal{H})$ is $w\nu\mathcal{H}$ -Lindelöf and μ -space \mathcal{Y}_G is countable, then $(\mathcal{Y}_G, \mu, g(\mathcal{H}))$ is $\mu g(\mathcal{H})$ -Lindelöf with a μ -codense hereditary class \mathcal{H} .

Proof. The proof follows immediately by Propositions 4.8, Propositions 4.2 and Propositions 4.4 (ii).

Proposition 4.14 Let $g : (\mathcal{X}_G, \nu) \rightarrow (\mathcal{Y}_G, \mu)$ be an almost (ν, μ) -continuous surjection, \mathcal{H} be a hereditary class on \mathcal{X}_G which is closed under countable union, if $(\mathcal{X}_G, \nu^*, \mathcal{H})$ is $n\nu^*\mathcal{H}$ -Lindelöf then $(\mathcal{Y}_G, \mu^*, g(\mathcal{H}))$ is $n\mu^*g(\mathcal{H})$ -Lindelöf.

Proof. By Proposition 4.7, $(\mathcal{X}_G, \nu^*, \mathcal{H})$ is $n\nu\mathcal{H}$ -Lindelöf. Since g is almost (ν, μ) -continuous surjection, then $(\mathcal{Y}_G, \mu, g(\mathcal{H}))$ is $n\mu g(\mathcal{H})$ -Lindelöf. But \mathcal{H} is closed under countable union thus $g(\mathcal{H})$ is closed under countable union on \mathcal{Y}_G . Again by applying Proposition 4.7 the proof is completed.

2. Conclusion

Our work aims to define and study the notion of weakly ν -Lindelöf with respect to a hereditary class $\mathcal{H} : w\nu\mathcal{H}$ -Lindelöf, its properties and its relation to known concepts are showed.

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