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## APPLIED & INTERDISCIPLINARY MATHEMATICS | RESEARCH ARTICLE

# Periodic wave shock solutions of Burgers equations

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**Abstract:** In this paper, we present a new approach for the study of Burgers equations. Our purpose is to describe the asymptotic behavior of the solution in the Cauchy problem for viscous equation with small parameter  $\varepsilon$  and to discuss in particular the case of periodic wave shock. We show that the solution of this problem approaches the shock-type solution for the Cauchy problem of the inviscid Burgers equation. The results are formulated in classical mathematics and proved with infinitesimal techniques of nonstandard analysis.

**Subjects:** Advanced Mathematics; Applied Mathematics; Foundations & Theorems

**Keywords:** nonstandard analysis; boundary problem; viscous Burgers equation; inviscid Burgers equation; heat equation; periodic wave shock

**AMS subject classifications:** 35XX; 35LXX; 58J45

### 1. Introduction

One of the most important PDEs in the theory of nonlinear conservation laws is Burgers equation, which combines both nonlinear propagation effects and diffusive effect. This equation is the approximation for the one-dimensional propagation of weak shock waves in a fluid. It can also be used in the description of the variation in vehicle density in highway traffic. The equation is one of the fundamental model equations in fluid mechanics. Burgers introduced the equation to describe the behavior of shock waves, traffic flow and acoustic transmission. It is well known that Burgers' equation plays a relevant role in many different areas of mathematical physics, especially in Fluid Mechanics. Moreover, the simplicity of its formulation, in contrast with the Navier–Stokes system, makes of the Burgers' equation a suitable model equation to test different numerical algorithms and results of a varied nature (Samokhin, 2013, 2014). This equation is parabolic when the viscous term is included. It has the form (Hopf, 1950):

$$u_t + uu_x = \varepsilon u_{xx} \quad (1.1)$$

### ABOUT THE AUTHOR

Saida Bendaas is a researcher of "Faculty of Sciences, Department of Mathematics, University of Setif1, Setif, Algeria". She received her PhD degrees on nonstandard analysis from the University of Haute Alsace in Mulhouse, France. She published more than 08 papers in international refereed journals. Her research interests include new techniques of nonstandard analysis and its application on ordinary and partial differential equations, linear systems as well as quasi-linear first-order partial differential equations. She has served her institute in several administrative capacities.

### PUBLIC INTEREST STATEMENT

Many phenomena in the world have been described by nonlinear partial differential equations that can be solved by several methods. Burgers equation is one of the famous nonlinear equations. In the presented work, our general purpose is to describe the asymptotic behavior of solutions in boundary value problem with a small parameter and to discuss in particular the case of periodic wave shocks with new techniques of infinitesimal nonstandard analysis. Also, the author compared the present work via another published method and concluded that the present results are more accurate and efficient than standard methods studied.

If the viscous term is null the remaining equation is hyperbolic this is the inviscid Burgers equation

$$u_t + uu_x = 0 \quad (1.2)$$

If the viscous term is dropped from the Burgers equation, discontinuities may appear in finite time, even if the initial condition is smooth they give rise to the phenomenon of shock waves with important application in physics (Kida, 1979). This properties make Burgers equation a proper model for testing numerical algorithms in flows where severe gradients or shocks are anticipated (Samokhin, 2014; Sinal, 1992; Wazwaz, 2005). Recently, Shah and Singh (2015) used mixture of new integral transform and Homotopy Perturbation Method to find the solution of Burgers' equation arising in the longitudinal dispersion phenomenon in fluid flow through porous media. Discretization methods are well-known techniques for solving Burgers equation. Ascher and McLachlan established many methods as multisymplectic box scheme. Olayiwola, Gbolagade and Akinpelu (2010), also presented the modified variational iteration method for the numerical solution of generalized Burger's-Huxley equation. Pochekeeta and Popovych (2017) used enhanced classification techniques for the numerical solution of generalized Burgers equation. To establish numerical solutions for the modified Burgers equation, Ucar, Yagmurlu and Tasbozan (2017) were used in finite difference methods. For the boundary value problem, Sinal (1992) is interested to the initial condition case: null on  $R_-$  and Brownian on  $R_+$ . She, Aurell and Frisch (1992) with a numerical calculus particularly examine the initial conditions of Brownian fractional type.

This paper completes recent works on the study of boundary value problems of Burgers equations for different initial conditions (Bendaas, 2015a, 2015b, 2016). In the presented work, our general purpose is to describe the asymptotic behavior of solutions in boundary value problem with a small parameter  $\epsilon$  and to discuss in particular the case of periodic wave shocks with new techniques infinitesimal of nonstandard analysis.

We can conclude that the solution of the cauchy problem of inviscid equation is infinitely close to the solution of the cauchy problem of viscid equation as  $\epsilon$  is a parameter positif sufficiently small. We introduce the infinitesimal techniques to give a simple and direct formulation for the asymptotic behavior, unlike classical methods are based on search of solutions of the reduced problem to deduce existence and asymptotic behavior of the solutions as  $\epsilon$  tends to 0, the passage of the limit is very complicated, but in general the limit exist and it's a solution for the reduced problem (when  $\epsilon = 0$ ). For  $\epsilon$  small the solution  $u(x, t)$  is approximated by this limit (Kida, 1979). Other methods are based on a weak formulation of burgers equation seen as a conservation law satisfied on each of the computational domain called cell or finite volume. Stochastic particle method is so used for with different initial conditions. It is worth noting that our contribution is an elegant combination of infinitesimal techniques of nonstandard analysis and the Van Den Berg method (Bendaas, 2015a, 2016; Van Den Berg, 2006).

Historically, the subject nonstandard was developed by Lutz and Goze (1981). The idea to use nonstandard analysis in perturbation theory of differential equations goes back to the 1970s with the Reebian school. Relative to this use, among many works we refer the interested reader, for instance, to (Benoit, 1997; Diener & Diener, 1995; Lobry & Sari, 2008; Lutz & Sari, 1982; Nelson, 1977; Robinson, 1974) and the references therein. It has become today a well-established tool in asymptotic theory of differential equations. Among the famous discoveries of the nonstandard asymptotic theory of differential equations we can cite the canards which appear in slow-fast vector fields and are closely related to the stability loss delay phenomenon in dynamical bifurcations

The paper is organised as follows: Section 2 concerns the boundary value problem of inviscid burgers equation, we start with the Fitting discontinuous shock then we describe the asymptotic behavior of solutions for this problem in the periodic wave case. Section 3 concerns the boundary value problem of viscid burgers equation, we give first the main result about periodic wave shock case,

then we give some basic preliminaries results and deals with our main result proof, we present it in a nonstandard form.

## 2. Inviscid Burgers equation

We will focus first on equation (1.2). Specifically, we will deal with the initial value problem

$$\begin{cases} u_t + uu_x = 0, \forall x \in R, t > 0 \\ u(\xi, 0) = f(\xi). \quad t = 0 \end{cases} \quad (2.1)$$

As it has been suggested previously, although (2.1) seems to be a very innocent problem a priori it hides many unexpected phenomena. This problem does not admit the regular solutions but some weak solutions with certain regularity exist.

The Burgers equation on the whole line is known to possess traveling waves solutions. Using the characteristic method, the solution of the problem (2.1) may be given in a parametric form:

$$\begin{cases} u = f(\xi) \\ x = \xi + f(\xi)t. \end{cases} \quad (2.2)$$

and shocks must be fitted in such that:

$$U = \frac{1}{2}(u_1 + u_2) = \frac{1}{2}(f(\xi_1) + f(\xi_2)) \quad (2.3)$$

where  $f: R \rightarrow R$  is a standard continuous function.  $\xi_1$  and  $\xi_2$  are the values of  $\xi$  on the two sides of the shock (Bendaas, 1994).

According to (2.2), the solution at time  $t$  is obtained from the initial profile  $u = f(\xi)$  by translating each point a distance  $f(\xi)t$  to the right. The shock cuts out the part corresponding to  $\xi_2 \geq \xi \geq \xi_1$ , if the discontinuity line is a straight line chord between the points  $\xi = \xi_1$  and  $\xi = \xi_2$  on the curve  $f(\xi)$ . Moreover since areas are preserved under the mapping, the equal area property still holds. The chord on the  $f$  curve cuts off lobes of equal area.

The shock determination can then be describe entirely on the fixed  $f(\xi)$  curve by drawing all the chords with the equal area property can be written analytically as

$$\frac{1}{2} \{ (f(\xi_1) + f(\xi_2)) (\xi_1 - \xi_2) \} = \int_{\xi_2}^{\xi_1} f(\xi) d\xi \quad (2.4)$$

This is the differential equation for the shock line chord which verifies the entropic condition such as Bendaas (1994). Since the left-hand side is the area under the chord and the right-hand side is the area under the  $f$  curve. If the shock is at  $x = s(t)$  at time  $t$ , we also have

$$s(t) = \xi_1 + f(\xi_1)t \quad (2.5)$$

$$s(t) = \xi_2 + f(\xi_2)t \quad (2.6)$$

From (2.5) and (2.6), we have

$$t = \frac{\xi_1 - \xi_2}{f(\xi_1) - f(\xi_2)} \quad (2.7)$$

### 2.1. Single hump

To describe the solutions of the problem (2.1), we assume that the initial condition  $f$  verify the following assumptions:

$(H_1)$ :  $f$  is equal to a constant  $u_0$  outside the range  $0 < \xi < L$ .

$(H_2)$ :  $f(\xi) > u_0$  in the range.

The theorem below gives the asymptotic behavior to the solution behind the shock and at the shock.

**THEOREM 1** Suppose that  $(H_1)$  and  $(H_2)$  are satisfied, the solution of the problem (2.1) is given by (2.2) with  $0 < \xi < \xi_2$  and the asymptotic form is:

$$u \sim \frac{x}{t}, \text{ for: } u_0 t < x < u_0 t + \sqrt{2At} \tag{2.8}$$

*Proof* We consider the problem (2.1) and we suppose that  $(H_1)$  and  $(H_2)$  are satisfied. Equation (2.4) may be written as

$$\frac{1}{2} \{f(\xi_1) + f(\xi_2) - 2u_0\} (\xi_1 - \xi_2) = \int_{\xi_1}^L (f(\xi) - u_0) d\xi \tag{2.9}$$

As time goes on  $\xi_1$  increase and eventually exceed  $L$ . At this stage  $f(\xi_1) = u_0$  and the shock is moving into the constant region  $u = u_0$ . The function  $\xi_1(t)$  can then be eliminated for we have.

$$\frac{1}{2} (f(\xi_2) - u_0) (\xi_1 - \xi_2) = \int_{\xi_2}^L (f(\xi) - u_0) d\xi, t = \frac{\xi_1 - \xi_2}{f(\xi_2) - u_0} \tag{2.10}$$

There for

$$\frac{1}{2} (f(\xi_2) - u_0)^2 t = \int_{\xi_2}^L (f(\xi) - u_0) d\xi \tag{2.11}$$

At this stage, the shock position and the value of  $u$  just behind the shock are given by :

$$\begin{cases} u = f(\xi_2), \\ s(t) = \xi_2 + f(\xi_2)t. \end{cases} \tag{2.12}$$

where  $\xi_2$  satisfies (2.11). As  $t$  is infinitely large we have  $\xi_2$  infinitesimal and  $f(\xi_2)$  approach  $u_0$ , hence the equation for  $\xi_2(t)$  takes the limiting form

$$\frac{1}{2} (f(\xi_2) - u_0)^2 t \sim A \tag{2.13}$$

where

$$A = \int_0^L (f(\xi) - u_0) d\xi \tag{2.14}$$

is the area of the hump above the undisturbed value  $u_0$ . We have  $\xi_2$  infinitesimal and

$$f(\xi_2) \sim u_0 + \sqrt{2A/t} \tag{2.15}$$

Therefore, the asymptotic formulas for  $s(t)$  and  $u$  in (2.12) are

$$s(t) \sim u_0 t + \sqrt{2At}. \tag{2.16}$$

$$u - u_0 \sim \sqrt{2A/t} \tag{2.17}$$

at the shock. The shock curve is asymptotically parabolic. The solution behind the shock is given by (2.2) with  $0 < \xi < \xi_2$ . Since  $\xi_2$  is small enough as  $t$  is small enough, all the relevant values of  $\xi$  also small enough and the asymptotic form is:

$$u \sim \frac{x}{t}, \quad \text{for: } u_0 t < x < u_0 t + \sqrt{2At}$$

The asymptotic solution and the corresponding  $(x, t)$  diagram are shown in Figure 1.

Another interesting problem is that of an initial distribution

$$f(\xi) = u_0 + a \cdot \sin \frac{2\pi\xi}{\lambda}$$

In this case, the shock equations (2.4) simplify considerably for all times  $t$ . Consider one period  $0 < \xi < \lambda$  as in Figure 2.

Relations (2.4) becomes

$$(\xi_1 - \xi_2) \sin \frac{\pi}{\lambda}(\xi_1 + \xi_2) \cos \frac{\pi}{\lambda}(\xi_1 - \xi_2) = \frac{\lambda}{\pi} \sin \frac{\pi}{\lambda}(\xi_1 - \xi_2) \sin \frac{\pi}{\lambda}(\xi_1 + \xi_2) \quad (2.18)$$

and the relevant choice is the trivial one

$$\sin \frac{\pi}{\lambda}(\xi_1 + \xi_2) = 0 \quad \text{that is, } \xi_1 + \xi_2 = \lambda.$$

From the difference and sum of (2.5) and (2.6), we have

$$t = \frac{\xi_1 - \xi_2}{2a \sin \frac{\pi}{\lambda}(\xi_1 - \xi_2)}$$

$$s = u_0 t + \frac{\lambda}{2}$$

respectively. The discontinuity in  $u$  at the shock is

$$\begin{aligned} u_2 - u_1 &= a \sin \frac{2\pi\xi_1}{\lambda} - a \sin \frac{2\pi\xi_2}{\lambda} \\ &= 2a \sin \frac{\pi}{\lambda}(\xi_1 - \xi_2) \end{aligned}$$

If we introduce

$$\xi_1 - \xi_2 = \frac{\lambda\theta}{\pi}, \quad \xi_1 + \xi_2 = \lambda$$

We have

$$t = \frac{\lambda}{2\pi a} \cdot \frac{\theta}{\sin \theta}$$

**Figure 1. The asymptotique triangular wave.**

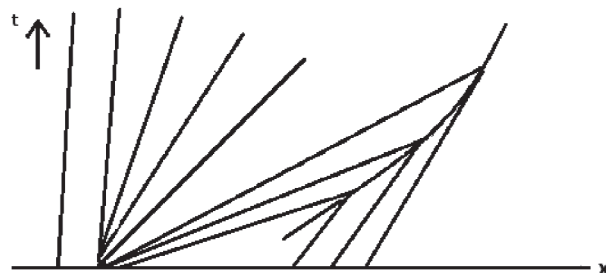
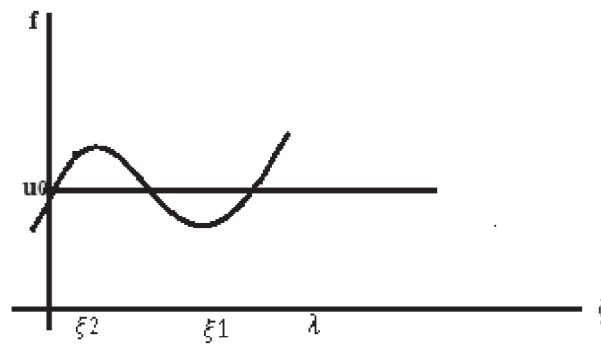


Figure 2. Shock construction for a periodic wave.



$$s = u_0 t + \frac{\lambda}{2} \tag{2.19}$$

$$\frac{u_2 - u_1}{u_0} = \frac{2a}{u_0} \sin \theta$$

The shock has constant velocity  $u_0$  and this result could have been deduced in advance from the symmetry of the problem. The shock starts with zero strength corresponding to  $\theta = 0$  at time  $t = \lambda/2\pi a$ . It reaches a maximum strength of  $2a/u_0$  for  $\theta = \pi/2$ ,  $t = \lambda/4a$  and decays ultimately with  $\theta$  approach  $\pi$ , when  $t$  is infinitely large

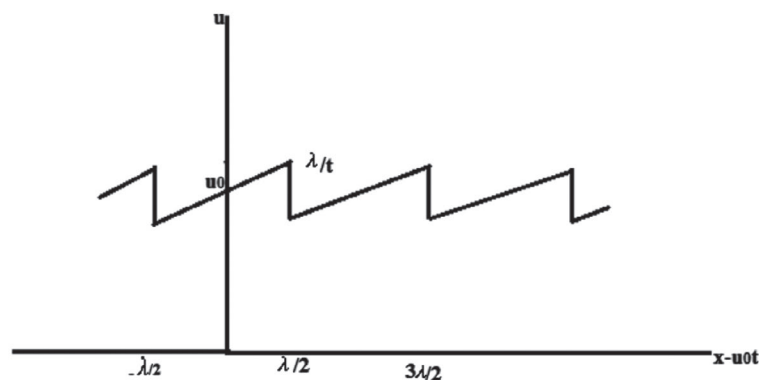
$$\frac{u_2 - u_1}{u_0} \sim \frac{\lambda}{u_0 t} \tag{2.20}$$

It is interesting that the final decay formula does not even depend explicitly on the amplitude  $a$ ; however, the condition for its application is  $t \gg \lambda/a$ . For any periodic sinusoidal  $f(\xi)$  or not  $\xi_1 - \xi_2 \rightarrow \lambda$  as  $t$  infinitely large; thence from (2.7).

$$\frac{u_2 - u_1}{u_0} = \frac{f(\xi_2) - f(\xi_1)}{u_0} \sim \frac{\lambda}{u_0 t} \tag{2.21}$$

Between successive shocks, the solution for  $u$  is linear in  $x$  with slope  $1/t$  as before, and the asymptotic form of the entire profile is the sawtooth shown in Figure 3.

Figure 3. Asymptotic form of a periodic wave.



### 3. Viscid Burgers equation

In this section, we shall present and prove our main result, we discuss the periodic wave case in the boundary value problem of viscid Burgers equation:

$$\begin{cases} u_t + uu_x = \varepsilon u_{xx}, \forall x \in R, t > 0 \\ u(\xi, 0) = f(\xi). \quad t = 0 \end{cases} \quad (3.1)$$

#### 3.1. Periodic wave. The main result

A periodic solution may be obtained by taking for  $\varphi$  a distribution of heat sources spaced a distance  $\lambda$  apart. Then

$$\varphi = \frac{1}{\sqrt{4\pi\varepsilon t}} \sum_{n=-\infty}^{\infty} \exp\left\{-\frac{(x-n\lambda)^2}{4\varepsilon t}\right\} \quad (3.2)$$

The main result of this work is:

**THEOREM 2. 1.** Assume that the initial data  $f$  have a profile shown in Figure 2 When  $\lambda^2/4\varepsilon t \gg 1$ , the problem (3.1) admit an unique solution for  $t > 0$  given by

$$u \sim \frac{x-m\lambda}{t}, \quad (m-1/2)\lambda < x < (m+1/2)\lambda$$

2. Such solution is the periodic wave chock, and for  $\varepsilon$  small enough, this solution is infinitely close to the solution of the inviscid problem (2.1) given in Section 2.

Before going further in the proof, we need the following proposition and lemma

#### 3.2. The Cole–Hopf transformation

Cole and Hopf noted the remarkable result (Hopf, 1950) that the viscid burgers equation (1.1) may be reduced to the linear heat equation

$$\varphi_t = \varepsilon \varphi_{xx} \quad (3.3)$$

by the nonlinear transformation:

$$u = -2\varepsilon[\log \varphi]_x \quad (3.4)$$

It is again convenient to do the trasformation in two steps. Firstly they are introduced

$$u = \psi_x$$

So that (1.1) may be integrated to

$$\psi_t + \frac{1}{2}\psi_x^2 = \varepsilon\psi_{xx}$$

Then we introduce

$$\psi = -2\varepsilon[\log \varphi]$$

To obtain (3.3).

The nonlinear transformation just eliminates the nonlinear term. The general solution of the heat equation (3.3) is well known and can be handled by a variety of methods. The basic problem considered in Section 2 is the initial value problem

$$u = f(\xi), \quad att = 0$$

this is transformed by (3.5) to the initial value problem

$$\varphi = \phi(x) = \exp \left\{ -\frac{1}{2\varepsilon} \int_0^x f(\eta) d\eta \right\}, t = 0 \tag{3.5}$$

For the heat equation the solution for  $\varphi$  is

$$\varphi = \frac{1}{\sqrt{4\pi\varepsilon t}} \int_{-\infty}^{+\infty} \phi(\eta) \exp \left\{ -\frac{(x-\eta)^2}{4\varepsilon t} \right\} d\eta \tag{3.6}$$

Through (3.4) the solution for  $u$  is

$$u(x, t) = \frac{\int_{-\infty}^{+\infty} \frac{x-\eta}{t} e^{-G/2\varepsilon} d\eta}{\int_{-\infty}^{+\infty} e^{-G/2\varepsilon} d\eta} \tag{3.7}$$

where

$$G(\eta, x, t) = \int_0^\eta f(v) dv + \frac{(x-\eta)^2}{2t} \tag{3.8}$$

### 3.3. The behavior of solutions as small enough

The behavior of the exact solution (3.7) is now considered as  $\varepsilon$  is small enough. For  $x, t$  and  $f(x)$  are held fixed as  $\varepsilon$  is small enough, the dominant contributions to the integrals in (3.7) come from the neighborhood of the stationary points of  $G$ . A stationary point is where

$$\frac{\partial G}{\partial \eta} = f(\eta) - \frac{x-\eta}{t} = 0$$

Let  $\eta = \xi(x, t)$  be such a point that is  $\xi(x, t)$  is defined as a solution of

$$f(\xi) - \frac{x-\xi}{t} = 0 \tag{3.9}$$

The contribution from the neighborhood of a stationary point  $\eta = \xi$  in an integral is given with the lemma 3.2

**LEMMA 3.1** (The Van Den Berg lemma (Van Den Berg, 2006)). Let  $G$  be a standard function defined and increasing on  $[0, +\infty[$  such that  $G(v) = av^r(1 + \delta)$  for  $v \approx 0$  and  $G(v) > m(v)^q$ . Let  $\varphi$  be an intern function defined on  $]0, +\infty[$  such that:  $\varphi(v) = bv^s(1 + \delta)$  for  $v \approx 0$  such that  $\forall d > 0, \exists$  standard  $k$  and  $c$  such that:  $|\varphi(v)| < k \exp(\cosh(v))$  for  $v > d$ . Then:

$$\int_0^\infty \varphi(v) \exp\left(-\frac{G(v)}{2\varepsilon}\right) dv = \frac{b\Gamma\left(\frac{s+1}{r}\right)}{ra^{\frac{s+1}{r}} \left(\frac{1}{2\varepsilon}\right)^{\frac{s+1}{r}}}, \tag{3.10}$$

where  $a$  and  $r$  are positifs standard,  $m$  and  $q$  are the both positifs.  $\delta$  is a positif real small enough.  $b$  and  $s$  are standard,  $b \neq 0$  and  $s > -1$ .

**LEMMA.3.2. The Nonstandard formula of the method of steepest descents.** Let  $\varepsilon$  be a positif rael small enough and let  $\varphi$  and  $G$  be two standard functions such that :

$G$ , is a  $C^2$  class function verified the lemma 3.1, and admits on the  $\xi$  point an unique absolute minimum ( $G'(\xi) = 0$  et  $G''(\xi) > 0$ ).

$\varphi(\xi) \neq 0$ , it is  $S$ -continuous on  $\xi$  and satisfied the conditions of the lemma 3.1 in the two sens. Then :



$\delta$  is a positif real small enough.

$$\int_{-\infty}^{+\infty} \varphi(\eta).e^{-G/2\epsilon} d\eta = \varphi(\xi) \frac{\sqrt{4\pi\epsilon}}{\sqrt{G''(\xi)}}.e^{-G/2\epsilon}(1 + \delta) \tag{3.11}$$

*Proof* Suppose first that there is only one stationary point  $\xi(x, t)$  wich satisfies (3.9) then

$$\int_{-\infty}^{+\infty} \frac{x - \eta}{t} e^{-G/2\epsilon} d\eta = \frac{x - \eta}{t} \frac{\sqrt{4\pi\epsilon}}{\sqrt{G''(\xi)}} e^{-G/2\epsilon}(1 + \delta) \tag{3.12}$$

$$\int_{-\infty}^{+\infty} e^{-G/2\epsilon} d\eta = \frac{\sqrt{4\pi\epsilon}}{\sqrt{G''(\xi)}} e^{-G/2\epsilon}(1 + \delta) \tag{3.13}$$

and in (3.7) we have

$$u \simeq \frac{x - \xi}{t} \tag{3.14}$$

where  $\xi(x, t)$  is difinied by (3.9).It is exactly the solution of (2.1) which was discussed in section 2. The stationary point  $\xi(x, t)$  becomes the characteristic variable

*Proof* To obtain a periodic wave for  $u$ , we choose for  $\varphi$  a distribution of heat sources spaced a distance  $\lambda$  given by (3.14). Then the corresponding solution for  $u$  is

$$u = -2\epsilon \frac{\varphi_x}{\varphi} = \frac{\sum_{n=-\infty}^{\infty} \left( \frac{x - n\lambda}{t} \right) \exp \left\{ -\frac{(x - n\lambda)^2}{4\epsilon t} \right\}}{\sum_{n=-\infty}^{\infty} \exp \left\{ -\frac{(x - n\lambda)^2}{4\epsilon t} \right\}} \tag{3.15}$$

For  $\lambda^2/4\epsilon t \gg 1$ , this implies that  $\sqrt{\epsilon t} \ll \lambda/2$ , and  $\sum_{n=-\infty}^{\infty} = 2 \sum_{n=0}^{\infty}$ , then for  $n = m$  we have

$$\frac{(x - n\lambda)^2}{4\epsilon t} = \frac{(x - m\lambda)^2}{\epsilon t} \ll 1$$

which gives  $|x - m\lambda| \ll \sqrt{\epsilon t}$ . and the exponential with the minimum value of  $(x - n\lambda)^2/4\epsilon t$  will dominate over all the others. Therefore, the term which will dominate for

$$(m - 1/2)\lambda < x < (m + 1/2)\lambda$$

and (3.15) is approximately

$$u \sim \frac{x - m\lambda}{t}, \text{ for } (m - 1/2)\lambda < x < (m + 1/2)\lambda$$

This is a sawtooth wave with a periodic set of shocks a distance  $\lambda$  apart, and  $u$  jumps from  $-\lambda/2t$  to  $\lambda/2t$  at each shock. The result agrees with the inviscid solution given by (2.20).

### 3.4. Conclusions

This paper completes recent works on the study of boundary value problems of **Burgers equations** for different initial conditions (Bendaas, 2015a, b, 2016). Our general purpose is to describe the asymptotic behavior of solutions in boundary value problem with a small parameter  $\epsilon$  and to discuss in particular the case of periodic wave shocks. The originality of this work consists in introducing new infinitesimal techniques of nonstandard analysis.

We can conclude that the solution of the cauchy problem of inviscid equation is infinitely close to the solution of the cauchy problem of viscid equation as  $\epsilon$  is a parameter positif sufficiently small.

These techniques gives us a simple and direct formulation for the asymptotic behavior. It is worth noting that our contribution is an elegant combination of infinitesimal techniques of nonstandard analysis and the Van Den Berg method (Bendaas, 2015a, 2016; Van Den Berg, 2006).

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