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STATISTICS | RESEARCH ARTICLE

An improved generalized class of estimators for population variance using auxiliary variables

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Abstract: This paper proposed an improved generalized class of estimator for estimating population variance using auxiliary variables based on simple random sampling without replacement. The expression of mean square error of the proposed estimator is obtained up to the first order of approximation. We have derived the conditions for the parameters under which the proposed estimator performs better compared to the usual estimator and other existing estimators. An empirical study and simulation study are also carried out with the support of theoretical results.

Subjects: Science; Mathematics & Statistics; Statistics & Probability

Keywords: population variance; auxiliary information; simple random sampling; mean square error; bias; simulation

1. Introduction

Through sampling literature it is well established that the use of Auxiliary information in Sampling Survey at the estimation stage improve the efficiency of the estimators of the population parameters. viz. population mean, variance, median, population correlation coefficient, etc. Ratio, regression and product method of estimation are good examples in this context. Several author including Ahmed, Raman, and Hossain (2000), Gupta and Shabbir (2008), Singh and Solanki (2009–2010, 2013), Yadav and Kadilar (2013), Solanki and Singh (2013) proposed estimators of population variance using information of auxiliary variable under different sampling schemes. Asghar, Sanaullah, and Hanif (2014) proposed generalized exponential-type estimators using information on auxiliary variable for estimating the population variance S_y^2 . Singh, Chauhan, Sawan, and Smarandache (2009)

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PUBLIC INTEREST STATEMENT

Variance estimation plays an important role in survey sampling. It is used for two purposes. One is the analytic purpose such as for constructing confidence intervals or performing hypothesis testing and another one is for descriptive purpose such as to evaluate the efficiency of the survey designs or to provide estimates for planning surveys. Several authors have paid their attention towards formation of estimators for the estimation of population variance using auxiliary information. Motivated by their works, we have proposed an improved generalized class of estimators for estimation of population variance using auxiliary information. The expression of mean square error is derived up to the first order of approximation. We have compared our proposed estimator with the other existing estimators. The efficiencies of the estimators are validated using real population data-sets and simulation study.

and Adichwal, Sharma, and Singh (2015) proposed estimator of population variance using two auxiliary variables and suggested to use two auxiliary variables, if they are made available. In this paper, we have also proposed an improved estimator for the population variance S_y^2 based on simple random sampling without replacement utilizing information of two auxiliary variables X and Z .

Let us consider a simple random sample (SRS) of size n is drawn from the given population of N units. Let the value of the study variable Y and the auxiliary variables X and Z for the i th units ($i = 1, 2, 3, \dots, N$) of the population be denoted by Y_i, X_i and Z_i for the i th unit in the sample ($i = 1, 2, 3, \dots, n$) by y_i, x_i and z_i , respectively. From the sample observation we have

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i, \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i, \bar{z} = \frac{1}{n} \sum_{i=1}^n z_i, s_y^2 = \frac{1}{n-1} \sum_{i=1}^n (y_i - \bar{y})^2, s_x^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \text{ and } s_z^2 = \frac{1}{n-1} \sum_{i=1}^n (z_i - \bar{z})^2$$

Let us define

$$\epsilon_y = \frac{s_y^2 - S_y^2}{S_y^2}, \epsilon_x = \frac{s_x^2 - S_x^2}{S_x^2}, \epsilon_z = \frac{s_z^2 - S_z^2}{S_z^2}$$

such that $E(\epsilon_x) = E(\epsilon_y) = E(\epsilon_z) = 0$

$$E(\epsilon_x^2) = \frac{1}{n}(\partial_{040} - 1), E(\epsilon_y^2) = \frac{1}{n}(\partial_{400} - 1), E(\epsilon_z^2) = \frac{1}{n}(\partial_{004} - 1)$$

$$E(\epsilon_x \epsilon_y) = \frac{1}{n}(\partial_{220} - 1), E(\epsilon_y \epsilon_z) = \frac{1}{n}(\partial_{202} - 1), E(\epsilon_z \epsilon_x) = \frac{1}{n}(\partial_{022} - 1)$$

where, $\partial_{pqr} = \frac{\mu_{pqr}}{(\mu_{200}^{p/2} \mu_{020}^{q/2} \mu_{002}^{r/2})}$, $\mu_{pqr} = \frac{1}{N} \sum_{i=1}^N (y_i - \bar{Y})^p (x_i - \bar{X})^q (z_i - \bar{Z})^r$; p, q, r being the non-negative integers.

2. Estimators in literature

In order to estimate population variance of the study variable Y , using the information of two auxiliary variables X and Z , Singh et al. (2009) proposed a general class of exponential estimator given as

$$t_1 = s_y^2 \left[k \exp \left\{ \frac{S_x^2 - S_x^2}{S_x^2 + S_x^2} \right\} + (1 - k) \exp \left\{ \frac{S_z^2 - S_z^2}{S_z^2 + S_z^2} \right\} \right] \tag{2.1}$$

Adichwal et al. (2015) proposed the following two generalized class of estimators for estimation of population variance using two auxiliary variables as

$$t_N = w s_y^2 + (1 - w) s_y^2 \left(\frac{S_x^2}{S_x^2} \right) \exp \left[\alpha \frac{S_z^2 - S_z^2}{S_z^2 + S_z^2} \right] \tag{2.2}$$

$$t_M = \delta s_y^2 \left(\frac{S_x^2}{S_x^2} \right)^\beta \exp \left[\gamma \frac{S_z^2 - S_z^2}{S_z^2 + S_z^2} \right] + (1 - \delta) s_y^2 \left(\frac{S_x^2}{S_x^2} \right)^\beta \exp \left[\gamma \frac{S_z^2 - S_z^2}{S_z^2 + S_z^2} \right] \tag{2.3}$$

The mean square error (MSE) expressions of the estimators t_1, t_N and t_M are, respectively, given by

$$\text{MSE}(t_1) = \frac{S_y^4}{n} \left[(\partial_{400} - 1) + \frac{k_0^2}{4} (\partial_{040} - 1) + \frac{(1 - k_0)^2}{4} (\partial_{004} - 1) - k_0 (\partial_{220} - 1) + (1 - k_0) (\partial_{202} - 1) - \frac{k_0(1 - k_0)}{2} (\partial_{022} - 1) \right] \tag{2.4}$$

$$\text{where, } k = \frac{\{\partial_{004} + 2(\partial_{220} + \partial_{202}) + \partial_{022} - 6\}}{(\partial_{040} + \partial_{004} + 2\partial_{022} - 4)} = k_0$$

$$\text{MSE}(t_N) = \frac{1}{n} S_y^4 \left[\frac{\alpha^2}{4} (w_0 - 1)^2 g^2 (\partial_{004} - 1) + (\partial_{400} - 1) + (w_0 - 1)^2 g^2 (\partial_{040} - 1) + 2g(w_0 - 1)(\partial_{220} - 1) \right. \\ \left. + \alpha g(w_0 - 1)(\partial_{202} - 1) + \alpha g^2 (w_0 - 1)^2 (\partial_{022} - 1) \right] \tag{2.5}$$

$$\text{where, } w_0 = \left[1 - \frac{2g(\partial_{220} - 1) + \alpha g(\partial_{202} - 1)}{\frac{\alpha^2 g^2}{2} (\partial_{004} - 1) + 2g^2 (\partial_{040} - 1) + 2\alpha g^2 (\partial_{022} - 1)} \right]$$

$$\text{MSE}(t_M) = \frac{S_y^4}{n} \left[\beta^2 (\delta_0 g - g - \delta_0)^2 (\partial_{040} - 1) + (\partial_{400} - 1) + \frac{\gamma^2}{4} (\delta_0 g - g - \delta_0)^2 (\partial_{004} - 1) + 2\beta (\delta_0 g - g - \delta_0)(\partial_{220} - 1) \right. \\ \left. + \gamma (\delta_0 g - g - \delta_0)(\partial_{202} - 1) + \gamma \beta (\delta_0 g - g - \delta_0)^2 (\partial_{022} - 1) \right] \tag{2.6}$$

$$\text{where, } \delta_0 = \left[\frac{4\beta^2 g(\partial_{040} - 1) + \gamma^2 g(\partial_{004} - 1) - 4\beta(\partial_{220} - 1) - 2\gamma(\partial_{202} - 1) + 4\gamma\beta g(\partial_{022} - 1)}{(g - 1) \{ 4\beta^2 (\partial_{040} - 1) + \gamma^2 (\partial_{004} - 1) + 4\gamma\beta (\partial_{022} - 1) \}} \right]$$

3. Proposed estimators

In this paper, we have proposed a generalized exponential-type estimator for population variance for the study variable Y based on simple random sampling without replacement using information of two auxiliary variable X and Z given by

$$t = \lambda S_y^2 \exp \left[\eta \left\{ \frac{S_x^2 - s_x^2}{S_x^2 + (a - 1)s_x^2} \right\} + \psi \left\{ \frac{S_z^2 - s_z^2}{S_z^2 + (b - 1)s_z^2} \right\} \right] \tag{3.1}$$

where η , ψ , a , and b are suitable chosen constants to be determined such that the MSE of t is minimum.

Expanding Equation (3.1) in terms of e 's up to the first order of approximation, we have

$$t = \lambda S_y^2 (1 + \epsilon_y) \exp \left[\frac{-\eta \epsilon_x}{a + (a - 1) \epsilon_x} + \frac{-\psi \epsilon_z}{b + (b - 1) \epsilon_z} \right] \tag{3.2}$$

$$\text{or, } t - S_y^2 = (\lambda - 1) S_y^2 + \lambda S_y^2 \left\{ \epsilon_y - \frac{\psi \epsilon_z}{b} - \frac{\eta \epsilon_x}{a} - \frac{\eta \epsilon_y \epsilon_z}{b} - \frac{\eta \epsilon_x \epsilon_y}{a} \right\} \tag{3.3}$$

$$\text{or, } t - S_y^2 = (\lambda - 1) S_y^2 + \lambda S_y^2 \left\{ \epsilon_y - \psi^* \epsilon_z - \eta^* \epsilon_x - \psi^* \epsilon_y \epsilon_z - \eta^* \epsilon_x \epsilon_y \right\} \tag{3.4}$$

$$\text{where, } \frac{\eta}{a} = \eta^* \text{ and } \frac{\psi}{b} = \psi^*$$

Squaring both sides of Equation (3.4) and taking its expectations we get the MSE expressions of the estimator t as,

$$\text{MSE}(t) = (\lambda - 1)^2 S_y^4 + \frac{\lambda^2}{n} S_y^4 \left[(\partial_{400} - 1) + \psi^{*2} (\partial_{004} - 1) \right. \\ \left. + \eta^{*2} (\partial_{040} - 1) - 2\psi^* (\partial_{202} - 1) - 2\eta^* (\partial_{220} - 1) + 2\eta^* \psi^* (\partial_{022} - 1) \right] \tag{3.5}$$

Partially differentiating Equation (3.5) with respect to λ , η^* , and ψ^* and equating it to zero, we get the optimum value of λ , η^* and ψ^* as

$$\lambda_{\text{opt}} = \frac{1}{1 + \frac{1}{n} \left[(\partial_{400} - 1) + \psi^{*2} (\partial_{004} - 1) + \eta^{*2} (\partial_{040} - 1) - 2\psi^* (\partial_{202} - 1) - 2\eta^* (\partial_{220} - 1) + 2\eta^* \psi^* (\partial_{022} - 1) \right]}$$

$$\eta_{\text{opt}}^* = \frac{(\partial_{220} - 1) - \psi^* (\partial_{022} - 1)}{(\partial_{040} - 1)}$$

$$\psi_{\text{opt}}^* = \frac{(\partial_{202} - 1)(\partial_{040} - 1) - (\partial_{220} - 1)(\partial_{022} - 1)}{(\partial_{040} - 1)(\partial_{004} - 1) - (\partial_{022} - 1)^2}$$

Substituting the optimum value of λ_{opt} , η_{opt}^* and ψ_{opt}^* in Equation (3.5), we obtain the minimum MSE associated with the estimators t as,

$$\text{MSE}(t)_{\min} = (\lambda_{\text{opt}} - 1)^2 S_Y^4 + \frac{\lambda_{\text{opt}}^2}{n} S_Y^4 \left[(\partial_{400} - 1) + \psi_{\text{opt}}^{*2} (\partial_{004} - 1) + \eta_{\text{opt}}^{*2} (\partial_{040} - 1) - 2\psi_{\text{opt}}^* (\partial_{202} - 1) - 2\eta_{\text{opt}}^* (\partial_{220} - 1) + 2\eta_{\text{opt}}^* \psi_{\text{opt}}^* (\partial_{022} - 1) \right] \quad (3.6)$$

4. Efficiency comparison

In this section, we are comparing the minimum MSE of the proposed estimator t with usual estimator S_y^2 and other existing estimators.

The variance of the usual estimator S_y^2 under SRSWOR is given by

$$V(S_y^2) = \frac{S_Y^4}{n} [\partial_{400} - 1] \quad (4.1)$$

$$V(S_y^2) - \text{MSE}(t)_{\min} = \frac{(1 - \lambda_{\text{opt}}^2)}{n} S_Y^4 (\partial_{400} - 1) - (\lambda_{\text{opt}} - 1)^2 S_Y^4 - \frac{\lambda_{\text{opt}}^2}{n} S_Y^4 \left[\psi_{\text{opt}}^{*2} (\partial_{004} - 1) + \eta_{\text{opt}}^{*2} (\partial_{040} - 1) - 2\psi_{\text{opt}}^* (\partial_{202} - 1) - 2\eta_{\text{opt}}^* (\partial_{220} - 1) + 2\eta_{\text{opt}}^* \psi_{\text{opt}}^* (\partial_{022} - 1) \right] \geq 0 \quad (4.2)$$

$$\begin{aligned} \text{MSE}(t_N) - \text{MSE}(t)_{\min} &= \frac{1}{n} S_Y^4 \left[\frac{\alpha^2}{4} (w_0 - 1)^2 g^2 (\partial_{004} - 1) + (\partial_{400} - 1) + (w_0 - 1)^2 g^2 (\partial_{040} - 1) \right. \\ &\quad \left. + 2g(w_0 - 1)(\partial_{220} - 1) + \alpha g(w_0 - 1)(\partial_{202} - 1) + \alpha g^2 (w_0 - 1)^2 (\partial_{022} - 1) \right] \\ &\quad - \left[(\lambda_{\text{opt}} - 1)^2 S_Y^4 + \frac{\lambda_{\text{opt}}^2}{n} S_Y^4 \left\{ (\partial_{400} - 1) + \psi_{\text{opt}}^{*2} (\partial_{004} - 1) \right. \right. \\ &\quad \left. \left. + \eta_{\text{opt}}^{*2} (\partial_{040} - 1) - 2\psi_{\text{opt}}^* (\partial_{202} - 1) - 2\eta_{\text{opt}}^* (\partial_{220} - 1) \right. \right. \\ &\quad \left. \left. + 2\eta_{\text{opt}}^* \psi_{\text{opt}}^* (\partial_{022} - 1) \right\} \right] \geq 0 \quad (4.3) \end{aligned}$$

$$\begin{aligned}
 \text{MSE}(t_M) - \text{MSE}(t)_{\min} = & \frac{S_y^4}{n} \left[\beta^2 (\delta_0 g - g - \delta_0)^2 (\partial_{040} - 1) + (\partial_{400} - 1) \right. \\
 & + \frac{\gamma^2}{4} (\delta_0 g - g - \delta_0)^2 (\partial_{004} - 1) + 2\beta (\delta_0 g - g - \delta_0) (\partial_{220} - 1) \\
 & \left. + \gamma (\delta_0 g - g - \delta_0) (\partial_{202} - 1) + \gamma \beta (\delta_0 g - g - \delta_0)^2 (\partial_{022} - 1) \right] \\
 & - \left[(\lambda_{\text{opt}} - 1)^2 S_y^4 + \frac{\lambda_{\text{opt}}^2}{n} S_y^4 \left\{ (\partial_{400} - 1) + \psi_{\text{opt}}^{*2} (\partial_{004} - 1) \right. \right. \\
 & \left. \left. + \eta_{\text{opt}}^{*2} (\partial_{040} - 1) - 2\psi_{\text{opt}}^* (\partial_{202} - 1) - 2\eta_{\text{opt}}^* (\partial_{220} - 1) \right. \right. \\
 & \left. \left. + 2\eta_{\text{opt}}^* \psi_{\text{opt}}^* (\partial_{022} - 1) \right\} \right] \geq 0
 \end{aligned} \tag{4.4}$$

When the conditions (4.2) to (4.4) are satisfied, our suggested estimator t will be more efficient as compared to S_y^2 , t_1 , t_N and t_M respectively.

5. Empirical study

To illustrate the performance of various estimators of S_y^2 , we consider the following data sets

Population I [Source: Sarjinder Singh (2003, p. 1116)].

y : Fish caught in year 1995, x : Fish caught in year 1993, z : Fish caught in year 1994,

$N = 69, n = 25$.

$\partial_{400} = 7.7685, \partial_{040} = 9.9860, \partial_{004} = 9.9851, \partial_{220} = 8.3107, \partial_{202} = 8.1715, \partial_{022} = 9.6631,$
 $S_y^2 = 37199578$.

Population II [Source: Murthy (1967, p. 399)].

y : Area under wheat in 1964, x : Area under wheat in 1963, z : Cultivated area in 1961,

$N = 34, n = 15$.

$\partial_{400} = 3.7879, \partial_{040} = 2.9123, \partial_{004} = 2.8082, \partial_{220} = 3.1046, \partial_{202} = 2.9790, \partial_{022} = 2.7379,$
 $S_y^2 = 22564.55704$.

From Table 1, it is clear that the MSE of the estimator “ t ” is less as compared to the estimators S_y^2 , t_N , t_M and t_1 . In terms of PRE’s, it is observed that the PRE of the proposed estimator t is higher as

Table 1. PRE of various estimators with respect to S_y^2

Estimators	PREs	
	Population I	Population II
S_y^2	100.0000	100.0000
t_N	848.7573	698.2471
t_M	848.7573	698.2471
t_1	848.7573	698.2471
T	876.6598	716.6599

compared to PRE's of the estimators S_y^2 , t_N , t_M , and t_1 for both the given data-sets. So, the estimator t is more efficient than the estimator S_y^2 and other existing estimators.

6. Simulation study

This section presents the computational procedure for the comparison of proposed estimator with other existing estimators. The simulation study is based on the algorithm proposed by Reddy, Rao, and Boiroju (2010) to illustrate the performance of various estimators of S_y^2 . The following algorithm explain the simulation procedure used in this paper.

Step-1: Generate two independent random variable X from $N(\mu, \sigma^2)$ and X_1 from $N(\mu_1, \sigma_1^2)$ using box-Muller method (Jhonson, 1987).

Step-2: Set $Y = \rho X + \sqrt{1 - \rho^2} X_1$ where $0 < \rho = 0.4, 0.6, 0.8 < 1$.

Step-3: return the pair (Y, X) .

Step-4: Consider the population-I with the parameters $\mu = 5, \sigma = 3, \mu_1 = 5$ and $\sigma_1 = 3$ in step-1 and repeat the steps 1 to 3 for 1000 times. This population will contain the same variance for the variable Y and X .

Step-5: Similarly, generate the population-II with the parameters $\mu = 3, \sigma = 2, \mu_1 = 5$ and $\sigma_1 = 3$ in step-1 and repeat the steps 1 to 3 for 1000 times. This population will have different variances for the variable Y and X .

Step-6: From the population of size $N = 1000$, draw 2000 SRSs (y_i, x_i) ($i = 1, 2, \dots, n$) without replacement of size $n = 40, 50$ and 60 .

Step-7: The Average mean squared error (MSE) of the estimators are defined by

$$\text{Average MSE}(t) = \frac{1}{2000} \sum_{k=1}^{2000} E(t_k - S_y^2)^2$$

The PRE of an estimator t with respect to the usual estimator S_y^2 is defined by

$$\text{PRE}(t) = \frac{\text{MSE}(S_y^2) \times 100}{\text{MSE}(t)}$$

The obtained results of the simulation study are as follows.

From Tables 2 and 3 we observe that as the sample size and the value of correlation coefficient increases, the average PRE's of the estimators also increases for both the population I and II. The average PRE of the estimator " t " is higher in all cases indicating that the proposed estimator t is more efficient as compared to usual estimator S_y^2 and other existing estimators t_N , t_M , and t_1 .

Table 2. Comparison of proposed estimator t with other existing estimators for Population-I

ρ	N	Average percent relative efficiency (PRE)				
		s_y^2	t_N	t_M	t_1	t
0.5	40	100.0000	112.0037	111.6313	110.9877	117.5637
	50	100.0000	112.1265	111.5847	110.9795	116.5967
	60	100.0000	112.1866	111.5948	110.9865	115.847
0.6	40	100.0000	122.3532	121.7895	122.9326	128.3793
	50	100.0000	122.5582	121.7388	122.9584	127.4171
	60	100.0000	122.6414	121.7331	122.9711	126.6652
0.8	40	100.0000	179.4806	177.6442	177.4084	189.413
	50	100.0000	180.2122	177.5656	177.5064	188.5566
	60	100.0000	180.3817	177.3901	177.3211	187.7472

Table 3. Comparison of proposed estimator t with other existing estimators for Population-II

ρ	N	Average per cent relative efficiency (PRE)				
		s_y^2	t_N	t_M	t_1	t
0.5	40	100.0000	104.493	104.2726	100.7379	109.7692
	50	100.0000	104.557	104.2354	100.6977	108.8092
	60	100.0000	104.5891	104.2507	100.6683	108.0547
0.6	40	100.0000	108.471	108.1663	106.4562	113.8899
	50	100.0000	108.5663	108.1228	106.434	112.9245
	60	100.0000	108.6153	108.1362	106.4296	112.1738
0.8	40	100.0000	133.0232	132.2563	134.1661	139.6054
	50	100.0000	133.3169	132.2032	134.2187	138.6565
	60	100.0000	133.4178	132.1748	134.2144	137.8979

7. Conclusion

This paper proposed an improved generalized class of estimator for estimating population variance based on simple random sampling without replacement using information of two auxiliary variables. The performance of the proposed estimator is verified by using two real population data-sets and by simulation study. Tables 1–3 clearly show that the proposed estimator t is more efficient as compared to the usual estimator and other existing estimators. Hence, it is recommended for use in practice.

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