Abstract: The purpose of this study was to investigate the behavior of various indices of Tehran Stock Exchange firstly in the boom period from 2018-03-21 to 2018-11-02 and secondly in the recession period from 2016-03-20 to 2016-12-20 using double-sided balanced conditional Sharpe ratio. The results of this study showed the best performance for the insurance index with a double-sided balanced conditional Sharpe ratio of 0.123 and later for the metallic minerals index with a measure of 0.1215. Moreover, on the basis of the double-sided balanced conditional Sharpe ratio, the food except for sugar index with a measure of 0.035 showed the worst performance.

1. Introduction
When looking into investment, you need to look at both risk and return. While return can be easily quantified, risk cannot. Today, standard deviation is the most commonly referenced risk measure, while the Sharpe ratio is the most commonly used return/risk measure. For any risky asset or portfolio, the Sharpe ratio is defined as the ratio of the excess return to the standard deviation of that return. The Sharpe ratio has been around since 1966 but its life has not passed without controversy. Even its founder, Nobel laureate William Sharpe, has admitted the ratio is not without its problems.

Sharpe ratio has been used for performance evaluation by many researchers such as Bernardo and Ledoit (2000), Brooks and Kat (2002), Malkiel and Saha (2005), Spurgin (2001), Goetzmann, Ingersoll, Spiegel, and Welch (2002). Therefore, many researchers have replaced standard...
deviation in the Sharpe ratio by an alternative risk measure. Some researchers such as Sortino and Price (1994) and Ziemba (2005) have replaced standard deviation by downside deviation and some others such as Dowd (2000) and Gregoriou and Gueyie (2003) have used Value-at-Risk measure instead of standard deviation.

Due to Sharpe ratio shortages, this study used conditional standard deviation instead of unconditional standard deviation. In this study, in order to overcome other shortcomings of traditional Sharpe ratio introduced, double-sided balanced conditional Sharpe ratio was used to solve the problem of asymmetry of time in the boom-recession cycle. The research sample includes the daily price information of six different indexes, including insurance, food except sugar, sugar, chemicals, metallic minerals, and price index (TEPIX) of Tehran stock market in two periods, firstly, in the boom period from 2018-03-21 to 2018-11-02 and secondly in the recession period from 2016-03-20 to 2016-12-20.

2. Literature

2.1. Literature of conditional risk

2.1.1. GARCH
If an autoregressive moving average (ARMA) model is assumed ARCH for the error variance, the model is a generalized autoregressive conditional heteroskedasticity (GARCH) model.

In that case, the GARCH \((p, q)\) model (where \(p\) is the order of the GARCH terms \(\sigma_t^2\) or the variance of the previous day and \(q\) is the order of the ARCH terms \(\epsilon_{t-1}\) or the error terms of the previous day) following the notation of the original paper (Bollerslev, 1986) is given by:

\[
\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i} + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2
\]

(1)

2.1.2. GJR-GARCH
Using this model, we can model the leverage effects proposed by Black (1976) and French, Schwert, and Stambaugh (1987). The Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model by Glosten, Jagannathan, and Runke (1993) also models asymmetry in the ARCH process. The leverage effect is modeled in the GARCH process. If \(\epsilon_{t-1} > 0\), then \(I = 0\) and if \(\epsilon_{t-1} < 0\), then \(I = 1\) and leverage effects can be tested assuming \(\gamma > 0\).

\[
\sigma_t = \omega + \sum_{i=1}^{p} (\alpha_i + \gamma_I \epsilon_{t-1}) \epsilon_{t-j} + \sum_{j=1}^{q} \beta_i \sigma_{t-j}
\]

(2)

2.1.3. EGARCH
The exponential generalized autoregressive conditional heteroskedastic (EGARCH) model by Nelson and Cao (1992) is another form of the GARCH model. Formally, an EGARCH(p,q):

If \(\gamma \neq 0\) is significant, then the effects of the shocks on the conditional variance are asymmetric. In this model, leverage effects can be tested assuming \(\gamma < 0\).

\[
\log(\sigma_t^2) = \omega + \alpha_1 \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \gamma \log \left( \frac{\epsilon_{t-1}}{\sigma_{t-1}} \right) + \beta_1 \log(\sigma_{t-1}^2)
\]

(3)

2.1.4. PGARCH
The Power GARCH (PGARCH) model by Taylor (1986) and Schwert (1989) use the conditional standard deviation as a measure of volatility instead of the conditional variance. This model is generalized by Ding, Granger, and Engle (1993) using the PGARCH model as follows:
\[ \sigma_t^4 = \omega + \sum_{i=1}^{q} \alpha_i (|\mu_{t-i} - \gamma \mu_{t-i}|)^\delta + \sum_{j=1}^{p} \beta_j \sigma_j^4 \]  

(4)

In this equation, \( \gamma \) denotes asymmetry. In the symmetric model, \( \gamma \) is zero for all values and the coefficient.

For real risk series measurement by formula 5, the residual series was used to calculate the difference between real daily return and average returns.

\[ \sigma^2 = (r - \mu)^2 \]  

(5)

In this research, according to the research objectives, to determine the conditional risk of optimal from the root mean square error or RMSE criterion was used.

\[ \text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}} \]  

(6)

where \( e_i \) stands for the difference between actual and predict series for Conditional variance.

Zhang (2006) used moving average model, exponential moving average, random walk, and various GARCH models to predict Shanghai and Shenzhen indices in the stock exchange of China. He concluded no single model could have the best performance in all conditions. For example, asymmetric models like GJRGARCH and EGARCH in Shenzhen index had a better performance than other GARCH models, but asymmetric models were not appropriate for conditional risk forecast in the Shanghai index.

Abdelaal (2011) investigated the Egyptian stock exchange from 1998 to 2009. They found that EGARCH model predicted volatility better than other models. Liu and Hung (2009) tested EGARCH, GARCH, ARCH, and GJR-GARCH models in S&P index and reported that asymmetric models such as GJR-GARCH and EGARCH were more important than the type of error distribution for more accurate prediction of volatility. Dritsaki (2017) studied the daily returns of stock in Stockholm stock exchange and concluded that asymmetric GARCH models like EGARCH with student distribution along with ARIMA (0, 0, 1) model provided a more precise prediction of GARCH models. Andreea-Cristina and Stelian (2017) investigated volatility in Euro exchange rate versus the Romanian currency and found that asymmetric EGARCH and PGARCH models were more powerful than symmetric GARCH models for estimation of risk and return. The results of studies by Guo (2017b, 2017a) on Hong Kong stock exchange, Sarkar and Banerjee (2006), Liu and Hung (2009), Intaz, Subhrabaran, and Niranjan (2016) on Netherlands stock exchange, Coffie, Tackie, Bedi, and Aboagye-Otchere (2017) on North and East Africa stock exchange, Nilsson (2017) on Sweden stock exchange, and Dritsaki (2017) on the daily return of stock in Stockholm stock exchange indicated that asymmetric GARCH modes like GJR-GARCH as well as the other models considering leverage effects for prediction of risk had a better performance than symmetric GARCH models in prediction of risk.

2.2. The Sharpe ratio

In a series of Sharpe (1966, 1975, 1994) introduced and developed risk-adjusted measure of investment's performance. This measure, generally known as the Sharpe ratio (SR), and is used to help investors understand the return of an investment compared to its risk. In finance, the Sharpe ratio (also known as the Sharpe index, the Sharpe measure, and the reward-to-variability ratio) is a method to examine the performance of an investment by adjusting for its risk. The ratio measures the excess return (or risk premium) per unit of deviation in an investment asset or a trading strategy, typically referred to as risk, named after William F. Sharpe. Performance measurement is an integral part of investment analysis and risk management. The Sharpe ratio is one of the most prominently used measures for performance evaluation of an investment with respect
to return and risk. The Sharpe ratio is calculated by subtracting the risk-free rate from the return of
the stock and dividing that result by the standard deviation of the stock’s excess return.

$$SR_i = \frac{R_i - R_f}{\sigma_i}$$  \hspace{1cm} (7)

where $R_i$ is return of stock and $R_f$ is risk-free rate and $\sigma_i$ is standard deviation of the portfolio’s
excess return.

Bayley and López de Prado (2012) showed that Sharpe ratios tend to be overstated in the case of
hedge funds with short track records. These authors have proposed a probabilistic version of the
Sharpe ratio that takes into account the asymmetry and fat-tails of the return distribution. With
regard to the selection of portfolio managers on the basis of their Sharpe ratios, these authors
have proposed a Sharpe ratio indifference curve (Bailey and Lopez de Prado 2013). This curve
illustrates the fact that it is efficient to hire portfolio managers with low and even negative Sharpe
ratios as long as their correlation to the other portfolio managers is sufficiently low.

Because it is a dimensionless ratio, laypeople find it difficult to interpret the Sharpe ratios of
different investments. For example, how much better is an investment with a Sharpe ratio of 0.5
than one with a Sharpe ratio of −0.2? This weakness was well addressed by the development of
the Modigliani risk-adjusted performance measure, which is in units of percent return—universally
understandable by virtually all investors. In some settings, the Kelly criterion can be used
to convert the Sharpe ratio into a rate of return. The Kelly criterion gives the ideal size of the
investment, which when adjusted by the period and expected rate of return per unit, gives the
rate of return (Wilmott, 2007). Chow and Lai (2014) showed that CSR is able to discriminate the
downside performance of funds, something that the conventional Sharpe ratio generally fails to
do. A large out-of-sample analysis of US mutual fund shows that CSR has predictability for
future portfolio performance. Jones and O’Steen (2017) evaluated time-varying correlations
between multiple asset classes using an asymmetric-DCC GARCH model. Specifically, they
focused on the changes in these correlations during quantitative easing. They then used
these conditional correlations along with conditional means and variances to find optimal
investment portfolios using Markowitz mean-variance minimization. Lastly, they computed
time-varying Sharpe ratios. Their results showed increasing Sharpe ratios during the period of
quantitative easing, suggesting that the Federal Reserve’s programs were successful in increas-
ing the returns and minimizing the risk—i.e. volatility across several asset classes during the
financial crisis. Barillas, Kan, Robotti, and Shanken (2017) showed how to conduct asymptoti-
cally valid tests of model comparison when the extent of model mispricing is gauged by the
squared Sharpe ratio improvement measure. This is equivalent to the ranking models on their
maximum Sharpe ratios, effectively extending the GRS test to accommodate the comparison of
non-nested models. Mimicking portfolios can be substituted for any nontraded model factors,
and estimation error in the portfolio weights is taken into account in the statistical inference.

3. Methodology

3.1. Conditional Sharpe ratio

Although volatility tends towards a constant amount in the long run, which is the same as uncondi-
tional variance, sometimes shocking the market causes more volatility, and in order to consider
the shock effect, the conditional variance should be used. Also, the standard deviation assumes that price
movements in either direction are equally risky, due to these problems, instead of the standard
deviation from the conditional standard deviation and instead of the traditional Sharpe ratio (uncondi-
tional Sharpe ratios) from the conditional Sharpe ratio was used. To measure this ratio, we first reduce
the risk-free rate of interest in one day from the daily average return of an index. Then, we divide the
output by the average conditional standard deviation of the return of that index in the review period.
CSR_i = \frac{\bar{R}_i - R_f}{\bar{\sigma}_{i,t}} \tag{8}

where CSR_i is Conditional Sharpe Ratio of any index and \bar{R}_i is daily average return of an index and \bar{R}_f is risk-free rate and \bar{\sigma}_{i,t} denotes average conditional standard deviation of any index.

### 3.2. Conditional Sharpe ratio in boom conditions

\[CSR_i(\text{in boom}) = \frac{\bar{R}_i(\text{in boom}) - R_f}{\bar{\sigma}_{i,t}}\tag{9}\]

### 3.3. Conditional Sharpe ratio in recession conditions

\[CSR_i(\text{in recession}) = \frac{\bar{R}_i(\text{in recession}) - R_f}{\bar{\sigma}_{i,t}}\tag{10}\]

### 3.4. Double-Sided Conditional Sharpe Ratio (DSCSR)

To calculate the double-sided conditional Sharpe ratio, the simple average of the conditional Sharpe ratio was used for two periods of boom and recession.

\[\text{DSCSR}_i = \frac{CSR_i(\text{in boom}) + CSR_i(\text{in recession})}{2}\tag{11}\]

### 3.5. Double-Sided Balanced Conditional Sharp Ratio (DSBCSR)

In line with investment defensive strategies such as buy and hold, in addition to the average of the conditional Sharpe ratio in two periods of boom and recession, their standard deviation was also measured by formula 12.

\[\sigma_{\text{double-sided}} = \sqrt{\frac{(CSR_i(\text{in boom}) - \text{DSCSR}_i)^2 + (CSR_i(\text{in recession}) - \text{DSCSR}_i)^2}{2}}\tag{12}\]

where \sigma_{\text{double-sided}} is the standard deviation of the double-sided conditional Sharpe ratio, CSR_i (in boom) is conditional Sharpe ratio in boom conditions, CSR_i (in recession) is conditional Sharpe ratio in recession conditions, and DSCSR_i is the double-sided conditional Sharpe ratio.

In the next step, the double-sided balanced conditional Sharpe ratio (DSBCSR_i) is modeled in the form of formula 13.

\[\text{DSBCSR}_i = \frac{\text{DSCSR}_i}{\text{EXP}(\sigma_{\text{double-sided}})}\tag{13}\]

in formula 13, where DSCSR_i represents a double-sided conditional Sharpe ratio and \text{EXP}(\sigma_{\text{double-sided}}) represents the adjusted dispersion rate of two Sharpe ratios in both recession and boom conditions.

The superiority of the DSBCSR model compared to DSCSR model, simultaneous simulation of two factors, first is the average of two conditional Sharpe ratios in two periods of boom and recession and second the dispersion rate or standard deviation of two conditional Sharpe ratios in two periods of boom and recession. In other words, the DSBCSR model, in addition to having the features of the DSCSR model, models the dispersion or standard deviation of two conditional Sharpe ratios in two periods of boom and recession.

### 4. Results

Since the price index (TEPIX) represents the prices and gain of all companies accepted in Tehran Stock Exchange, average return of price index was used to measure the boom or recession (high and positive average return of price index Indicates boom conditions and low and negative average return of price index Indicates recession conditions).
As shown in Table 1, in the boom period, the average daily return of price index was 0.00437 and the standard deviation and median daily return were 0.0155, 0.0011, respectively. However, in the recession period, the average daily return of price index was –0.00007 and the standard deviation and median daily return were 0.0045, 0.0003, respectively.

To determine the optimized conditional risk model in six indexes, including price, insurance, except for sugar, sugar, chemicals, and metallic minerals, a total of 48 estimates were made by EViews8 software.

As indicated in column 6 of Table 2–7, gamma coefficient (γ) is either insignificant (i.e. good or bad news has equal effects on volatility) or negative for the GJR-GARCH model and positive for the EGARCH model (i.e. good news or positive returns have more effect on volatility than bad news or negative returns), which is in line with the results of previous studies such as the study of Mehrara and Abduli (2008). In this situation, leverage models, e.g., semi-standard deviation could not be a good measurement for risk.

Also, Root Mean Square Error (RMSE criterion) was used to find the optimized conditional risk model. The optimized conditional risk is presented by (*) in column 8 of Table 2–7.

As shown in Table 2 and Figures 1 and 2, the insurance index is optimized in both boom and recessions periods in the GARCH (1,1) model.

As shown in Table 3 and Figures 3 and 4, the food except sugar index is optimized in both boom and recessions periods in the GARCH (1,1) model.

As shown in Table 4 and Figures 5 and 6, the sugar index is optimized in both boom and recessions periods in the EGARCH (1,1) model.

In anywhere coefficients ω, α, β, γ and δ are significant. The symbol * denotes significance at the 10% levels.

As shown in Table 5 and Figures 7 and 8, the chemicals index is optimized in both boom and recessions periods in the GJR-GARCH (1,1) model.

As in Table 6 and Figures 9 and 10, the metallic minerals index is optimized in both boom and recessions periods in the GARCH (1,1) model.

As indicated in Table 7 and Figures 11 and 12, EGARCH (1,1) is optimized in the boom period and GARCH (1,1) is optimized in the recession period in the price index.

According to the data in Table 8, during the boom period, the metallic minerals index had the highest average daily return of 0.74%, followed by the chemical index (0.49%), and the food

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>Average daily return</th>
<th>Median daily return</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>In the boom period from 21 March 2018 to 2 November 2018</td>
<td>0.00437</td>
<td>0.0011</td>
<td>0.0155</td>
</tr>
<tr>
<td>In the recession period from 20 March 2016 to 20 December 2016</td>
<td>–0.00007</td>
<td>0.0003</td>
<td>0.0045</td>
</tr>
</tbody>
</table>
except for sugar index had the lowest average daily return of 0.21%. Also, the highest unconditional Sharpe ratios belonged to the metallic minerals index (0.29) and price index (0.25), chemicals index (0.217). The unconditional Sharpe ratios of the insurance index, sugar index, and food except for sugar index were 0.155, 0.14, and 0.102, respectively.

The type of optimal condition variance model (among GARCH family models) is selected using the RMSE criterion for variance equation whose results are shown as optimal conditional risk and optimal conditional standard deviation in the columns 5 and 6 of Table 8. In this regard, the highest conditional Sharpe ratios belonged to the price index (0.289), metallic minerals index (0.265), and chemicals index (0.243). The conditional Sharpe ratios of the sugar index, insurance index, and food except for sugar index were 0.147, 0.146, and 0.103, respectively.

Considering the daily risk-free rate of 0.0005 and using the formula 7 the conditional Sharpe ratio for insurance index in the boom period was calculated as follows:

Table 2. Conditional risk in the insurance index

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>Conditional risk model</th>
<th>ω</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>δ</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom period</td>
<td>GARCH (1,1)</td>
<td>0.000*</td>
<td>0.45*</td>
<td>0.58*</td>
<td>-</td>
<td>-</td>
<td>0.00252 (*)</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH (1,1)</td>
<td>0.000*</td>
<td>0.5*</td>
<td>0.59*</td>
<td>-0.22</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.014</td>
<td>0.007</td>
<td>0.000</td>
<td>0.27</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH (1,1)</td>
<td>−1.3*</td>
<td>0.6*</td>
<td>0.9*</td>
<td>0.09</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.000</td>
<td>0.003</td>
<td>0.000</td>
<td>0.36</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>0.000</td>
<td>0.41*</td>
<td>0.51*</td>
<td>−0.13</td>
<td>2.9</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.9</td>
<td>0.05</td>
<td>0.011</td>
<td>0.38</td>
<td>0.13</td>
<td>-</td>
</tr>
<tr>
<td>Recession period</td>
<td>GARCH</td>
<td>0.000*</td>
<td>0.2*</td>
<td>0.75*</td>
<td>-</td>
<td>-</td>
<td>0.0017 (*)</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.02</td>
<td>0.009</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH (1,1)</td>
<td>0.000*</td>
<td>0.25*</td>
<td>0.8*</td>
<td>−0.245*</td>
<td>-</td>
<td>0.0018</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.004</td>
<td>0.007</td>
<td>0.000</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH (1,1)</td>
<td>−1.2*</td>
<td>0.26*</td>
<td>0.89*</td>
<td>0.17*</td>
<td>-</td>
<td>0.00175</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.0017</td>
<td>0.048</td>
<td>0.000</td>
<td>0.007</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>1.3</td>
<td>0.087</td>
<td>0.79*</td>
<td>−0.6</td>
<td>2.3*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.88</td>
<td>0.5</td>
<td>0.000</td>
<td>0.5</td>
<td>0.069</td>
<td>-</td>
</tr>
</tbody>
</table>

In anywhere coefficients ω, α, β, γ and δ are significant. The symbol * denotes significance at the 10% levels.
According to the data of Table 9, in the recession period, the highest returns were reported for the insurance index (0.17%), the metallic minerals, food excerpt sugar, and sugar indices (0.7%), and the price index (−0.007%). In the end, the chemicals index was found to be (−0.026%). Also, the type of optimal condition variance model (among GARCH family models) is selected using the RMSE criterion for variance equation whose results are shown as optimal conditional risk and optimal conditional standard deviation in the columns 5 and 6 of Table 9. Moreover, in order of preference the highest conditional Sharpe ratios during the recession were reported for the insurance index (0.1043), the sugar index (0.0113), the minerals index (0.011), the food except sugar index (−0.028), and the price index (−0.11) and finally, the chemicals index was

$$CSR_{(inboom)} = \frac{0.0026 - 0.0005}{0.0144} = 0.146$$

Figure 3. Optimal Conditional risk in the food except sugar index during a boom.

Figure 4. Optimal Conditional risk in the food except sugar index during a recession.

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>Conditional risk model</th>
<th>(\omega)</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\delta)</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom period</td>
<td>GARCH (1,1)</td>
<td>0.000*</td>
<td>0.188*</td>
<td>0.77*</td>
<td>-</td>
<td>-</td>
<td>0.0029</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH (1,1)</td>
<td>0.000*</td>
<td>0.16*</td>
<td>0.83*</td>
<td>-0.097</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH (1,1)</td>
<td>0.03</td>
<td>0.04</td>
<td>0.000</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH (1,1)</td>
<td>0.009</td>
<td>0.01</td>
<td>0.000</td>
<td>0.37</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>0.000</td>
<td>0.021</td>
<td>0.85*</td>
<td>-0.52</td>
<td>3.6</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.95</td>
<td>0.9</td>
<td>0.000</td>
<td>0.86</td>
<td>0.31</td>
<td>-</td>
</tr>
<tr>
<td>Recession period</td>
<td>GARCH (1,1)</td>
<td>0.000*</td>
<td>0.086*</td>
<td>0.82*</td>
<td>-</td>
<td>-</td>
<td>0.00065</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH (1,1)</td>
<td>0.000*</td>
<td>0.13*</td>
<td>0.8*</td>
<td>-0.12*</td>
<td>-</td>
<td>0.00068</td>
</tr>
<tr>
<td></td>
<td>EGARCH (1,1)</td>
<td>-6*</td>
<td>0.72*</td>
<td>0.43*</td>
<td>0.37*</td>
<td>-</td>
<td>0.00097</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>2.3</td>
<td>0.046</td>
<td>0.77*</td>
<td>-0.3</td>
<td>3.1*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.9</td>
<td>0.4</td>
<td>0.000</td>
<td>0.2</td>
<td>0.07</td>
<td>-</td>
</tr>
</tbody>
</table>

In anywhere coefficients \(\omega\), \(\alpha\), \(\beta\), \(\gamma\) and \(\delta\) are significant. The symbol * denotes significance at the 10% levels.
calculated to be $-0.149$. Considering the daily risk-free rate of 0.0005 and using the formula 8, the conditional Sharpe ratio for insurance index in the recession period was calculated as follows:

\[
\text{CSR}_{(\text{in recession})} = \frac{0.0017 - 0.0005}{0.0115} = 0.1043
\]

According to the findings of the column 4 of Table 10, the highest priorities of double-sided conditional Sharpe ratio (DSCSR) were found for metallic minerals index (0.138), the insurance index (0.1252), the sugar index (0.07915), the price index (0.0845), the chemicals index (0.047), and the food except for sugar index (0.037). Furthermore, according to the data in column 6 of Table 10, the highest priorities of double-sided balanced conditional Sharpe ratio were reported for...
the insurance index (0.123), the metallic minerals index (0.1215), the sugar index (0.074), the price index (0.069), the chemicals index (0.0386), the food except for sugar index (0.035).

For example, we used formula 11 to calculate the double-sided conditional Sharpe ratio in the insurance and metallic minerals indexes.

\[
DSCSR_i = \frac{0.146 + 0.1043}{2} = 0.1252 \text{ (insurance index)}
\]

\[
DSCSR_i = \frac{0.265 + 0.011}{2} = 0.138 \text{ (metallic minerals index)}
\]
To calculate the double-sided balanced conditional Sharpe ratio, we first computed the standard deviation of two conditional Sharpe ratios in two periods of boom and recession and then divided the double-sided conditional Sharp ratio into the exponential function of the standard deviation. For example, with using formulas 12 and 13, the double-sided balanced conditional Sharpe ratio in the insurance and metallic minerals indexes were calculated as follows:

$$
\sigma_{\text{double-sided}} = \left(\frac{(0.146 - 0.1252)^2 + (0.1043 - 0.1252)^2}{2}\right)^{\frac{1}{2}} = 0.0208 \text{ (insurance index)}
$$

$$
DSBCSR_i = \frac{0.1252}{\exp(0.0208)} \approx 0.123 \text{ (insurance index)}
$$

### Table 6. Conditional risk in the metallic minerals index

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>Conditional risk model</th>
<th>$\omega$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\delta$</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom period</td>
<td>GARCH(1,1)</td>
<td>0.000*</td>
<td>0.23*</td>
<td>0.77*</td>
<td>-</td>
<td>-</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.004</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH(1,1)</td>
<td>0.000*</td>
<td>0.09*</td>
<td>1*</td>
<td>-0.29*</td>
<td>-</td>
<td>0.0092</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH(1,1)</td>
<td>-0.23*</td>
<td>-0.23*</td>
<td>0.95*</td>
<td>0.26*</td>
<td>-</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>0.0047</td>
<td>0.089*</td>
<td>0.89*</td>
<td>-0.99*</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.6</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>Recession period</td>
<td>GARCH(1,1)</td>
<td>0.000*</td>
<td>0.19*</td>
<td>0.7*</td>
<td>-</td>
<td>-</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.048</td>
<td>0.03</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>GJR-GARCH(1,1)</td>
<td>0.000*</td>
<td>0.26*</td>
<td>0.64*</td>
<td>-0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.036</td>
<td>0.06</td>
<td>0.000</td>
<td>0.5</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EGARCH(1,1)</td>
<td>-1.9*</td>
<td>0.37*</td>
<td>0.79*</td>
<td>0.098</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.02</td>
<td>0.01</td>
<td>0.000</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>PGARCH (1,1)</td>
<td>0.009</td>
<td>0.19*</td>
<td>0.68*</td>
<td>-0.37</td>
<td>0.7</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>P-value</td>
<td>0.7</td>
<td>0.03</td>
<td>0.000</td>
<td>0.1</td>
<td>0.25</td>
<td>-</td>
</tr>
</tbody>
</table>

In anywhere coefficients $\omega$, $\alpha$, $\beta$, $\gamma$ and $\delta$ are significant. The symbol * denotes significance at the 10% levels.

Figure 9. Optimal Conditional risk in the metallic minerals index during a boom.

Figure 10. Optimal Conditional risk in the metallic minerals index during a recession.
The performance of two insurance and metallic minerals indexes is shown in columns 4 and 6 of Table 10. Based on the DSCSR, the superiority of the minerals index is 0.138, and based on the DSBCSR model, the superiority of the insurance index is 0.123.

### Table 7. Conditional risk in the price index

<table>
<thead>
<tr>
<th>Economic cycle</th>
<th>ω</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>δ</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 21 March 2018 to 2 November 2018</td>
<td>GARCH(1,1)</td>
<td>0.000*</td>
<td>0.27*</td>
<td>0.71*</td>
<td>-</td>
<td>0.0032</td>
</tr>
<tr>
<td>P-value</td>
<td>0.000</td>
<td>0.001</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GJR-GARCH(1,1)</td>
<td>0.000*</td>
<td>0.046*</td>
<td>1.02*</td>
<td>-0.28*</td>
<td>-</td>
<td>0.00295</td>
</tr>
<tr>
<td>P-value</td>
<td>0.000</td>
<td>0.02</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>-0.23*</td>
<td>-0.12*</td>
<td>0.96*</td>
<td>0.15*</td>
<td>-</td>
<td>0.0028 (*)</td>
</tr>
<tr>
<td>P-value</td>
<td>0.000</td>
<td>0.02</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PGARCH(1,1)</td>
<td>0.000</td>
<td>0.1</td>
<td>0.82</td>
<td>-0.99</td>
<td>1.3</td>
<td>-</td>
</tr>
<tr>
<td>P-value</td>
<td>0.8</td>
<td>0.87</td>
<td>0.000</td>
<td>0.9</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>Recession period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 20 March 2016 to 20 December 2016</td>
<td>GARCH(1,1)</td>
<td>0.000*</td>
<td>0.32*</td>
<td>0.43*</td>
<td>-</td>
<td>0.0003 (*)</td>
</tr>
<tr>
<td>P-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>GJR-GARCH(1,1)</td>
<td>0.000*</td>
<td>0.27*</td>
<td>0.43*</td>
<td>0.09</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P-value</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.45</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>EGARCH(1,1)</td>
<td>-3.9*</td>
<td>0.4*</td>
<td>0.66*</td>
<td>-0.05</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>P-value</td>
<td>0.007</td>
<td>0.000</td>
<td>0.000</td>
<td>0.33</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PGARCH(1,1)</td>
<td>0.002</td>
<td>0.199*</td>
<td>0.56*</td>
<td>0.21</td>
<td>0.96</td>
<td>-</td>
</tr>
<tr>
<td>P-value</td>
<td>0.8</td>
<td>0.07</td>
<td>0.000</td>
<td>0.3</td>
<td>0.17</td>
<td>-</td>
</tr>
</tbody>
</table>

In anywhere coefficients ω, α, β, γ and δ are significant. The symbol * denotes significance at the 10% levels.

Figure 11. Optimal Conditional risk in the price index during a boom.

Figure 12. Optimal Conditional risk in the price index during a recession.
<table>
<thead>
<tr>
<th>Index</th>
<th>Average daily return</th>
<th>Unconditional standard deviation</th>
<th>Unconditional Sharpe ratio</th>
<th>Optimal conditional risk model</th>
<th>Optimal conditional risk (conditional standard deviation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance</td>
<td>0.0026</td>
<td>0.0135</td>
<td>0.155</td>
<td>GARCH (1,1)</td>
<td>0.146</td>
</tr>
<tr>
<td>Food except sugar</td>
<td>0.0021</td>
<td>0.0135</td>
<td>0.155</td>
<td>GARCH (1,1)</td>
<td>0.155</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.0005</td>
<td>0.0157</td>
<td>0.102</td>
<td>GARCH (1,1)</td>
<td>0.140</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.0004</td>
<td>0.0178</td>
<td>0.14</td>
<td>EGARCH (1,1)</td>
<td>0.0017</td>
</tr>
<tr>
<td>Metallic mineral</td>
<td>0.0004</td>
<td>0.0203</td>
<td>0.217</td>
<td>GARCH (1,1)</td>
<td>0.0381</td>
</tr>
<tr>
<td>Price index</td>
<td>0.0004</td>
<td>0.0157</td>
<td>0.25</td>
<td>EGARCH (1,1)</td>
<td>0.0334</td>
</tr>
<tr>
<td>Price index</td>
<td>0.0004</td>
<td>0.0155</td>
<td>0.25</td>
<td>EGARCH (1,1)</td>
<td>0.0334</td>
</tr>
</tbody>
</table>
Table 9. Conditional Sharpe ratio during the recession period from 20 March 2016 to 20 December 2016

<table>
<thead>
<tr>
<th>Index</th>
<th>Average daily return</th>
<th>Unconditional standard deviation</th>
<th>Unconditional Sharpe ratio</th>
<th>Optimal conditional risk model</th>
<th>Optimal conditional risk (conditional standard deviation)</th>
<th>Conditional Sharp ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurance</td>
<td>0.0017</td>
<td>0.011</td>
<td>0.11</td>
<td>GARCH (1,1)</td>
<td>0.0115</td>
<td>0.1043</td>
</tr>
<tr>
<td>Food except sugar</td>
<td>0.0003</td>
<td>0.007</td>
<td>-0.028</td>
<td>GARCH (1,1)</td>
<td>0.0071</td>
<td>-0.0282</td>
</tr>
<tr>
<td>Sugar</td>
<td>0.0007</td>
<td>0.0174</td>
<td>0.0115</td>
<td>EGARCH (1,1)</td>
<td>0.0177</td>
<td>0.0113</td>
</tr>
<tr>
<td>Chemicals</td>
<td>-0.00026</td>
<td>0.005</td>
<td>-0.152</td>
<td>GJR GARCH (1,1)</td>
<td>0.0051</td>
<td>-0.149</td>
</tr>
<tr>
<td>Metallic minerals</td>
<td>0.0007</td>
<td>0.018</td>
<td>0.011</td>
<td>GARCH (1,1)</td>
<td>0.018</td>
<td>0.011</td>
</tr>
<tr>
<td>Price index</td>
<td>-0.00007</td>
<td>0.0045</td>
<td>-0.11</td>
<td>GARCH (1,1)</td>
<td>0.0048</td>
<td>-0.12</td>
</tr>
</tbody>
</table>
In general, the superiority of the DSBCSR model compared to DSCSR model, simultaneous simulation of two factors, first is the average of two conditional Sharpe ratios in two periods of boom and recession and second the dispersion rate or standard deviation of two conditional Sharpe ratios in two periods of boom and recession. But, in the DSCSR model, only the average of two conditional Sharpe ratios in two periods of boom and recession is calculated.

5. Conclusion and suggestions
A defensive stock is a stock that provides a constant dividend and stable earnings regardless of the state of the overall stock market. Because of the constant demand for their products, defensive stocks tend to remain stable during various phases of the business cycle. Defensive stocks tend to perform better than the broader markets during recessions. However, they tend to perform below the market during an expansion phase. This is attributed to their low beta or relative risk and performance in the market. Defensive stocks typically have betas of less than 1.

In this study, three defensive indexes of Tehran Stock Exchange, including insurance index, food except sugar index and sugar index and two invasive indexes with betas larger than one in boom conditions, namely chemicals index and metallic minerals index, were studied firstly in the boom period from 2018-03-21 to 2018-11-02 and secondly in the recession period from 2016-03-20 to 2016-12-20 using double-sided balanced conditional Sharpe ratio. Overall, the double-sided conditional Sharpe ratio indicated the superiority of each index in both the boom and recession periods, and the double-sided balanced conditional Sharpe ratio simultaneously represented both the superiority and stability of both the boom and recession periods. The results of this study showed that the best performance among the six indexes studied in this research was related to the insurance index with a double-sided balanced conditional Sharpe ratio of 0.123, the metallic minerals index (0.1215), and the sugar index (0.074). In other words, based on the new model of double-sided balanced conditional Sharpe ratio introduced in this study, the best performance was reported for the insurance, metallic minerals, and sugar indexes. So, based on the double-sided balanced conditional Sharpe ratio model, the defensive indexes including the insurance index and the sugar index as well as the invasive index of metallic minerals had the best performance in the long run using investment defensive strategies such as buy and hold. In general, investors have different investment styles but can basically be divided into two camps: active management like market timing and passive management like buy and hold strategy. Hence, this study introduced a new practical model in order to make use of the investment passive strategy. In other words, based on the double-sided balanced conditional Sharpe ratio, investors can achieve continuous and sustainable profits in both boom and recession periods.
Funding
The authors received no direct funding for this research.

Author details
Saeid Tajdini
e-mail: saeidtajdini1@yahoo.com1
ORCID ID: http://orcid.org/0000-0001-9446-4524
Mehsen Mehrara
e-mail: mmehrarad@ut.ac.ir2
Reza Tehrani
e-mail: tehrani@ut.ac.ir3

1Ph. D. Financial Management, University of Tehran, Tehran, Iran.
2Department of Economics, Faculty of Economics, University of Tehran, Tehran, Iran.

Citation information
Cite this article as: Double-sided balanced conditional Sharpe ratio, Saeid Tajdini, Mehsen Mehrara & Reza Tehrani, Cogent Economics & Finance (2019), 7: 1630931.

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doi:10.1016/0927-5398(93)90006-D


