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## ECONOMIC METHODOLOGY, PHILOSOPHY & HISTORY | RESEARCH ARTICLE

# Preference shocks in an RBC model with intangible capital

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**Abstract:** In this paper, we develop and simulate an RBC model that includes intangible capital as a third factor of production. We study the effects of intra-temporal preference shocks on economic aggregates, employing the undetermined coefficient method of to solve the model. Impulse response functions to preference shocks reveal that demand-side shocks are important in explaining the variations in macroeconomic aggregates.

**Subjects:** Economic Theory & Philosophy; Macroeconomics; Mathematical Economics

**Keywords:** Intangible capital; real business cycle; preference/demand shocks

**JEL Classification:** E32; E37

### 1. Introduction

Post-war U.S. data suggests that consumption growth “Granger-causes” gross domestic product (GDP) but not vice versa, and that GDP in turn “Granger-causes” business investment growth but not the other way around (Wen, 2007). This result suggests that consumption contains better information about the sources of shocks and that output contains better information than investment about the sources of shocks. Standard RBC models cannot be used to explain these causal relationships since they are based on the assumption that most aggregate economic fluctuations are due to technology (supply-side) shocks.<sup>1</sup> The seminal work of Kydland and Prescott (1982)

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### PUBLIC INTEREST STATEMENT

The article shows that in the presence of intangible capital, preference shocks such as leisure can also help explain movements in macroeconomic aggregates. This an important result since it entails that demand shocks in the economy also play an important role in explaining short-term business cycle fluctuations. An important conclusion is that demand-side shocks are also relevant in explaining aggregate fluctuations in macroeconomic variables which goes against the prediction of standard Real Business Cycle models that major fluctuations in economic aggregates are due to technology shocks.

documented a key role for productivity shocks in mimicking the majority of variations in economic aggregates such as output, consumption, hours worked, real wages, and average productivity. Their main findings are that (i) investment is almost three times more volatile than output; (ii) nondurable consumption is less volatile than output; (iii) hours worked and output are positively correlated; (iv) almost all macroeconomic variables are positively correlated with output; and (v) movements in macroeconomic variables show persistency (Rebelo, 2005).

Kydland and Prescott's (1982) results are supported by Cooley and Prescott (1995), Prescott (1986), and King and Rebelo (1999). However, the assertion that supply-side shocks are the main sources of economic fluctuations are challenged by Keynesian economists. The Keynesian school argues that economic fluctuations are a result of demand-side shocks (Gali and Rabanal, 2005). For example, Gali's (1999) empirical study, which is based on an SVAR model estimation with short-term and long-term restrictions, shows that conditional on a technology shock, there exists a negative correlation between hours worked and productivity and that the contribution of productivity shocks in explaining variations in economic aggregates are quite weak. Gali and Rabanal (2004) argue that the effects of productivity shocks on economic aggregates are contingent upon the degree of price flexibility and the systematic response of monetary policy to the shock. They argue that if the central bank accommodates the productivity shock by further reducing the interest rate; then, only hours worked will increase. In short, they argue that a mere supply-side shock cannot explain the observed data. Subsequent work, such as Gali and Rabanal (2005) and Basu, Fernald, and Kimball (2006), supports these findings.<sup>2</sup>

Baxter and King (1991) also highlight the importance of the demand-side shocks. Like Solow (1957), they observe that factor input growth is insufficient to explain the growth in output, and argue that positive externalities and increasing returns to scale resolve this puzzle. They show that a model with increasing returns to scale, which is driven by measured demand-side shocks, mimics the actual fluctuations of business cycles in the U.S. They also find that when preference shocks are combined with productivity shocks, both the increasing returns and the constant returns models show data consistent weak correlation between hours-worked and wages.

Using standard RBC models driven by demand shocks, Wen (2007) attempts to establish a causal relationship between consumption and output<sup>3</sup> and between output and investment by assuming that: (i) employment and output cannot respond to demand shocks immediately; and (ii) investment cannot immediately respond to demand shocks. Wen (2007) surprisingly finds that consumption growth negatively causes output growth and also that output growth negatively causes investment growth. Wen (2007) mitigates this problem by adding to the model capacity utilization and mild production externalities, and shows that output growth "Granger causes investment", but fails to establish the result that consumption "Granger causes output". Nakamura (2009), while questioning the importance of productivity shocks, shows that monetary, cost-push, and preference shocks are also capable of replicating business cycle fluctuations.

Bencivenga (1992) assumes no intertemporal substitution and full depreciation of the capital stock, and documents that preference shocks are an important source of variation in economic aggregates<sup>4</sup> In particular, she finds that hours are more volatile than productivity and that consumption is more volatile than output. She also finds a weak negative correlation between working hours and productivity.

In this paper, we develop an RBC model with intangible capital to investigate the effect of preference shock (such as preference shocks to leisure demand) on economic aggregates. To the best of our knowledge, our work is unique in analyzing the effect of demand shocks in an RBC model with intangible capital. We introduce intangible capital in our model as a third input in the production function, along with labor and physical capital. We assume throughout that, just like any other good, intangible capital is produced by means of labor and capital. Limited by multiple constraints, we assume throughout that firms always face a trade-off between current

production and future production whenever they increase the production of intangible capital. Allocating more labor and capital for current production means higher output and higher profits in the current time period. In contrast, if the firm chooses to invest in the creation of intangible capital, it will earn more profit in the future but at the cost of low output and low profit in the current time period.

Our model is a standard RBC model with the exception of the inclusion of intangible capital in the production function. We assume no government and that all agents operate in competitive markets. To simplify our algebra and given the main focus of our paper, we assume that all agents are homogenous, which helps us to develop our model using a single representative household and a single representative firm. The inclusion of intangible capital in the model is expected to take into account crowding out effects, which generally occur in the event of demand-side shocks such as changes in government expenditure and exogenous shifts in consumption. An increase in aggregate demand normally increases the real rate of interest, which—by crowding out private investment—lessens the impact of the shock on key economic aggregates. Many authors in the recent past attempted to overcome this problem by assuming habit persistent in consumption and increasing returns to scale in production (see, for example, Baxter and King, 1991 and Benhabib & Wen, 2004). We attempted in this paper to highlight the role of intangible capital in materializing the demand-side shock effects on real variables without appealing to consumption persistence and increasing returns to scale assumptions.

Using our intangible capital augmented RBC model, we study impulse response functions of macroeconomic aggregates to a preference shock. We find that demand-side shocks are important because they can cause variations in macroeconomic aggregates. In particular, we find that a preference shock causes variations in consumption that leads to variations both in output and investments. The findings of our study are consistent with postwar U.S. data.

The rest of the paper is organized as follows. The second section discusses the model and its solution and calibration. The third section discusses results and the final section concludes.

## 2. Model

In this section, we develop our model, which is related to that of Malik, Ali, and Khalid. (2014). The novelty of our model is in the inclusion of intangible capital in the production function. We assume throughout that the economy comprises two types of agents: households and firms. We further assume that all agents are homogenous and operate in a competitive environment. As is the case in many standard RBC models, we assume no government and that agents form expectations rationally.

### 2.1. Households

The economy is populated by a “representative household”; all households are identical, both ex ante and ex post. We assume that agents live infinitely and that a representative household wishes to maximize the following sum of present and future discounted utility, derived from the consumption of good and leisure:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - N_t, B_t) \quad (1)$$

where  $\beta$  is a discount rate,  $C_t$  measures current consumption,  $N_t$  measures labor supply and  $B_t$  measures preferences shocks. We assume that preferences shock follows a first-order autoregressive process with an *iid* error term ( $\varepsilon_{bt}$ ) and inertial coefficient  $0 < \rho_b < 1$ :

$$\ln B_t = \rho_b \ln B_{t-1} + \varepsilon_{bt} \quad (2)$$

To proceed further and to derive results through model calibrations, we assume that each labor endowed with one unit of labor supply and her preferences can be mapped with the following functional form of the utility function:

$$U(C_t, N_t) = \ln C_t + B_t \eta (1 - N_t) \tag{3}$$

where the parameter  $\eta$  is the relative weight of leisure ( $L_t = 1 - N_t$ ) in the consumer utility function. We may use equation (3) to show that  $-B_t \eta C_t$  is the marginal rate of substitution between consumption and leisure.<sup>5</sup> In each period, the representative household has three sources of income: (i) labor income ( $w_t N_t$ ) after supplying  $N_t$  number of hours at the real wage rate  $w_t$ ; (ii) rental income ( $r_t K_t$ ), where  $r_t$  is rental rate and  $K_t$  is physical capital; and (iii) profit income  $\pi_t$ . In each period the household consumes ( $C_t$ ) and invests ( $I_t$ ) in physical capital. Using the above information, we may write the representative household budget constraint as:

$$C_t + I_t = w_t N_t + r_t K_t + \pi_t \tag{4}$$

We further assume that investment  $I_t$  augments the capital stock over the time as follows:

$$K_{t+1} = I_t + (1 - \delta) K_t \tag{5}$$

where  $\delta \in (0, 1)$  is the rate of depreciation of physical capital. To rule out Ponzi schemes, we assume a suitable transversality condition.

Equations (4) and (5) can be combined to yield a single budget constraint:

$$C_t + K_{t+1} = w_t N_t + r_t K_t + (1 - \delta) K_t + \pi_t \tag{6}$$

Using equations (3) and (6) the Bellman equation for the household utility maximization problem can be written as follows<sup>6</sup>:

$$V(k) = \max_{c, n, k'} E[\ln c + B \eta (1 - N) + \theta EV(k')] \\ \text{s.t.} \\ C + K' = wN + rK + (1 - \delta)K + \pi$$

Which yields the following Euler equations commonly known as first-order conditions of utility maximization when the household chooses  $C_t, N_t$ , and  $K_{t+1}$ , :

$$\frac{1}{C} = \lambda \tag{7}$$

$$\frac{B \eta}{1 - N} = w \lambda \tag{8}$$

$$\theta EV(k') = \lambda \tag{9}$$

$$V_k = (1 + r - \delta) \lambda \tag{10}$$

We can combine equations (7) to (10) to yield the following equations:

$$w = \frac{CB \eta}{1 - N} \tag{11}$$

$$1 = \theta E \left\{ \frac{C}{C'} (r' + (1 - \delta)) \right\} \tag{12}$$

Equation (11) and (12) can be solved to determine the time path of consumption and labor supply (or leisure) which are subject to shift as  $B, \delta$ , and  $\theta$  changes.

## 2.2. Firms

We assume that to produce the final good ( $Y_t$ ) the representative firm hires labor ( $N_t$ ), rents physical capital ( $K_t$ ), uses intangible capital ( $Z_t$ ) and operates under conditions of perfect competition. The firm's production function is defined as follows, where we assume constant returns to scale technology:

$$Y_t = e^{\ln \theta_t} (A_t s_{tN} N_t)^\phi (s_{tK} K_t)^{1-\phi-\tau} Z_t^\tau \quad (13)$$

In equation (13) intangible capital is added as a third factor of production that is produced with the help of labor and physical capital. Accordingly,  $s_N$  and  $s_K$  denote respectively the fraction of labor and physical capital which the firm allocates to produce the final good ( $Y_t$ ). The remainder of the labor and physical capital ( $1 - s_N$ ) and ( $1 - s_K$ ), respectively, is used by the firm to produce intangible capital ( $Z_t$ ). In equation (13) we also model two shocks: a permanent technology shock ( $A_t$ ) and a transitory productivity shock ( $\theta_t$ ). The dynamics of both these shocks are defined by equations (14) and (15) respectively, where parameter  $\rho_a$  and  $\rho_\theta$  measure persistent in the shocks and the parameter  $v_a$  measures a drift process:

$$\ln A_t = v_a + \rho_a \ln A_{t-1} + \varepsilon_{at} \quad (14)$$

$$\ln \theta_{t+1} = \rho_\theta \ln \theta_t + \varepsilon_{\theta,t+1} \quad (15)$$

The stock of intangible capital available in period  $t + 1$  depends on the amounts of labor and physical capital devoted to the production of intangible capital in the current period as well as the stock of intangible capital produced in period  $t - 1$  for period  $t$  ( $Z_t$ ):

$$Z_{t+1} = \left[ (A_t (1 - s_{tN}) N_t)^\mu ((1 - s_{tK}) K_t)^{1-\mu} \right]^{1-\omega} Z_t^\omega \quad (16)$$

Following Bankard's (2000) organizational forgetting idea, we assume that knowledge depreciates with time as the economic environment goes through various transformations. The parameter  $\omega \in (0, 1)$  captures the contribution of past intangible capital, which decays the further back in time it was created.  $\omega = 1$  implies that intangible capital is constant over time. In contrast,  $\omega = 0$  implies that intangible capital decays fully in the current period and makes no contribution to intangible capital in future periods. The productivity shock  $A_t$ , which appears both in equation (13) and equation (16), ensures a balanced growth path where increases in labor productivity over time occur in both the final good and intangible capital good sectors. It can be shown that  $\mu(1 - \omega)$  measures the elasticity of labor hours that are used in the current period to create intangible capital with respect to intangible capital in the next period  $\left( \frac{d \ln Z_{t+1}}{d \ln N_t} \right)$ . In the case of  $\mu = 1$ , physical capital is not used in the creation of intangible capital. On the other hand,  $\mu = 0$  implies that the firm allocates only labor to the production of the final good ( $Y_t$ ).

## 2.3. Normalization

In order to solve the model for the steady-state and to study the optimum choices of representative agents, we normalize all the variables of the model to a permanent technology shock ( $A_t$ ) such that the transformed system does not exhibit growth.<sup>7</sup> Our transformed variable is defined by the following equations, where a tilde is placed over each of the normalized variables<sup>8</sup>:

$$\tilde{C} + a' \tilde{K}' = w \tilde{N} + r \tilde{K} + (1 - \delta) \tilde{K} + \tilde{\pi} \quad (17)$$

$$\tilde{I} = a' \tilde{K}' (1 - \delta) \tilde{K} \quad (18)$$

$$a' = \beta E \left\{ \frac{\tilde{C}}{\tilde{C}'} (r' + (1 - \delta)) \right\} \quad (19)$$

$$\tilde{Y} = e^{\ln \theta} (s_N \tilde{N})^\phi (s_K \tilde{K})^{1-\phi-\tau} \tilde{Z}^\tau \quad (20)$$

$$\tilde{Z}'\alpha' = \left[ \left( (1 - s_N)\tilde{N} \right)^\mu \left( (1 - s_K)\tilde{K} \right)^{1-\mu} \right]^\omega \tilde{Z}^\omega \quad (21)$$

#### 2.4. Firm optimization

Facing two constraints in each period (the production function for output in equation (13) and the production function for intangible capital equation (16)), the firm maximizes the present value of its real profits. The firm's optimization problem can be written as:

$$V(Z) = \underset{N, K, s_N, s_K, Z'}{\text{Max}} E \left[ U_c \left[ (s_N\tilde{N})^\phi (s_K\tilde{K})^{1-\phi-\tau} \tilde{Z}^\omega - w\tilde{N} - r\tilde{K} \right] + \theta V(Z') \right] \\ + \lambda \left[ \left[ \left( (1 - s_N)\tilde{N} \right)^\mu \left( (1 - s_K)\tilde{K} \right)^{1-\mu} \right]^\omega \tilde{Z}^\omega - \tilde{Z}'\alpha' \right] \quad (22)$$

The first order optimization conditions for the above problem are:

$$\theta E(V_z') = \lambda \alpha' \quad (23)$$

$$wU_c = U_c \left[ \phi \frac{\tilde{Y}}{\tilde{N}} \right] + \lambda \left[ \frac{\mu(1-\omega)\alpha'\tilde{Z}'}{\tilde{N}} \right] \quad (24)$$

$$U_c \left[ \frac{\phi\tilde{Y}}{s_N} \right] = \left[ \frac{\lambda\mu(1-\omega)\alpha'\tilde{Z}'}{1-s_N} \right] \quad (25)$$

$$rU_c = (1-\phi-\tau) \left[ \frac{\tilde{Y}}{\tilde{K}} \right] + \lambda \left[ \frac{(1-\mu)(1-\omega)\alpha'\tilde{Z}'}{\tilde{N}} \right] \quad (26)$$

$$U_c \left[ \frac{(1-\phi-\tau)\tilde{Y}}{s_K} \right] = \left[ \frac{\lambda(1-\mu)(1-\omega)\alpha'\tilde{Z}'}{1-s_K} \right] \quad (27)$$

The envelope condition is as follows:

$$V_z = U_c \left[ \tau \frac{\tilde{Y}}{\tilde{Z}} \right] + \lambda \left[ \omega \alpha' \frac{\tilde{Z}'}{\tilde{Z}} \right] \quad (28)$$

Substituting equation (28) into equation (23) gives equation (29), which represents the marginal value of an extra unit of intangible capital:

$$\theta \left[ U_c \left[ \tau \frac{\tilde{Y}}{\tilde{Z}'} \right] + \lambda' \left[ \omega \alpha' \frac{\tilde{Z}''}{\tilde{Z}'} \right] \right] = \lambda \alpha' \quad (29)$$

We note that due to the inclusion of intangible capital in the production function, firms are not equating the marginal productivities of labor and capital to their respective factor prices. It is interesting to note here that equations (24) and (25) imply that firm allocates capital and labor to the production of intangible capital in such a way that marginal decreases in the output of the final good offset the marginal increase in intangible capital available to the firm. Formally, combining equation (26) with (24) and equation (27) and (25) results in the following equations:

$$w = \phi \frac{\tilde{Y}}{s_N \tilde{N}} = \frac{1}{s_N} MP_N \quad (30)$$

$$r = (1-\phi-\tau) \frac{\tilde{Y}}{s_K \tilde{K}} = \frac{1}{s_K} MP_K \quad (31)$$

Since  $0 < s_N < 1$  and  $0 < s_K < 1$ , it is clear from the above two equations that factor prices exceed their marginal product in the production of the final good.<sup>9</sup>

*Proposition 1: In the presence of intangible capital, factor prices exceed their respective marginal productivities.*

Equation (32) captures investment in intangible capital:

$$\tilde{I}_z = w\tilde{N}(1 - s_N) + r\tilde{K}(1 - s_K) \tag{32}$$

Combining equations (30), (31), and (32) gives:

$$\frac{\tilde{I}_z}{\tilde{Y}} = \left(\frac{1 - s_N}{s_N}\right)\phi + \left(\frac{1 - s_K}{s_K}\right)(1 - \phi - \tau) \tag{33}$$

Equation (33) implies that investment in intangible capital is increasing in output but decreasing in the shares of factors of labor and capital. The firm's profit in the presence of intangible capital is:

$$\tilde{\pi} = \tilde{Y} - w\tilde{N} - r\tilde{K} \tag{34}$$

Using equations (30), (31) and (33) in equation (34) gives us the following relationship between profit and intangible capital investment:

$$\tilde{\pi} = \tau\tilde{Y} - \tilde{I}_z \tag{35}$$

Equation (35) depicts a trade-off between the current period's profit ( $\tilde{\pi}$ ) and investment in intangible capital ( $\tilde{I}_z$ ). The above discussion gives us our second proposition.

*Proposition 2: Firms sacrifice their present profit to create intangible capital for future production.*

### 2.5. The steady state

In the steady state, all variables are invariant to time changes, i.e.  $C' = C$ ,  $Y' = Y$ ,  $K'' = K' = K$ , and  $Z'' = Z' = Z$ . We can use equation (12) to solve for the steady-state value of the real rate of interest ( $r$ ) as follows:

$$\bar{r} = \frac{a}{\theta} - (1 - \delta) \tag{36}$$

Similarly, while using equation (29) we can show that:

$$\lambda a \bar{Z} = \theta \tau \frac{U_c \tilde{Y}}{1 - \theta \omega} \tag{37}$$

and by using equation (37) in (24), (25), (26) and (27) we get the following expressions:

$$\bar{w} = \frac{\tilde{Y}}{\tilde{N}} \left[ \phi + \frac{\mu(1 - \omega)\theta\tau}{1 - \theta\omega} \right] \tag{38}$$

$$\bar{r} = \frac{\tilde{Y}}{\tilde{K}} \left[ (1 - \phi - \tau) + \frac{(1 - \mu)(1 - \omega)\theta\tau}{1 - \theta\omega} \right] \tag{39}$$

$$\frac{(1 - s_N)}{s_N} = \frac{\mu\theta\tau(1 - \omega)}{\phi(1 - \theta\omega)} \tag{40}$$

$$\frac{(1 - s_K)}{s_K} = \frac{\theta\tau(1 - \mu)(1 - \omega)}{(1 - \phi - \tau)(1 - \theta\omega)} \tag{41}$$

Equations (38) to (41) reveal that the steady-state wage rate, rent on physical capital, and income shares of both labor and capital are all independent of intangible capital. We can summarize this result as follows:

*Proposition 3: In the steady state, the wage rate ( $\bar{w}$ ), rental on physical capital ( $\bar{r}$ ), income share of labor, and income share of capital are all independent of the stock of intangible capital.*

The factor shares of labor and capital in the steady state can easily be recovered from equations (34) and (35). Plugging equations (40) and (41) into equation (33), we get the steady state intangible investment to output ratio as:

$$\frac{\tilde{I}_z}{\tilde{Y}} = \frac{\beta\tau(1-\omega)}{1-\beta\omega} \quad (42)$$

We can note in equation (42) that the ratio of investment in intangible capital to the output of final goods depends on parameters  $\tau$  and  $\omega$ , where  $\tau$  is the key parameter that measures the elasticity of the final good to intangible capital (see equation (13) above). One can note in equation (42) that there exists a proportional relationship between the intangible investment-output ratio and  $\tau$ .<sup>10</sup>

The share of labor income to total output can be derived from equation (38) as:

$$\frac{w\tilde{N}}{\tilde{Y}} = \left[ \phi + \frac{\mu(1-\omega)\beta\tau}{1-\beta\omega} \right] \quad (43)$$

Similarly, using equation (33), we can solve for the capital-output ratio as:

$$\frac{\tilde{K}}{\tilde{Y}} = \frac{1}{r} \left[ (1-\phi-\tau) + \frac{(1-\mu)(1-\omega)\beta\tau}{1-\beta\omega} \right] \quad (44)$$

Equation (44) implies that the capital to output ratio decreases as  $\mu$  (the elasticity of intangible capital to labor used in the production of intangible capital, as seen in equation (16)) increases.<sup>11</sup> However, the increase in the same parameter increases the share of labor income in total output (see equation (43)). From equation (16) we can see that the permanent technology shock is increasing in labor. In this situation, an increase in  $\mu$  further increases the ability of the shock to create more intangible capital for a given amount of labor. On the other hand, from equation (44) we can see that an increase in  $\tau$  decreases the share of physical capital but the share of the capital good in the production of the final good increases (see equation (13) above). The above discussion leads us to our next proposition:

*Proposition 4: Investment in intangible capital is of higher value to the producer than investment in physical capital.*

Finally, we can write the following equation for intangible capital in the steady state:

$$\tilde{Z} = \left[ \left( (1-s_N)\tilde{N} \right)^\mu \left( (1-s_K)\tilde{K} \right)^{1-\mu} \right] \alpha^{1/1-\omega} \quad (45)$$

We can use equation (45) to eliminate  $\tilde{Z}$  in the production function (see equation (20) above). Since the capital-output ratio is already determined; equation (39) can be used to solve for the output of the final good in the steady state. Once the output is determined, capital can be calculated. Given the steady-state output, wage can be determined using equation (43). Similarly, consumption can be estimated from the goods market equilibrium condition ( $Y_t = C_t + I_t$ ). Profit in the steady state can be estimated using equation (34). Since the steady state values of capital, labor, and income shares of the factor of primary inputs are already determined, the stock of intangible capital and investment in intangible capital can be determined while using equations (45) and (32) respectively.

## 2.6. Stochastic model

We employ Christiano's (2002) method of undetermined coefficients to solve our dynamic model. We can use the two Euler equations from the household problem to study the time path of key variables in the event of preference shocks. The system of equations that describe the problem of the firm can be further simplified by using equation (26) in equations (24), (25), (26) and (29). This helps us to reduce the model into four equations that can be solved simultaneously for four endogenous variables: capital, intangible capital, the income share of labor, and the income share of capital. The relationship between the income share of capital and labor is as follows:

$$s_N = \frac{\phi(1-\mu)s_K}{\mu(1-\phi-\tau)(1-s_K) + \phi(1-\mu)s_K} \quad (46)$$

In equation (46) we note that the income share of labor ( $s_N$ ) is non-linearly related to the income share of capital ( $s_K$ ). Since  $s_N$  is known, the output of the final good can be written as follows:

$$Y = f(K, N, s_N, s_K, Z) \quad (47)$$

Likewise, the real wage rate ( $w$ ) can be determined as follows:

$$w = \phi \frac{\tilde{Y}}{s_N \tilde{N}} \quad (48)$$

The above equation suggests that the output of the final good depends both on labor as well as the income share of labor:  $w = f(Y, N, s_N)$ . We can also determine the rental rate ( $r$ ) by using equation (44):

$$r = \frac{\tilde{Y}}{\tilde{K}} \left[ (1-\phi-\tau) + \frac{\phi(1-\mu)(1-s_N)}{\mu s_N} \right] \quad (49)$$

Equation (49) suggests that the rental rate of physical capital is a function of output, capital, and the factor share of labor:  $r = f(Y, K, s_N)$ .

Consumption in a dynamic equilibrium can be determined either using the budget constraint or goods market clearing condition:

$$\tilde{C} = \tilde{Y} - \alpha' \tilde{K}' + (1-\delta)\tilde{K} \quad (50)$$

To summarize the above discussion, we can show that the dynamic system as laid out in our model consists of four equations that could be solved using the feedback part of the stochastic model.<sup>12</sup> Two equations are based on the household problem and the other two are based on the firm problem:

$$\tilde{w} = \frac{\tilde{C} B \eta}{1 - \tilde{N}} \quad (51)$$

$$\alpha' = \theta E \left\{ \frac{\tilde{C}}{\tilde{C}'} (r' + (1-\delta)) \right\} \quad (52)$$

$$\tilde{Z}' \alpha' = \left[ \left( (1-s_N) \tilde{N} \right)^\mu \left( (1-s_K) \tilde{K} \right)^{1-\mu} \right]^\omega \tilde{Z}^\omega \quad (53)$$

$$\theta \left( \frac{\tilde{C}' \tilde{Y}' s_N}{\tilde{C}' \tilde{Y}' (1-s_N)} \right) \left[ \frac{\mu(1-\omega)\tau}{\phi} + \omega \frac{(1-s'_N)}{s'_N} \right] = 1 \quad (54)$$

Equations (51) to (54) are the equilibrium conditions that can be used in our calibration exercise in the next section.

### 3. Model calibration

#### 3.1. Choice of parameter values

We calibrate eleven parameters as given on Table 1:  $\theta$ ,  $\eta$ ,  $\delta$ ,  $\rho_b$ ,  $\phi$ ,  $\tau$ ,  $\mu$ ,  $\omega$ ,  $\rho_a$ ,  $\rho_\theta$  and  $v_a$ . Since this is an extension of a standard RBC model, some of the parameters are easy to pin down, for instance, the discount rate is assumed to be  $\theta = 0.99$ , which is consistent with a 1% nominal rate of interest. Similarly, we assume a capital depreciation rate ( $\delta$ ) of 0.025, these values are identical to Favilukis and Lin (2013). The elasticity of labor ( $\phi$ ) is set at 0.535 and the elasticity of capital ( $1 - \phi - \tau$ ) is assumed to be 0.295. These parameter values are standard in the literature (see, for example, Plosser, 1989). The inclusion of intangible capital necessitates a choice of variables slightly different from the standard RBC model. For example, McGrattan and Prescott (2010) assume an income share of the capital of 0.26, which is on the lower side. The household intertemporal condition is used to pin down the value of utility parameter ( $\eta = 1.85$ ).

Since this model includes intangible capital, three parameters are of special importance:  $\tau$ ,  $\mu$ , and  $\omega$ . We alter the values of these parameters in between the maximum and minimum that are used in the literature to see how it affects the overall performance of the model. We also analyze how these changes affect the factor share allocation of labor and capital to the output production and creation of intangible capital.

The value of  $\tau$  used in the paper is 0.173. The minimum value is 0.076, which is used McGrattan and Prescott (2010). This value results in a higher steady state value for both labor and capital shares allocated to production. However, a lower value of  $\tau$  results in an investment to output ratio of approximately 32%, which is very high. The ratio of intangible investment to tangible investment is 23%, which is almost 50% less than what McGrattan and Prescott (2010) report. We have chosen 0.33 as the maximum value of  $\tau$ . The higher value of  $\tau$  results in lower labor and capital resources diverted towards output production and more used to produce intangible capital. A higher value of  $\tau$  brings the investment to output ratio to less than 14%, which means that consumption is more than 85% of output. This higher value also results in a very high ratio of investment in intangible capital to physical capital and output, which does not seem to be consistent with expectations.

The second important parameter is  $\mu$ . We have used a value that is close to the maximum value available in the literature. We have selected a minimum value of 0.4. The minimum value of  $\mu$  increases the labor share in production and decreases the capital share, as expected. Less labor is used in intangible capital creation and more capital is engaged in the creation of intangible capital. The minimum value of  $\mu$  also increases the investment to output ratio to 30%, which is very high in a closed economy model with no government. We have selected a median value of 0.62, which results in a higher labor share in production. The value of this parameter is very sensitive to the contribution of labor in the production and creation of intangible capital. The median value can only increase the investment to output ratio to 28%, which is still very high. The intangible investment to physical investment ratio is around 0.60. A higher value for  $\mu$  keeps the steady-state ratios close to the data.

The last parameter is  $\omega$ . If  $\omega = 1$ , this means that intangible capital is constant over time and a value of zero implies that current intangible capital does not contribute to produce intangible capital in the next period. We picked a minimum value of 0.30 and a maximum

Table 1. Choice of parameters value											
Parameters	$\theta$	$\eta$	$\delta$	$\phi$	$\tau$	$\mu$	$\omega$	$\rho_b$	$\rho_a$	$\rho_\theta$	$v_a$
Values	0.99	1.85	0.025	0.535	0.173	0.85	0.592	0.9767	0.5	0.95	0.0034

value of 0.90. A minimum value does significantly affect the capital and labor share allocation. It basically implies that current intangible capital is not contributing much to future creation. This number does not alter the steady-state ratios, however, the ratio of intangible to tangible capital increases. A very high value of  $\omega$  increases both the share allocation however, the increase is not more than 3%, and slightly increases the investment to output ratio. Overall, changes in the values of  $\omega$  are not very sensitive to the steady-state ratios.

Finally, we choose two steady-state ratios that are important for our model: the intangible investment to output ratio and the investment in intangible capital to investment in physical capital ratio. In the existing literature, a number of different values are used. For example, see McGrattan and Prescott (2010), Corrado, Hulten, and Sichel (2006) and Hou and Johri (2018). We assume an intangible investment to output ratio of 0.16. The investment in intangible capital to investment in physical capital ratio is assumed to be 0.68, which is higher than the ratio of 0.42 assumed by McGrattan and Prescott (2010), but lower than the value assumed by Corrado et al. (2006) and Hou and Johri (2018).

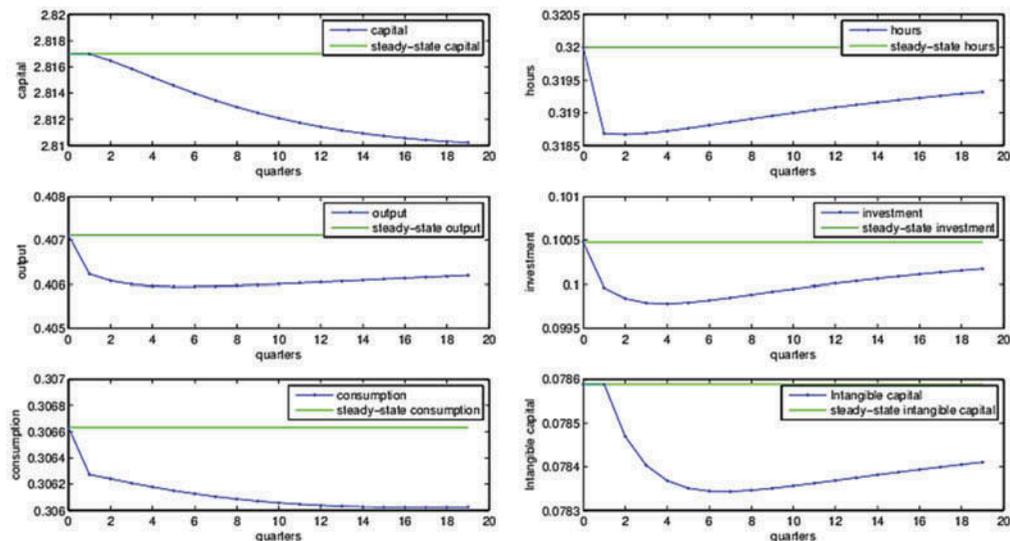
### 3.2. Dynamic effects of preferences shocks

In this section, we study the impulse responses of output, consumption, investment, intangible capital, and intangible investment to a preference shock ( $B_t$ ). From equation (1) we note that a preference shock is expected to increase leisure and lower consumption, due to the inherent trade-off between them. It is expected that—as consumption decreases—output decreases, which in turn results in lower physical as well as intangible capital. It is also expected that the intra-temporal preference shock produces co-movement between consumption and output, and between output and employment, as observed in real-world data. Under the RBC model, the characteristic equation of labor supply captures the difficulty for consumption and employment to move together unless technology changes at the same time. Technically, an increase in  $B_t$  functionally plays a similar role as changes in technology. What is needed to obtain co-movement between consumption and hours worked is for either the labor demand or the supply curves to shift for a reason other than a pure wealth effect. An increase in pure wealth increases consumption but lower hours worked. The preference shock tends to change hours worked for a reason other than a wealth effect. Closely related to our assertion, Bencivenga (1992) argues that preference shocks may be interpreted as resulting from shocks to household production or from changes in relative prices.

The impulse responses of normalized variables to a preference shock are shown in Figure 1m where we have generated the time path of key economic aggregates by giving a one standard deviation shock to leisure ( $B_t$ ). As expected, the increase in leisure causes a decrease in consumption. The fall in consumption is due to the marginal rate of substitution between consumption and leisure, as noted in Section 2. The decrease in consumption led to a decrease in output. This is an important result as is observed in the US data (see, for example, Wen, 2007). Wen (2007) attempted to track the co-movement between consumption and output (consumption “Granger” causes output) by giving a demand-shock but concludes that business cycle theory remains behind business cycle measurement. In other words, changes in productivity are the real cause of fluctuations in economic aggregates, rather than demand shocks. In contrast, our results show that in the presence of intangible capital, changes in consumption do cause changes in output.

The fall in output has a negative impact on the employment of both labor and physical capital, as can be seen in Figure 1. Likewise, we note that hours and capital devoted to the creation of intangible capital also decrease. The co-movement between output and physical investment is also observed in post-war U.S. data. It is interesting to note here that our model with intangible capital mimic these results in the event of preference shocks, which a standard RBC model is expected to replicate contingent on productivity shocks but not on demand shocks (Wen, 2007).

**Figure 1. Impulse response function to preference shocks-normalized variables.**



The change in output is probably due to factor price movements in the event of the shock. From equations (30) and (31) we note that factor prices are higher than their respective marginal productivities, which entails a less than proportional decrease in input prices in the event of fall in output. Firms are therefore left with no choice but to decrease the output of final goods and investment in intangible capital.

To summarize our above discussion, we may conclude that the study of changes in consumption is important because it predicts changes in output, which in turn predict changes in investment. This is consistent with the conclusion of Cochrane (1994).

#### 4. Conclusion

In this paper, we develop and simulate an RBC model that includes intangible capital as a third factor of production. We assume that in each period, the firm chooses to devote a certain fraction of labor and physical capital to the production of intangible capital, which becomes an input in the production of final good in the forthcoming period. However, the allocation of some resources to the production of intangible capital negatively affects current production and profit. We study the time path of consumption, output, physical capital, and intangible investment in the event of preference shock.

In the development of our model, we learn that due to the inclusion of intangible capital a perfectly competitive firm paying input prices more than their respective productivities. Similarly, in the steady state, we learn that the wage rate, rental on physical capital, income shares of both labor and capital are all independent of intangible capital. Our model calibrations indicate that demand-side shocks such as reference shock causes an increase in leisure but a decrease in consumption. The decrease in consumption causes a decrease in output, hours worked, and capital formation. These results are very similar to those depicted in post-war USA data, which reveals that changes in consumption cause changes in output and that changes in output cause changes in investment. The novelty of our result is that we mimic these results by calibrating the RBC through demand-side shocks, which goes against the prediction of standard RBC models that major fluctuations in economic aggregates are due to technology shocks.

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### Notes

1. See Plosser (1989) for a comprehensive discussion of RBC models and theory.
2. Plosser (1989, p. 57) argued that it is not always clear that which shock should be classified as demand-side shock and which one is supply-side shock since most of the shocks generally affect demand and supply-side of the model at the same time.
3. Cochrane (1994) argues that since changes in consumption predict changes in output, consumption shocks are important to study business cycles.
4. Bencivenga (1992) argues that preference shocks may be interpreted as resulting from shocks to household production or from changes in relative prices.
5. From (3) it can be shown that  $\frac{dC_t}{d\tau} = -B_t \eta C_t < 0$  i.e. to increase one unit of leisure the household is willing to forgo an amount  $B_t \eta C_t$  in consumption. The reader may note here that in case of preference shocks, the consumer is willing to sacrifice more consumption to get one unit of extra leisure.
6. Where  $K' = K_{t+1}$ .
7. Normalization of the variables in RBC models is done for reasons such as (i) the transformed system does not exhibit growth, (ii) steady state should be computed in normalized variables since steady state assume no occurrence of shock, and (iii) simulate the deterministic dynamic system (Malik. et al., 2014, page 35).
8. To normalize the variables we divide each equation by  $A_t$  and define  $\bar{C} = \frac{C_t}{A_t}$  and  $\frac{C_{t+1}}{A_{t+1}} = \frac{C_{t+1} A_{t+1}}{A_{t+1} A_t} = \bar{C}' \alpha'$ , where  $\alpha' = \frac{A_{t+1}}{A_t}$ .
9. Labor and capital shares act as a time-varying wedge between factor prices and marginal products.
10. From (42) it can be shown that  $\frac{d \ln(I_t/Y)}{d \ln \tau} = 1$ .
11. From (46) we can determine the capital share in total output as follows:

$$\frac{rK}{Y} = \left[ (1 - \phi - \tau) + \frac{(1-\mu)(1-\omega)\theta\tau}{1-\theta\omega} \right].$$

12. The feedback part characterizes the impact of the endogenous state variables on the current period endogenous variables. For further reference see Christiano (2002).

### References

- Basu, S., Fernald, J., & Kimball, M. (2006). Are technology improvements contractionary? *American Economic Review*, 96(5), 1418–1448. doi:10.1257/aer.96.5.1418
- Baxter, M., & Robert, G. (1991). *Productive externalities and business cycles*(Discussion Paper/Institute for Empirical Macroeconomics 53). Federal Reserve Bank of Minneapolis.
- Bencivenga, V. (1992). An econometric study of hours and output variation with preference shocks. *International Economic Review*, 33(2), 449–471. doi:10.2307/2526904
- Benhabib, J., & Wen, Y. (2004). Indeterminacy, aggregate demand, and the real business cycle. *Journal of Monetary Economics*, 51(3), 503–530. doi:10.1016/j.jmoneco.2003.08.011
- Benkard, L. (2000). Learning and forgetting: The dynamics of aircraft production. *American Economic Review*, 90(4), 1034–1054. doi:10.1257/aer.90.4.1034
- Christiano, L. (2002). Solving dynamic equilibrium models by a method of undetermined coefficients. *Computational Economics*, 20(1–2), 21–55. doi:10.1023/A:1020534927853
- Cochrane, J. (1994). Shocks. In: *Carnegie-Rochester Conference Series on Public Policy*. December 41, (pp. 295–364). North-Holland.
- Cooley, F., & Prescott, E. (1995). Economic growth and business cycles. In T. F. Cooley (Ed.), *Frontiers of business cycle research* (pp. 1–38). Princeton: Princeton University Press.
- Corrado, C., Hulten, C., & Sichel, D. (2006). *Intangible capital and economic growth*(Finance and Economics Discussion Series 2006-24). Board of Governors of the Federal Reserve System.
- Favilukis, J., & Lin, X. (2013). Long run productivity risk and aggregate investment. *Journal of Monetary Economics*, Elsevier, 60(6), 737–751. doi:10.1016/j.jmoneco.2013.05.002
- Gali, J. (1999). Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? *American Economic Review*, 89(1), 249–271. doi:10.1257/aer.89.1.249
- Gali, J., & Rabanal, P. (2004). Technology shocks and aggregate fluctuations: How well does the real business cycle model fit the postwar U.S. data? In M. Gertler & K. Rogoff (Eds.), *NBER macroeconomics annual 2004* (pp. 225–288). Cambridge: MIT press.
- Hou, K, & Johri, A. (2018). Intangible capital, the labor wedge and the volatility of corporate profits. *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, 29, 216–234.
- Gali, J., & Rabanal, P. (2005). Technology shocks and aggregate fluctuations: How well does the real business cycle model fit postwar U.S. data?. In *Nber macroeconomics annual 2004*, Vol. 19 (pp. 225–288). National Bureau of Economic Research, Inc.
- King, R., & Rebelo, S. (1999). Resuscitating real business cycles. In J. B. Taylor & M. Woodford (Eds.), *Handbook of macroeconomics* (pp. chap. 14, 927–1007). Amsterdam: Elsevier.
- Kydland, E., & Prescott, E. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6), 1345–1370. doi:10.2307/1913386

- Malik, K., Ali, S., & Khalid, A. (2014). Intangible capital in a real business cycle model. *Economic Modelling*, 39 (C), 32–48. doi:10.1016/j.econmod.2014.02.018
- McGrattan, R., & Prescott, E. (2010). Unmeasured investment and the puzzling US boom in the 1990s. *American Economic Journal: Macroeconomics*, 2(4), 88–123.
- Nakamura, E. (2009). Deconstructing the success of real business cycles. *Economic Inquiry*, 47(4), 739–754. doi:10.1111/j.1465-7295.2008.00141.x
- Plosser, C. (1989). Understanding real business cycles. *The Journal of Economic Perspectives*, 3(3), 51–77. doi:10.1257/jep.3.3.51
- Prescott, E. (1986). Theory ahead of business-cycle measurement. In: *Carnegie-Rochester Conference Series on Public Policy*, Minneapolis, Vol. 25 (pp. 11–44).
- Rebelo, S. (2005). Real business cycle models: Past, present, and future (NBER Working Paper 11401). Retrieved from <http://www.nber.org/papers/w11401>
- Solow, M. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics*, 39(3), 312–320. doi:10.2307/1926047
- Wen, Y. (2007). Granger causality and equilibrium business cycle theory. *Federal Reserve Bank of St. Louis*, 89(3), 195–205.



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