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*Corresponding author: Nixon S. Chekenya, Midland State University, Zimbabwe
E-mail: chekenyan@gmail.com

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FINANCIAL ECONOMICS | RESEARCH ARTICLE

Maximum likelihood estimation of stock volatility using jump-diffusion models

Nixon S. Chekenya*

Abstract: We investigate whether there are systematic jumps in stock prices using the Brownian motion approach and Poisson processes to test diffusion and jump risk, respectively, on Johannesburg Stock Exchange and whether these jumps cause asset return volatility. Using stock market data from June 2002 to September 2016, we hypothesize that stocks with high positive (negative) slopes are more likely to have large positive (negative) jumps in the future. As such, we expect to observe salient properties of volatility on listed stocks. We also conjecture that it is valid to use maximum likelihood procedures in estimating jumps in stocks.

Subjects: Financial Mathematics; Mathematical Finance; Quantitative Finance

Keywords: Merton jump diffusion model; Black scholes volatility (IV) curves; Weiner process; maximum likelihood estimation
Subjects: C12; C18

1. Introduction

We explore asset return volatility in actively traded stocks. Specifically, we investigate whether there are systematic jumps in stock returns using the Brownian motion and Poisson processes to test diffusion and jump risk, respectively, on the Johannesburg Stock Exchange and whether these jumps cause asset return volatility. Our hypothesis is that stocks with high positive (negative) slopes are more likely to have large positive (negative) jumps in the future.

Nixon S. Chekenya

ABOUT THE AUTHOR

Nixon S. Chekenya is an investment analyst at Steward Bank Limited and a research fellow at the Institute for New Economic Thinking at the Oxford Martin School, University of Oxford. Nixon has 1 year investment banking experience. He completed his honors degrees in mathematics, economics and actuarial science from Ghent University, Midlands State University. Nixon casually teaches in the economics department at Midlands State University. He studies under the Institute and Faculty of Actuaries, United Kingdom and the CFA Institute. His thesis "Selection Bias and the Performance of Black Fund Managers in South Africa", investigates the role of diversity in the investment industry. He has been an invited speaker at academic conferences in South Africa, Kenya, Nigeria, Belgium, Germany and the United States among other places.

PUBLIC INTEREST STATEMENT

This paper investigates whether there are systematic jumps in stock prices using the Brownian motion approach and Poisson processes to test diffusion and jump risk, respectively, on the Johannesburg Stock Exchange and whether these jumps cause asset return volatility. We develop and test sufficient conditions for a model when stock returns follow a jump-diffusion process. Based on daily stock price data, our results are that the stock market returns contain a jump component, although its magnitude is small. We measure the jump component over larger intervals in time (monthly interval) and find that the weekends and holidays tend to cover up the small jump component. The economic intuition is that jump risk is not diversifiable.

Asset return volatility is central to finance. High-frequency data exhibit fat-tailed distributions (excess kurtosis), skewness and volatility clustering. Excess kurtosis of stock can be attributed to jumps-sudden but infrequent movements-of large magnitude in stock prices (Campbell, Giglio, Polk, & Turley, 2018; Maheu & McCurdy, 2004; Yan, 2011). A number of studies document the presence of jump in stock returns in developed countries' contexts (Bollerslev, Todorov, & Li, 2013; Cremers, Halling, & Weinbaum, 2015; Jarrow & Rosenfeld, 1984; and Jorion, 1989). Systematic jumps in stock prices in developing countries have rarely been tested. Our study begs the question of whether there are jumps in stock prices using stock returns data from the Johannesburg Stock Exchange. Closest in spirit with our paper are those by Hanson, Westman, and Zhu (2004) which develops jump-diffusion stock-returns models and investigates the log-uniform distributed jump-amplitudes in the jump-diffusion model for fitting financial market distributions and Trautmann and Beinert (1995) whose study employs ML-techniques to estimate a Poisson type jump-diffusion model that describes the return behavior of actively traded German stocks and the DAX stock index as a proxy of aggregate wealth, respectively.

Randomness and persistence are two salient properties of stock volatility. Stocks with high positive (negative) slopes are more likely to have large positive (negative) jumps in the future. The statistical properties of stock returns have long been of curiosity to financial decision makers and academics (see Eraker, 2004). Positive kurtosis implies concave, U-shaped implied Black-Scholes volatility (IV) curves.

Diffusion processes are widely used to describe the evolution of asset returns over time. In option pricing, they enable the use of Black and Scholes-type formulae to price European options on stocks, foreign currencies, commodities, interest rates, and futures. The Jump Diffusion model is chosen in this study because it can potentially explain stock returns more accurately at the expense of making the market incomplete, since jumps in the stock prices cannot be hedged using trade securities.

One purpose of this paper is to provide answers to the question of whether there are systematic jumps in stock prices using stock returns data in developing countries, Johannesburg Stock Exchange in particular. We do this by extending Merton's (1973, 1975) jump-diffusion model and intertemporal asset pricing model.

The remainder of our paper is organized as follows. The next section generalizes the Merton Jump Diffusion (MJD) model to a developing country's case. Empirical tests for the presence of jump components in the stock returns are contained in section 3. Section 4 concludes the paper.

2. Theory

Our assumed model is similar to those contained in Jarrow and Rosenfeld (1984) and Merton (1975). It contains a finite number of assets and traders. There is one good which serves as a numeraire. The basic assumptions we make are:

(1) No transaction costs, no taxes, and frictionless markets.
(2) Competitive markets (traders are price takers)
(3) Continuous trading at equilibrium prices
(4) There are *m* risky assets whose prices satiate[1]

$$\frac{dSj}{Sj} = \alpha jdt + \ \sigma jdZj + (-\lambda jKjdt + \ \pi jdYj)\ j = 1, \ldots, m, \tag{1}$$

where $S_j(t)$ is the price of an asset *j* at time t; α_j, σ_j, λ_j, and K_j are constants; dZ_j is a Wiener process; dY_j is a Poisson process with parameter λ_j; π_j is the jump amplitude with expected value equal to K_j; and dZ_j, dY_j, and π_j are independent.

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(1) Traders have standardized opinions over $\{\alpha_j, \sigma_j, \lambda_j, K_j, j = 1, \ldots m\}$

(2) Traders' tastes are represented by a von Neumann-Morgenstern utility function which is strictly increasing and strictly concave.

Assumptions 1–3, 5, and 6 are standard in the literature (See Jarrow & Rosenfeld, 1984, Merton, 1975). Assumption number 4 is the key conjecture in our analysis.

It is fitting at this time to rewrite assumption 4 in an alternate but equivalent form which separates systematic and unsystematic risk components.

Consider the diffusion part of assumption 4,

$$dDj = \alpha jdt + \sigma jdZj, \; j = 1, \ldots, \; m. \tag{2}$$

Employing a comparable argument to Ross (1976, p.272), expression (2) implies that there exists $\{u_j, f_j, g_j, dØ, dꟺ_j\}$ $j = 1, \ldots$, such that

$$dDj = \alpha jdt + fjdØ + gjꟺdj, \tag{3a}$$

where $f^2_j + g_j^2 = \sigma_j^2$; $dØ$, $dꟺ_j$ are Wiener processes; $E(dØdꟺ_j) = 0$, $j = 1, \ldots, m$; and

$$\sum_{j=1}^{m} uj = 1, \; (x+a)^n = \sum_{j=1}^{m} uj(gjdꟺj) = 0, = \sum_{j=1}^{m} uj \; \alpha j > r. \tag{3b}$$

It is always likely to decompose a restricted number of normal arbitrary variables into a common factor, $dØ$, and error terms, $dꟺ_j$, which are normally distributed. The key property of normal returns employed is that covariance of zero implies numerical independence. This same assumption is confirmed in Fama (1973). Note that $dØ$, $dꟺ_i$ will be independent of dY_i and π_i by assumption 4. This disintegration gives $dØ$ the interpretation of being the unsystematic risk factor.

Substitution of expression (3) into (1) gives assumption 5a:

There are m risky assets whose prices satisfy

$$\frac{dSj}{Sj} = \alpha jdt + fjdØ + gjdꟺj + (-\lambda Kjdt + \pi jdYj), j = 1, \ldots, m, \tag{4}$$

where $S_J(t)$ is the price of asset j at time t; α_j, f_j, g_j, λ_j, K_j are constants; $dØ$, $dꟺ_j$ are Wiener processes; dY_j is a Poisson process with parameter λ_j; π_j is the jump amplitude with expected value equal to K_j; and $dØ$, $dꟺ_j$, dY_j, π_j are independent.

The jump component in expression (4), $(-\lambda Kjdt + \pi jdYj)$, infers that stock returns can have discontinuous ample paths. This generalizes existing models.

3. Data

The section tests for the stock returns to see if they contain jumps. If no jump component is present, then this would be consistent with the proposition of the previous deduction and the satisfaction of an instantaneous Capital Asset Pricing Model (CAPM). This section performs the following hypothesis tests:

H_0, jump risk is diversifiable

H_1, jump risk is non-diversifiable

We will survey the sample path of the stock price returns. To advance the testing procedure, note that under expression (4) the stock market returns dynamics are given by:

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$$\frac{dS}{S} = \sum_{j=1}^{m} sj\alpha jdt + \left(\sum_{j=1}^{m} sjfj\right)d\varnothing + \sum_{j=1}^{m} sj(gjd\text{Ш}j - \lambda jKjdt\pi jdYj + logVj \tag{5}$$

where S = $\sum_{j=1}^{n} mjSj$, $logVj$ ~ i.i.d. $N(\alpha, \sigma^2)$, normally distributed and models jumps in stocks.[2]

Under the null hypothesis, expression (5) reduces to

$$\frac{dS}{S} = \alpha dt + \sigma d\varnothing \tag{6}$$

where α = $\sum_{j=1}^{n} mj\alpha j$ and σ = $\sum_{j=1}^{n} mjfj$.

Under the alternative hypothesis, expression (5) reduces to:

$$\frac{dS}{S} = \alpha' dt + \sigma d\varnothing + dq \tag{7}$$

where dq = πdY denotes a Poisson process with parameter λ, π = jump amplitude with estimated value equal to K, and α' = α—λK.

To complete the method, another hypothesis is added to (7), that is, (π) has a lognormal distribution with parameters (a, b^2). This assumption is added in order that the Maximum Likelihood Estimation procedure developed in Rosenfeld can be used to estimate the parameters of Equations (6) and (7).

For ease of reference, we re-write the hypothesis to be tested:

H_0: jump risk is diversifiable,

$$\frac{dS}{S} = adt + \sigma d\varnothing; \tag{8}$$

H_1: jump risk not diversifiable,

$$\frac{dS}{S} = \alpha' dt + \sigma d\varnothing + dq; \tag{9}$$

and (π) is dispersed lognormal (a, b^2).

To properly test the null hypothesis, a likelihood ratio test can be used: A = −2(lnL_c—lnL_u), where L_c signifies the likelihood value for the reserved density function (i.e., the null hypothesis, Equation (8)) and L_u represents the likelihood function for the unconstrained density function (i.e., the alternative hypothesis, Equation (9)).[3]

We perform empirical tests on one index the index consists of daily observations for the stock market's total return index on the Johannesburg Stock Exchange from June 2002 to September 2016.

Table 1 presents estimates of parameters of the diffusion-only process for diverse observation intervals and time periods. The results suggest that the total return and volatility of the market are not constant over time. The total standard deviation of return on the market is measured by the Johannesburg Stock Exchange total return index over a 14-year period.

4. Methodology

Throughout this paper (as in Jarrow & Rosenfeld, 1984; Merton, 1976) we assume that St is the price of a financial asset whose return dynamics are given by;

$$\frac{dSt}{St-} = (\mu - \lambda\kappa)dt + \sigma dBt + [e\ J - 1]dNt \tag{10}$$

Table 1. Summary statistics

Column1	Column2
Constant drift	0.10%
Standard deviation of drift	1.00%
Jump probability	3.00%
Jump mean	2.00%
Jump standard deviation	4.00%

Table 1 presents estimates of parameters of the diffusion-only process for diverse observation intervals and time periods.

where μ is the instantaneous expected return per unit time, and σ is the instantaneous volatility per unit time. The stochastic process *Bt* is a standard Wiener process under the market measure P. The process *Nt* is a Poisson process, independent of the jump-sizes *J* and the Wiener process *Bt*, with arrival intensity λ per unit time under the measure P, so that its increments satisfy;

$$dNt = (1 \text{ with probability } \lambda dt,\ 0 \text{ with probability } 1 - \lambda dt) \tag{11}$$

The expected proportional jump size is;

$$\kappa \equiv \mathrm{EP}[\mathrm{e\ J} - \ 1] \tag{12}$$

Jumps arriving at different times are assumed to be independent of each other. A filtered probability measure space (Ω, F, {Ft}, P) is assumed where the filtration {Ft} is the natural filtration generated by the Wiener process Bt.

In the jump-diffusion model, the stock price S_t follows the random process;

$dS_t/S_t = \mu d_t + \sigma dW_t + (J\text{-}1)dN_t$. The first two terms are familiar from the Black-Scholes model: drift rate μ, volatility σ, and random walk (Wiener process) W_t The last term represents the jumps: *J* is the jump size as a multiple of stock price while *N*(*t*) is the number of jump events that have occurred up to time *t*. *N*(*t*) is assumed to follow the Poisson process;

$$P(N(t) = k) = \frac{(\lambda t)^k}{k!}\ \mathrm{E}^{-\lambda t}$$

where λt is the average number of jumps per unit time.

The jump size may follow any distribution, but a common choice is a log-normal distribution;

$$J \sim m\ exp\frac{(y^2}{2} + yN(0,1)$$

where *N (0,1)* is the standard normal distribution, *m* is the average jump size, and *v* is the volatility of jump size. The three parameters *λ, m, v* characterize the jump-diffusion model.

5. Model

We use the basic excel spreadsheet to model the effects of jump-diffusion on stock prices.[4] Our equation is as follows:

$$r_t = \alpha + \ \varepsilon_t + I_t u_t$$

where r_t is the log return, α is the mean drift, ε is the diffusion which follows a normal distribution calculated as σ*NORMSINV(RAND()), where σ (sigma) is the standard deviation of the jumps, I is equal to 0 (no jump) or 1 (presence of a jump). The value is determined by the jump probability; u_t is the value of the jump. This follows a normal distribution and is determined by E[u] + σ_u*NORMSINV(RAN()), where E[u] and σ_u are the mean and standard deviation of the jump, respectively.

Table 2. Stock price data statistics and jump estimation (Source: Author's computations)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
0					4.61	100.00	
1	0.94%	0	−1.67%	1.04%	4.62	101.04	
2	−0.24%	0	−0.29%	−0.14%	4.61	100.90	
3	−1.27%	0	0.95%	−1.17%	4.60	99.73	
4	0.02%	0	6.89%	0.12%	4.60	99.84	
5	−2.06%	0	3.46%	−1.96%	4.58	97.91	
6	−1.41%	0	0.31%	−1.31%	4.57	96.63	
7	0.87%	0	4.65%	0.97%	4.58	97.58	
8	−2.30%	0	1.50%	−2.20%	4.56	95.45	
9	−0.44%	0	3.74%	−0.34%	4.56	95.12	
10	−1.24%	0	2.49%	−1.14%	4.54	94.04	
11	−0.32%	0	−0.61%	−0.22%	4.54	93.83	
12	−0.09%	0	1.97%	0.01%	4.54	93.84	
13	1.61%	0	6.28%	1.71%	4.56	95.45	
14	0.47%	0	−2.45%	0.57%	4.56	96.00	
15	0.33%	0	0.19%	0.43%	4.57	96.42	
16	−0.04%	0	0.78%	0.06%	4.57	96.47	
17	−0.03%	0	2.30%	0.07%	4.57	96.54	
18	0.21%	0	−0.72%	0.31%	4.57	96.84	
19	0.13%	0	−1.33%	0.23%	4.58	97.07	
20	−1.18%	0	−4.81%	−1.08%	4.56	96.03	
21	1.30%	0	4.83%	1.40%	4.58	97.39	
22	−0.61%	0	3.83%	−0.51%	4.57	96.89	
23	−0.89%	0	−4.19%	−0.79%	4.57	96.13	
24	−0.43%	0	−1.39%	−0.33%	4.56	95.81	

(Continued)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
25	0.33%	0	−3.11%	0.43%	4.57	96.22	
26	0.70%	0	7.98%	0.80%	4.57	96.99	
27	−1.92%	0	−1.31%	−1.82%	4.56	95.25	
28	1.21%	0	5.57%	1.31%	4.57	96.51	
29	−1.32%	0	3.95%	−1.22%	4.56	95.34	
30	0.50%	0	6.07%	0.60%	4.56	95.91	
31	−1.20%	0	1.14%	−1.10%	4.55	94.86	
32	0.74%	1	4.52%	5.37%	4.61	100.09	100.0943
33	0.35%	0	−2.17%	0.45%	4.61	100.55	
34	−2.03%	0	2.03%	−1.93%	4.59	98.63	
35	−0.68%	0	9.19%	−0.58%	4.59	98.06	
36	0.94%	0	4.07%	1.04%	4.60	99.08	
37	−0.42%	0	−1.22%	−0.32%	4.59	98.76	
38	1.91%	0	−1.56%	2.01%	4.61	100.77	
39	−0.71%	0	2.43%	−0.61%	4.61	100.15	
40	−0.65%	0	−9.42%	−0.55%	4.60	99.61	
41	−0.22%	0	−1.01%	−0.12%	4.60	99.49	
42	−0.75%	0	−0.97%	−0.65%	4.59	98.85	
43	0.75%	0	−0.30%	0.85%	4.60	99.69	
44	2.33%	0	2.27%	2.43%	4.63	102.14	
45	−0.07%	0	1.62%	0.03%	4.63	102.17	
46	0.28%	0	7.79%	0.38%	4.63	102.56	
47	−0.66%	0	−0.03%	−0.56%	4.62	101.99	
48	−0.23%	0	−2.71%	−0.13%	4.62	101.85	
49	0.60%	0	9.75%	0.70%	4.63	102.57	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
50	0.89%	0	0.85%	0.99%	4.64	103.59	
51	1.44%	0	−0.72%	1.54%	4.66	105.20	
52	−0.01%	0	−1.15%	0.09%	4.66	105.29	
53	−1.56%	0	3.69%	−1.46%	4.64	103.76	
54	−0.36%	0	3.44%	−0.26%	4.64	103.50	
55	−1.14%	0	−0.18%	−1.04%	4.63	102.43	
56	0.14%	0	3.94%	0.24%	4.63	102.68	
57	−1.55%	0	8.55%	−1.45%	4.62	101.20	
58	1.43%	0	−7.36%	1.53%	4.63	102.76	
59	−0.35%	0	−4.24%	−0.25%	4.63	102.50	
60	0.61%	0	6.77%	0.71%	4.64	103.24	
61	−0.02%	0	4.44%	0.08%	4.64	103.32	
62	0.81%	0	3.97%	0.91%	4.65	104.27	
63	0.75%	0	3.64%	0.85%	4.66	105.15	
64	−2.60%	0	−3.75%	−2.50%	4.63	102.55	
65	0.68%	0	4.61%	0.78%	4.64	103.36	
66	−0.54%	0	1.08%	−0.44%	4.63	102.90	
67	0.19%	0	−1.24%	0.29%	4.64	103.20	
68	1.48%	0	2.21%	1.58%	4.65	104.84	
69	−1.21%	0	−0.64%	−1.11%	4.64	103.68	
70	2.07%	0	4.90%	2.17%	4.66	105.95	
71	0.29%	0	3.72%	0.39%	4.67	106.37	
72	−0.62%	0	−1.32%	−0.52%	4.66	105.81	
73	−0.67%	0	−6.91%	−0.57%	4.66	105.21	
74	1.63%	0	−3.88%	1.73%	4.67	107.04	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
75	0.14%	0	−1.70%	0.24%	4.68	107.30	
76	−1.34%	0	5.97%	−1.24%	4.66	105.97	
77	−0.63%	0	−3.70%	−0.53%	4.66	105.42	
78	1.06%	0	6.91%	1.16%	4.67	106.65	
79	−0.09%	0	−0.79%	0.01%	4.67	106.66	
80	−0.84%	0	5.31%	−0.74%	4.66	105.87	
81	0.87%	0	3.75%	0.97%	4.67	106.90	
82	−0.76%	0	0.48%	−0.66%	4.67	106.20	
83	0.52%	0	−3.06%	0.62%	4.67	106.86	
84	−1.82%	0	−3.71%	−1.72%	4.65	105.04	
85	−1.13%	0	0.17%	−1.03%	4.64	103.97	
86	−1.38%	0	7.88%	−1.28%	4.63	102.65	
87	−0.01%	0	2.77%	0.09%	4.63	102.74	
88	−0.03%	0	3.98%	0.07%	4.63	102.82	
89	−0.65%	0	5.66%	−0.55%	4.63	102.25	
90	1.49%	0	3.11%	1.59%	4.64	103.88	
91	−0.74%	0	−1.93%	−0.64%	4.64	103.22	
92	−1.29%	0	6.48%	−1.19%	4.62	102.00	
93	−0.89%	0	−0.41%	−0.79%	4.62	101.19	
94	−1.04%	0	−1.12%	−0.94%	4.61	100.24	
95	−1.80%	0	1.99%	−1.70%	4.59	98.56	
96	0.42%	0	5.81%	0.52%	4.60	99.07	
97	0.90%	0	−1.44%	1.00%	4.61	100.06	
98	−1.29%	0	−1.21%	−1.19%	4.59	98.88	
99	2.25%	0	−2.17%	2.35%	4.62	101.23	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
100	0.13%	0	0.08%	0.23%	4.62	101.47	
101	−1.08%	0	−0.46%	−0.98%	4.61	100.48	
102	1.60%	0	1.96%	1.70%	4.63	102.21	
103	−0.10%	1	4.56%	4.56%	4.67	106.97	106.9745
104	0.19%	0	6.63%	0.29%	4.68	107.29	
105	−1.60%	0	4.03%	−1.50%	4.66	105.70	
106	0.57%	0	4.90%	0.67%	4.67	106.40	
107	0.45%	0	0.71%	0.55%	4.67	106.99	
108	−0.61%	0	1.27%	−0.51%	4.67	106.44	
109	−0.10%	0	2.23%	0.00%	4.67	106.45	
110	0.49%	0	−4.42%	0.59%	4.67	107.07	
111	0.13%	0	−0.46%	0.23%	4.68	107.32	
112	−0.47%	0	−1.19%	−0.37%	4.67	106.92	
113	0.74%	0	5.13%	0.84%	4.68	107.81	
114	−1.09%	0	2.02%	−0.99%	4.67	106.75	
115	0.07%	0	2.82%	0.17%	4.67	106.93	
116	0.14%	0	4.42%	0.24%	4.67	107.19	
117	−0.84%	0	−2.71%	−0.74%	4.67	106.41	
118	−0.25%	0	−0.66%	−0.15%	4.67	106.25	
119	0.75%	0	5.23%	0.85%	4.67	107.15	
120	0.31%	0	11.25%	0.41%	4.68	107.59	
121	1.20%	0	7.98%	1.30%	4.69	108.99	
122	0.41%	0	−1.45%	0.51%	4.70	109.55	
123	0.32%	1	2.30%	2.72%	4.72	112.57	112.57
124	0.12%	0	9.92%	0.22%	4.73	112.82	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
125	−0.83%	0	−0.67%	−0.73%	4.72	111.99	
126	−0.33%	0	5.35%	−0.23%	4.72	111.74	
127	−0.03%	0	3.83%	0.07%	4.72	111.82	
128	−0.12%	0	−1.90%	−0.02%	4.72	111.79	
129	0.31%	0	−4.65%	0.41%	4.72	112.26	
130	−0.24%	0	2.88%	−0.14%	4.72	112.10	
131	1.14%	0	6.67%	1.24%	4.73	113.50	
132	0.44%	0	2.82%	0.54%	4.74	114.11	
133	1.30%	0	0.83%	1.40%	4.75	115.71	
134	1.08%	0	2.47%	1.18%	4.76	117.08	
135	0.49%	0	−3.66%	0.59%	4.77	117.77	
136	0.52%	0	−0.53%	0.62%	4.77	118.50	
137	−0.88%	0	−1.24%	−0.78%	4.77	117.58	
138	0.30%	0	2.99%	0.40%	4.77	118.05	
139	0.39%	0	3.42%	0.49%	4.78	118.63	
140	0.69%	0	−2.11%	0.79%	4.78	119.58	
141	−2.65%	0	5.90%	−2.55%	4.76	116.56	
142	−0.60%	0	6.53%	−0.50%	4.75	115.98	
143	0.90%	0	−0.19%	1.00%	4.76	117.14	
144	−1.81%	0	5.01%	−1.71%	4.75	115.16	
145	−0.99%	0	6.71%	−0.89%	4.74	114.14	
146	0.72%	0	8.05%	0.82%	4.75	115.07	
147	1.27%	0	3.45%	1.37%	4.76	116.66	
148	−1.92%	0	13.91%	−1.82%	4.74	114.55	
149	−0.75%	0	5.23%	−0.65%	4.73	113.81	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
150	−0.09%	0	5.59%	0.01%	4.73	113.82	
151	−0.92%	0	0.86%	−0.82%	4.73	112.89	
152	0.80%	0	8.61%	0.90%	4.74	113.92	
153	0.88%	0	−0.84%	0.98%	4.75	115.04	
154	−0.07%	0	−7.57%	0.03%	4.75	115.07	
155	−0.76%	1	1.19%	0.53%	4.75	115.67	115.6747
156	0.07%	0	−2.03%	0.17%	4.75	115.87	
157	0.06%	0	1.18%	0.16%	4.75	116.06	
158	0.57%	0	3.64%	0.67%	4.76	116.83	
159	1.55%	0	1.28%	1.65%	4.78	118.78	
160	−0.40%	0	−0.66%	−0.30%	4.77	118.43	
161	−0.68%	0	5.14%	−0.58%	4.77	117.75	
162	1.26%	0	3.93%	1.36%	4.78	119.36	
163	−0.50%	0	−0.46%	−0.40%	4.78	118.89	
164	−0.64%	0	5.03%	−0.54%	4.77	118.25	
165	−0.77%	0	−5.14%	−0.67%	4.77	117.46	
166	−0.98%	0	2.85%	−0.88%	4.76	116.44	
167	−1.00%	0	3.21%	−0.90%	4.75	115.40	
168	0.88%	0	3.25%	0.98%	4.76	116.53	
169	0.70%	0	−4.89%	0.80%	4.77	117.47	
170	−0.64%	0	3.08%	−0.54%	4.76	116.84	
171	1.85%	0	−1.65%	1.95%	4.78	119.14	
172	−0.27%	0	6.15%	−0.17%	4.78	118.93	
173	−1.02%	0	−2.56%	−0.92%	4.77	117.85	
174	0.39%	0	−3.06%	0.49%	4.77	118.43	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
175	0.00%	0	−0.16%	0.10%	4.78	118.55	
176	2.18%	0	1.24%	2.28%	4.80	121.28	
177	−1.12%	0	4.21%	−1.02%	4.79	120.04	
178	1.06%	0	5.18%	1.16%	4.80	121.44	
179	−0.05%	0	6.86%	0.05%	4.80	121.50	
180	2.48%	0	0.98%	2.58%	4.83	124.67	
181	−0.30%	0	7.20%	−0.20%	4.82	124.43	
182	−1.34%	0	0.66%	−1.24%	4.81	122.89	
183	−1.53%	0	−3.02%	−1.43%	4.80	121.15	
184	0.95%	0	−3.21%	1.05%	4.81	122.43	
185	1.08%	0	2.39%	1.18%	4.82	123.89	
186	−0.77%	0	4.59%	−0.67%	4.81	123.06	
187	−1.48%	0	8.92%	−1.38%	4.80	121.37	
188	1.33%	0	8.50%	1.43%	4.81	123.12	
189	0.19%	0	4.31%	0.29%	4.82	123.48	
190	−0.83%	0	9.01%	−0.73%	4.81	122.58	
191	1.32%	0	2.85%	1.42%	4.82	124.33	
192	−1.97%	0	6.24%	−1.87%	4.80	122.02	
193	0.40%	0	−0.14%	0.50%	4.81	122.63	
194	−0.39%	0	0.98%	−0.29%	4.81	122.27	
195	0.03%	0	2.33%	0.13%	4.81	122.43	
196	−0.37%	0	0.13%	−0.27%	4.80	122.10	
197	−0.82%	0	−2.06%	−0.72%	4.80	121.22	
198	−1.17%	0	4.98%	−1.07%	4.79	119.93	
199	1.78%	0	−0.08%	1.88%	4.81	122.21	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
200	2.11%	0	7.93%	2.21%	4.83	124.93	
201	−0.54%	0	−0.39%	−0.44%	4.82	124.39	
202	−0.63%	0	−0.41%	−0.53%	4.82	123.73	
203	0.31%	0	0.77%	0.41%	4.82	124.24	
204	−0.19%	0	−5.61%	−0.09%	4.82	124.14	
205	−0.26%	0	−1.61%	−0.16%	4.82	123.94	
206	−0.76%	1	−4.58%	−5.24%	4.77	117.61	117.6123
207	1.09%	0	3.25%	1.19%	4.78	119.02	
208	0.75%	0	9.19%	0.85%	4.79	120.03	
209	−0.36%	0	5.35%	−0.26%	4.79	119.72	
210	−0.37%	0	1.00%	−0.27%	4.78	119.40	
211	−0.21%	0	−3.45%	−0.11%	4.78	119.26	
212	2.27%	0	0.03%	2.37%	4.80	122.12	
213	−1.39%	0	7.67%	−1.29%	4.79	120.56	
214	0.10%	0	−0.21%	0.20%	4.79	120.79	
215	0.60%	0	7.84%	0.70%	4.80	121.65	
216	0.83%	0	7.64%	0.93%	4.81	122.78	
217	−0.01%	1	−0.64%	−0.55%	4.80	122.10	122.1049
218	−0.50%	0	6.48%	−0.40%	4.80	121.61	
219	−1.98%	0	−0.01%	−1.88%	4.78	119.35	
220	−1.86%	0	−3.40%	−1.76%	4.76	117.27	
221	−1.86%	0	0.14%	−1.76%	4.75	115.23	
222	0.12%	0	0.23%	0.22%	4.75	115.48	
223	1.42%	0	−3.16%	1.52%	4.76	117.24	
224	−0.48%	0	6.99%	−0.38%	4.76	116.80	

(*Continued*)

Table2. (*Continued*)

Time Period t	Diffusion ε	Jump?	Jump Value u	log(Return)	log(Price)	Price	Jump
225	0.78%	0	−0.29%	0.88%	4.77	117.83	
226	0.59%	0	3.70%	0.69%	4.78	118.65	
227	−0.13%	0	2.40%	−0.03%	4.78	118.61	
228	0.02%	0	3.80%	0.12%	4.78	118.75	
229	0.04%	0	1.44%	0.14%	4.78	118.91	
230	−0.97%	0	−0.17%	−0.87%	4.77	117.88	
231	0.03%	0	6.92%	0.13%	4.77	118.03	
232	0.24%	0	2.44%	0.34%	4.77	118.44	
233	−0.27%	0	1.36%	−0.17%	4.77	118.24	
234	1.37%	1	2.11%	3.58%	4.81	122.55	122.5533
235	−0.43%	0	2.58%	−0.33%	4.81	122.15	
236	1.68%	0	2.32%	1.78%	4.82	124.34	
237	1.68%	0	5.75%	1.78%	4.84	126.58	

We then enter our parameters in the excel sheet to calculate the path of the stock prices. We then click on the recalculate button to generate a new path.

6. Conclusion

This paper develops and tests sufficient conditions for a model when stock returns follow a jump-diffusion process. Based on daily stock price data, our results are that the stock market returns contain a jump component, although its magnitude is small. We measure the jump component over larger intervals in time (monthly interval) and find that the weekends and holidays tend to cover up the small jump component. The economic intuition is that jump risk is not diversifiable.

Supplementary material

Supplemental data for this article can be accessed here.

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Author details

Nixon S. Chekenya
E-mail: chekenyan@gmail.com
Midland State University, Gweru, Zimbabwe.

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Notes

1. The random variable π_j must satisfy certain technical requirements in order that a solution to (1) exists. (See Benaim & Friz, 2009; Johannes, Eraker, & Polson, 2000; Kushner, 1967; Jarrow & Rosenfeld, 1984).
2. To follow the general literature of jump-diffusion modeling, our variable is assumed to be an independent and identically distributed random variable. In this paper, we characterize stock returns as i.i.d in order to estimate jumps in stocks without violating the assumption of i.i.d and randomness in the variable (see, for example, Andersen, Bollerslev, & Dobrev, 2007; Camara & Li, 2008; Jiang, 1999).
3. The likelihood ratio statistic “A” is a Chi-Square distribution with 3 degrees of freedom.
4. We conjecture that stock prices are normally dictated by a Brownian drift.

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