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Capital and innovation aggregation with environmental pressure: An optimal evolution

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Abstract: Based on the advance-retreat course model, a growth model under environmental pressure, this paper builds an economic growth model that focuses on the aggregation of capital and innovation with environmental pressure. Importantly, the paper presents methods for computing the optimal quantity of capital-goods and innovation-goods. The paper makes the empirical researches using US GDP data (1940–2010 and 1969–2010). The findings include that the aggregations of capital and innovation promote economic growth, the optimal number of capital-goods decreases with innovation growth, the optimal number of innovation-goods decreases with capital expansion, both capital-goods and innovation-goods aggregate with environmental pressure increasing, and innovation is quicker than capital in aggregation.

Subjects: Development Studies; Economics, Finance, Business & Industry; Development Studies, Environment, Social Work, Urban Studies

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JEL classifications: E25; F21; O21; P23

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PUBLIC INTEREST STATEMENT

This paper, based on the advance-retreat course (ARC) model—a growth model under environmental pressure—focuses on the optimal aggregation of capital and innovation with environmental pressure, where the environmental pressure comes from the political, economic, and natural environments and may cause the cost and consumption. The paper presents two methods for computing the optimal quantity of capital-goods and innovation-goods. Using the methods, the economic resources, like capital and investment on innovation, can be optimally allocated or reallocated. The findings include that the aggregations of capital and innovation will promote economic growth; innovation is quicker aggregating than capital; the aggregation promotes the capital-goods and (or) innovation-goods to be in survival of the fittest and to come to the efficient industries; the government or authorities are able to give full play to the efficiency of socioeconomic resources with economic policy and the method related in the paper.

1. Introduction

As a society and its economy gradually progress and resolve problems, human beings develop a larger capacity to resolve problems. However, as problems become more complex, social development and economic growth have to face greater and greater pressure, until significant change occurs, at which time the original course ends and a new course begins. This is the key insight of advance-retreat course (ARC) theory (Dai, Liang, & Wu, 2013; Dai, Qi, & Liang, 2011).

Economic growth and change in the structure of production factors are important areas of research in social and economic fields. There are many distinguished lines of research, such as the business cycle theory (King, Plosser, &, Rebelo, 1988; Lucas, 1987; Plosser, 1989), the economic growth theory based on the *R&D* (Barro & Sala-i-Martin, 1995; Jones, 1995, 1998; Solow, 1956, 1957), and the new growth theory (Romer, 1986, 1990). In recent years, economists have made significant progress in understanding economic growth and development, developing theories such as the following: the dynamics-based economic growth theory (de la Croix & Michel, 2002), the analysis of firm regionalization clustering and innovation (Pouder & John, 2003), the non-linear economic theory (Fiaschi & Lavezzi, 2007), the economic sustainable development based on the invariance in growth theory (Martinet & Rotillon, 2007), economic markets as functions of downsizing and specialization (Mirowski, 2007), endogenous recombinants in economic growth (Tsur & Zemel, 2007), and so on. Still, environmental factors and economic policy have yet to be integrated into a growth model. Such a model and related analysis will be of high practical value.

Capital flow and change in innovation, the aggregation of market factors, and the allocation (and reallocation) of economic resources are problems of great concern to economists and governmental authorities. These phenomena play an important role in efforts to reform economic structures and design policies that promote economic growth and development. Coyle (2007) extends the economic theory of index numbers to the aggregation of price risk over commodities in production. Axelrod, Kulick, Plott, and Roust (2009) study how to improve information aggregation performance by removing disinformation due to strategic behavior and misleading disequilibrium behavior. Metcalfe (2010) studies the technology and technological change in the theory of resource allocation as well as the reasons why the development of technology and its application are so uneven over time and place. Jansen (2010) studies the efficient allocation of labor by job auctions.

However, the economic resources such as capital, labor, innovation, and technology are aggregated and allocated by natural selection. We need to analyze and understand this process to improve socioeconomic efficiency. Expanding upon the Solow growth model (1956, 1957) and the Cobb–Douglas production function, Dai et al. (2011, 2013) build an economic growth model with environmental pressure, namely the ARC model. Based on the literatures (Dai et al., 2011, 2013), this paper focuses on the optimal quantity of capital-goods and innovation-goods in an economy and analyzes the features of the optimal numbers in changing conditions. The main findings are that innovation growth is the driving force behind capital aggregation and that capital expansion is the driving force behind innovation aggregation.

2. Models

2.1. Categorized production function

The industries in an economy can generally be categorized as traditional and emerging. Traditional industries are those that mostly involve labor and manufacture, whereas emerging industries are those that mostly involve new science and technology. Traditional industries require large quantities of labor and equipment; these resources constitute the essential foundation of the industry. In a traditional industry, capital often takes a material form (e.g. in production equipment or buildings), whereas labor is produced by workers with standardized skills and effort. Technical progress may be measured by the number of technologies that are used in production equipment, processing techniques, and products. Traditional industries usually employ advanced processing techniques and complete equipment systems and enjoy stable product markets. Traditional industries often require

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a higher cost of capital and better technology. Furthermore, their technology levels tend to remain stable for long periods of time. In contrast, high technology is fundamental to emerging industries. In an emerging industry, capital may present in materialized or non-materialized form; it may include equipment, patents, software, intangible assets, and workers with standardized professional skills and effort levels. Technology develops rapidly in emerging industries, and technical level tends to change relatively rapidly.

For the sake of convenience, the capital input is the value of the capital required for production, the labor input is the number of workers required for production, and the technology input is the funds required for the research and development of technologies for production. Thus, the production function with technological progress (Plosser, 1989; Solow, 1956, 1957) for an economy can be expressed as follows:

$$Y = A \cdot F(K, L)$$

where Y is the output of the economy, K is the capital, L is the labor, and A is the technology level. For the given quantities of capital and labor, the increase in the technology level will yield an increase in output. Thus, economies with higher technology levels have greater productivity efficiency.

In reality, because capital, labor, and technology change over time, $K \equiv K(t)$, $L \equiv L(t)$, $A \equiv A(t)$. Technology level $A(t)$, because it changes with time, can also be expressed as $\frac{dA(t)}{dt} = h(t)$ if it is differentiable; thus, $A(t) = \int h(t)dt = H(t) + a$ with a as a constant. Therefore, the output of an economy can be expressed as:

$$Y = Y_1 + Y_2 \tag{1}$$

where $Y_1 = a \cdot F[K(t), L(t)]$, $Y_2 = H(t) \cdot F[K(t), L(t)]$.

In Model (1), the technology level of the output $Y_1 = a \cdot F[K(t), L(t)]$, a, is a constant, which means that the level of technology is stable, as in traditional industries. Therefore, Y_1 represents the output of traditional industries, and is referred to as the basic output. The technology level of the output $Y_2 = H(t) \cdot F[K(t), L(t)]$, $H(t)$, is a time function, which signifies that the level of technology is variable, as in emerging industries. Therefore, Y_2 represents the output of emerging industries, and is referred to as the emerging output. Model (1) is referred to as the categorized production function (CFP) for traditional and emerging industries, and $A(t) = H(t)+a$ is the categorized total factor productivity. CFP (1) indicates that traditional industries have two input factors, capital and labor, whereas emerging industries have three input factors: capital, labor, and technology. Furthermore, CFP (1) can also be concisely expressed as follows:

$$Y = \mu + \sigma$$

where $\mu = a \cdot F[K(t), L(t)]$ is the production function of traditional industries, $\sigma = q(t) \cdot \mu$ is the production function of emerging industries, and $q(t) = \frac{H(t)}{a}$ is the ratio of the technology level of emerging industries to that of traditional industries, indicating the degree of innovation of the former, referred to as innovation efficiency, or simply as innovation. Innovation efficiency is a dimensionless quantity. It embodies the advantage and the production efficiency of emerging industries on traditional industries.

Generally, traditional industries and emerging industries have different capital and labor requirements. CPF (1), however, shows that the input factors of both traditional and emerging industries stem from the economy's overall capital and labor. This finding can be explained as follows. Each unit of capital can be divided into two parts, with one used for traditional industries and another for emerging industries. Similarly, each unit of labor can have two skill types: one applicable to traditional industries and another applicable to emerging industries. Thus, capital and labor can flow between traditional industries and emerging industries. When the production efficiency of emerging industries increases, capital and labor will flow toward those industries.

CPF (1) points out that a part of total output comes from the traditional industries for an economy, and the rest comes from the emerging industries. If emerging industries is low in innovation level, economic output is mainly contributed by traditional industries. If the innovation level for an emerging industry is higher, economic output is mainly contributed by the emerging industries. At this time, the economy should be a developed one, because its growth is mainly promoted by innovations.

2.2. An economic growth model with environmental pressure

Economic growth requires production inputs and consumes a variety of economic resources. Additionally, economic growth can be made more difficult by a range of issues, including resource scarcity, market competition, investment risk, environmental pollution, policy institutions, financial risk, environmental crises, social unrest, natural disasters, diseases, and war, all of which generate environmental pressure on economic growth. It is worth noting that in addition to promoting economic growth, innovation input may generate environmental pressure; it may increase resource consumption, ecological pollution, or investment risk, which will increase the innovation cost and the consumption of the real output. The consumption of social and economic resources due to the environmental pressure generated during economic growth is referred to as the environmental cost.

We denote the environmental cost in traditional industries as ϕ and the environmental cost in emerging industries as κ. The environmental cost is caused by environmental pressure, which is related to the input factors. When the input factors change, the environmental cost will also change. Following the literatures (Reed, 2001; Schoenberg, Peng, & Woods, 2003), the ratio of the change in the environmental cost to the change in the input factors is the power function for the current inputs of the industry, that is,

$$\frac{d\phi}{d\mu} = v\mu^{\varphi} \text{ and } \frac{d\kappa}{d\theta} = w\sigma^{\varphi}$$

where $\varphi > 0$ indicates the existence of environmental pressure. Hence, the environmental costs of basic output and emerging output can be expressed as:

$$\phi = \frac{v}{\theta}\mu^{\theta} \text{ and } \kappa = \frac{w}{\theta}\sigma^{\theta}$$

respectively, where $\theta = \varphi + 1$ is the environmental pressure index (EPI). For the sake of convenience, the integral constants are equal to zero in general.

Following literature (Sanchez, Gonzalez-Estevez, Lopez-Ruiz, & Cosenza, 2007), we assume that the government is the agent that can adjust the EPI through economic policy. The government can reduce the EPI through a free and open policy, and increase the EPI through a closed and protective policy, though it cannot eliminate environmental pressure completely, that is, $\theta > 1$. The formula for environmental cost indicates that an increase in production input will be accompanied by an increase in the environmental pressure or environmental cost. Generally, the environment for economic production is likely to improve gradually; however, to ensure long-term development and growth, an economy has to face an increasing number of challenges of growing complexity. It will be more difficult to solve these new problems (which will arise from issues such as institutional policy, financial risk, environmental crisis, social unrest, natural disaster, disease, and war) using current approaches. These challenges can be collectively described as increasing environmental pressure. In reality, traditional industries and emerging industries may have different EPI to increase specificity. This means that traditional industries face the different environmental pressure from emerging industries, and the government can carry out different policy, so that traditional and emerging industries make a difference in development and growth.

Given that economic growth can never be separated from environmental pressure and the resulting environmental cost, Model (1) can be expressed as:

$$Y = \mu + \sigma - (\phi + \kappa) = \mu \cdot \Gamma \tag{2}$$

where $\mu = a{\cdot}F(K, L)$, $\sigma = q{\cdot}\mu$, $\phi = (v/\theta)\mu^{\theta}$, $\kappa = (w/\theta)\sigma^{\theta}$, $\Gamma = 1 + q - \left[(v + wq^{\theta})/\theta\right]\mu^{\theta-1}$ is total factor productivity with environmental pressure. Model (2) may be referred to as the economic growth model with environmental pressure, is a normal ARC economic growth model.[1]

Model (2) indicates that during the process of economic growth, the environmental pressure continues to increase as the output increases. This process continues until a recession begins. Only significant and successful policy and institutional reform can generate a fundamentally new economic environment, relieve the existing environmental pressure, and initiate a new cycle of economic growth. As the economy continues to grow, however, additional environmental pressure will accrue. Hence, ARC model reflects the cyclical features of economic growth.

According to Cobb–Douglas production function, the basic output can be especially expressed as $\mu = a{\cdot}F[K(t), L(t)] = \mu_0 K^{\alpha} L^{\beta}$, where K and L represent capital and labor, respectively, $\mu_0 = a{\cdot}c_0$, c_0 is a initial value, $\alpha + \beta = 1$, $\alpha, \beta > 0$. The basic output $\mu = a{\cdot}c_0 K^{\alpha} L^{\beta}$ will be used in the following discussion.

3. Methods

3.1. The method for computing the optimal number of capital-goods

There are many capital-goods (denoted as capital-goods) in socioeconomic development. Romer (1990) concludes that an unlimited increase in the quantity of capital-goods will lead to an infinite increase in output. In reality, the quantity of capital-goods is limited and restricted by the total social capital, market capacity, production conditions, and economic environment. This fact raises questions, for example, what is the optimal quantity of capital-goods? Suppose social production capital consists of n capital-goods—that is, total capital $K = \sum_{i=1}^{n} K_i$, where K_i is ith capital-goods, $i = 1, \ldots, n$. According to Model (2) and Appendix A, the actual output (2) can be expressed as follows:

$$Y = n^{\alpha}\bar{\mu} - n^{\theta\alpha}\bar{\phi} + n^{\alpha}\bar{\sigma} - n^{\theta\alpha}\bar{\kappa} \tag{3}$$

where $\bar{\mu} = a \cdot c_0 \bar{K}^{\alpha} L^{\beta}$, $\bar{\phi} = \frac{v}{\theta}\bar{\mu}^{\theta}$, $\bar{\sigma} = q \cdot \bar{\mu}$, $\bar{\kappa} = \frac{w}{\theta}\bar{\sigma}^{\theta}$.

If the capital-goods are homogeneous—that is, all K_i are similar in scale, structure, and character, such that $K_i = \bar{K}$—the optimal quantity of capital-goods is as follows from Appendix A:

$$n^{*} = \frac{1}{\sqrt[\alpha]{\bar{\mu}}}\left(\frac{1+q}{v + wq^{\theta}}\right)^{\frac{1}{(\theta-1)\alpha}} \tag{4}$$

Equation 4 also indicates that the optimal number, n^*, will decrease as capital-goods or total in novation increases. Thus, if total capital, K, is invariant, $\bar{K}$ will increase as n^* decreases and capital is aggregating. If capital-goods, $\bar{K}$, is invariant and n^* is decreasing, the capital-goods will be determined by a process of natural selection. Equation 4 also indicates that the optimal number of capital-goods will decrease with innovation growth. Therefore, innovation growth causes capital to aggregate.

Note that if environmental pressure is not taken into account, that is, if $v = w = 0$, then $n^* = \infty$, which corresponds to the Romer's conclusion.

3.2. The method for computing the optimal quantity of innovation-goods

Innovation is the main factor driving economic growth. We divide innovations into categories, one of which is referred to as innovation-goods, by their utility. There is a high correlation between these goods. Computer and chip technology, for example, are used in industrial control, weather analysis, aerospace engineering, communication engineering, and so on. Modern textile, paper, steel, mechanical, and electrical industries are inseparable from control technologies and innovation. We suppose that innovation consists of m innovation-goods and that the growth rate of innovation is a linear combination of all innovation-goods. According to Model (2) and Appendix B, the actual output (2) can be expressed as follows:

$$Y = \mu - \frac{v}{\theta}\mu^{\theta} + \mu \cdot \bar{q}^{m\rho} - \frac{w}{\theta}\mu^{\theta}\bar{q}^{m\theta\rho} \tag{5}$$

where $\bar{q} = \left(q_1^{\rho_1}, \ldots, q_m^{\rho_m}\right)^{\frac{1}{m\rho}}$ is the geometric average of all innovation-goods, $\rho = \frac{1}{m}\sum_{i=1}^{m}\rho_i$, q_i is the ith innovation-goods, and ρ_i is the competitive coefficient of q_i, $\rho_i > 0$, $\sum_{i=1}^{m}\rho_i = 1$, $i = 1, \ldots, m$. If the competitive coefficient, ρ_i, is larger, the innovation-goods, q_i, have a competitive advantage in all of the innovation-goods.

If the innovation-goods are homogeneous, that is, if all q_i are similar in character, structure and competitive advantage such that $q_i = \bar{q}$ and $\rho_i = \rho$, then the optimal number of innovation-goods is as follows from Appendix B:

$$m^* = \frac{1}{\rho \ln \bar{q}} \ln\left[\frac{1}{\mu}\left(\frac{1}{w}\right)^{\frac{1}{\theta-1}}\right] \tag{6}$$

Further analysis of Equation 6 indicates that the optimal number, m^*, will decrease as total capital or innovation-goods increase. Thus, if total innovation, q, is invariant, $\bar{q}$ will increase as n^* decreases, which indicates that innovation is aggregating. If innovation-goods, $\bar{q}$, is invariant, decreasing m^* implies that the capital-goods are in survival of the fittest. Equation 6 also indicates that the optimal number of innovation-goods will decrease with capital expansion. Therefore, capital expansion causes innovations to aggregate.

If environmental pressure is not accounted for in Model (5) and Equation 6, that is, if $w = 0$, then $m^* = \infty$, which indicates that an unlimited increase in the quantity of innovation-goods will cause an infinite increase in output.

3.3. The conservation equation and optimal allocation of economic resources

Based on Model (3) and Equation 4, if total capital is conserved before and after an aggregating process, that is, $n\bar{K} = n^*K^*$, where n is the number of capital-goods before the aggregation, n^* is the optimal number of capital-goods after the aggregation, and K^* is the average value of capital-goods after the aggregation, then $K^* = \frac{n}{n^*}\bar{K}$. Thus, if $n/n^* > 1$, the capital-goods are aggregated, and K^* increases. Each capital-good should involve even more capital. To improve the efficiency of capital allocation, the capital-goods, which encourage economic growth and are now smaller in value, should be allocated even more capital.

If $n/n^* < 1$, the capital-goods are diffused, and K^* decreases. Some capital-goods should be allocated less capital. To reduce capital consumption and improve the efficiency of capital allocation, the quantity of capital-goods should be increased, and the amount of capital in the capital-goods should be reduced well unless economic growth is encouraged.

The approach outlined above allocates investment across innovation, labor, and other economic resources.

4. Results

In the following section, the aggregation includes the survival of the fittest capital-goods and innovation-goods. The allocation includes the allocation of economic resources, reallocation of economic resources, or both.

4.1. The optimal aggregation and allocation of capital

According to Equation 4, if environmental pressure is invariable, then the optimal quantity of capital-goods will decrease with an increase in the average capital value of each goods. Thus, capital is aggregating, and some mainstream capital-goods arise. This phenomenon is the optimal aggregation of capital.

Additionally, the optimal quantity of capital-goods will decrease with an increase in innovations, which indicates that an increase in innovations may promote the aggregation of capital. In addition, the decrease in capital-goods may increase the value of capital-goods and innovation. Therefore, Conclusion 1 is as follows.

Conclusion 1. There is an optimal aggregation of capital in social and economic development. The optimal number of capital-goods decreases with an increase in innovation. Thus, innovation growth is the primary determinant of capital aggregation. Decreasing the number of capital-goods promotes capital expansion and increases innovation.

Conclusion 1 demonstrates the relationship between the core production factors (capital and innovation) and the quantity of capital-goods. The optimal quantity of capital-goods, calculated by formula 4, provides a basis for the optimal allocation of production capital. Therefore, Strategy 1 is obtained as follows.

Strategy 1 (The optimal allocation of production capital). If the current quantity of capital-goods is larger than the optimal one, the authorities should reduce the quantity of capital-goods through policy and increase the average value of the capital-goods by the allocation or reallocation of production resources. If the current quantity of capital-goods is smaller than the optimal one, the authorities should increase the quantity of capital-goods through policy and decrease the average capital value of some goods.

In using Strategy 1, the two following principles should be noted:

(1) The optimal quantity and average value of capital-goods are different across economies, industries, and enterprises. The optimal quantity of capital-goods can be calculated using Equation 4.

(2) When the quantity of capital-goods decreases, capital expansion will occur across the remaining capital-goods, which have a potential advantage and are smaller in value. When the quantity of capital-goods increases, capital reduction will occur across the capital-goods, which are larger in value and have no potential.

4.2. The optimal aggregating and allocating of innovations

According to Equation 6, if environmental pressure is invariable, the optimal quantity of innovation-goods will decrease with an increase in the average quantity of innovation-goods. Thus, the innovations are aggregating, and some mainstream innovations goods arise. This phenomenon is the optimal aggregation of innovations. However, the optimal quantity of economic innovation-goods will decrease with an increase in basic output, which indicates that an increase of basic output may promote innovation aggregation. In addition, the decrease in the quantity of innovation-goods will increase the value of capital-goods or the average quantity of innovation-goods. Therefore, Conclusion 2 is as follows.

Conclusion 2. There is an optimal aggregation of innovations in social and economic development. The optimal quantity of innovation-goods decreases with an increase in basic output. Thus, capital expansion is the driving force behind innovation aggregation.

Conclusion 2 demonstrates the relationship between the core production factors (capital and innovation). The optimal quantity of innovation-goods, calculated using Equation 6, provides a basis for the optimal allocation of innovation resources. Therefore, Strategy 2 is obtained as follows.

Strategy 2 (The optimal allocation of innovation resources). If the current quantity of innovation-goods is larger than the optimal one, the government or authority should decrease the quantity of innovation-goods through policy and increase the scale of the remaining innovations by the redistributing innovation resources. If the current quantity of innovation-goods is smaller than the optimal one, the government or authority should increase the quantity through policy and decrease the scale of some innovation-goods.

In using Strategy 2, the two following principles should be noted:

(1) The optimal quantity and average quantity of innovation-goods are different across economies, industries, and enterprises. The optimal quantity of innovation-goods can be calculated using Equation 6.

(2) When the quantity of innovation-goods decreases, the remaining innovation-goods, which have a developing advantage and are smaller in quantity, should receive greater investment. When the number of innovation-goods is increased, the innovation-goods, which are larger in quantity and have no potential, should decrease in quantity.

4.3. Environmental pressure and aggregation

In Equations 4 and 6, if the EPI is smaller, the optimal quantity is larger and decreases quickly. If the EPI is larger, the optimal quantity is smaller and decreases slowly. Environmental pressure impacts the optimal quantity of capital-goods and innovation-goods, that is, the aggregating speeds for capital-goods and innovation-goods.

Conclusion 3. A liberal and open economic environment helps to diversify capital-goods and innovation-goods, and the natural selections of capital-goods and innovation-goods are quicker. A restricted environment slows the aggregation of capital-goods and innovation-goods and reduces the survivals of the fittest in speed.

Thus, Strategy 3 is obtained by controlling the survival of the fittest.

Strategy 3 (The controlling strategy for survival of the fittest). The authorities should decrease environmental pressure with liberal economic policy to quicken the survival of the fittest economic resources and their optimal allocation, reallocation, or both. On the contrary, if the authorities hope to slow down the survival of the fittest, environmental pressure should be increased by restricted economic policy.

5. Discussion

5.1. The economic growth model

Departing from the current economic growth model, the Model (2) introduces environmental pressure, economic policy, and environmental costs. The Cobb–Douglas production function, the Solow model (Solow, 1956), and the AK model (Barro & Sala-i-Martin, 1995; Jones, 1995) are special cases of Model (2). It is important that policy-makers compute the optimal quantity of capital-goods and innovation-goods in an economy and analyze the features of the optimal quantities in change based on Model (2).

Solow (2000) finds Romer's conclusion that an "unlimited increase capital-goods will result in an infinite output" surprising. From Equation 4 and the explanations that follow, if environmental pressure is not taken into account, the optimal quantity of capital-goods will be infinity, and Romer's conclusion is true. However, this circumstance is a special case. In fact, most resources are limited. The optimal number of capital-goods will gradually decrease with an increase in innovations. If the average value of capital-goods does not equal zero, the optimal number of capital-goods must be finite. Therefore, the Romer growth model does not seem to describe the aggregation of capital and innovations with environmental pressure.

5.2. The optimal aggregation of capital

Analyzing Equation 4 in depth, the discussion on the optimal aggregation of capital is as follows.

The optimal quantity of capital-goods is larger in an economy if the environmental pressure is lower and the number of mainstream industries or enterprises in which capital aggregates is correspondingly larger. Currently, economic growth is at its initial or intermediate stage. If environmental pressure is higher, the optimal quantity of capital-goods is smaller; that is, the number of industries or enterprises in which capital aggregates is correspondingly smaller. Economic growth is generally at its latter stage, and monopolies arise. However, if innovation is larger in quantity, the optimal number of capital-goods will be smaller, which implies that capital is more concentrated and levels of socioeconomic specialization and standardization are higher. If one regards the products of an enterprise as capital-goods, innovation and technology growth will encourage mainstream products.

Conclusion 1 indicates that innovation growth is the driving force behind capital aggregation. Capital chases higher returns, and technical progress and innovation improve productivity and increase production returns. Thus, innovation growth is attractive to capital and causes capital to aggregate.

In Model (3), if labor is classified according to job contents, the discussions concerning labor (and the conclusion) are similar to those of capital-goods.

5.3. The optimal aggregation of innovations

Innovation and technology are the key factors promoting economic growth. The number of innovation-goods will significantly affect the economic structure and production manner. Analyzing Equation 6 in depth, a discussion on the optimal aggregation of innovations is as follows.

When environmental pressure is lower, the optimal quantity of innovation-goods is larger, and economic growth is in its initial or intermediate stage. If environmental pressure is higher, the optimal quantity of innovation-goods is smaller, and economic growth is in its latter stage. Innovation plays an important role in economic growth, and the concentration of innovations is higher in degree.

If the average value of the competitive coefficient of innovations is smaller, the optimal quantity of innovation-goods is larger, which implies that the average innovation competitiveness is weaker and innovations are scattered. This condition is also a basic feature of the innovation cycle in its initial stage. If innovation competitiveness is stronger, that is, if the average value of the competitive coefficient of innovations is larger, then the optimal number of innovation-goods is smaller, the innovations are concentrated, and the innovation cycle is in its latter stage.

Conclusion 2 indicates that capital expansion is the driving force for innovation aggregation. Technical progress and innovation are able to improve productivity and increase production returns. However, capital is the necessary foundation for technological progress and innovation growth. The capital expansion will be of a higher degree in supporting innovation growth. Therefore, capital expansion is attractive to technology and innovation and causes technology and innovation to aggregate.

6. Empirical analysis

6.1. Fitting the US GDP

Without a loss of generality, in Model (2), we assume that $\mu(t) = \mu_0 e^{\lambda t}$, $q(t) = q_0 e^{st}$ and $\sigma(t) = \mu(t) \cdot q(t) = \mu_0 q_0 e^{(\lambda + s)t}$, where λ is the growth rate of basic output, s is the growth rate of innovation. We use Model (2) to fit US GDP (1940–2010 and 1969–2010), and the sample data[2] are recorded as $D(t)$, $t = 1940, \ldots, 2010$. The basic output growth rate $\lambda = 0.110$,[3] and the economic innovation growth rate $s = 0.097$.[4] Model (2) is expressed as $Y_\theta(t) = \mu_0 e^{0.11t} + \sigma_0 e^{0.207t} - \frac{v}{\theta}\mu_0^\theta e^{0.11\theta t} - \frac{w}{\theta}\sigma_0^\theta e^{0.207\theta t}$, which can be estimated by the regression method. We employ the fitting function **Fit[*Y*(*t*), *D*(*t*), *t*]** in the MAPLE software system. When the error is small and the coefficient of determination is large, two fitting results are below based on $\min_{\theta>1} \sum_{i=1940}^{2010} [Y_\theta(i) - D(i)]^2$:

Case 1: Fitting the US GDP (1940–2010). Based on real US GDP data (1940–2010), the data estimated from fitness are listed in Table 1.

Case 2: Fitting the US GDP (1969–2010). Because of the growth rates of utility patent applications in US (1969–2010), this is a shorter term fitness than Case 1. Based on real US GDP data (1969–2010), the data estimated from fitness are also listed in Table 1.

Table 1 shows that $\theta = 1.36$ in Case 2 is less than $\theta = 1.412$ in Case 1, and that means that US economic environment during 1969–2010 is more improved than ever. The GDP data and Model (2)-based US GDP fitting curves for both Case 1 and Case 2 are depicted in Figure 1.

Figure 1 shows that ARC Model (2) may fit the real US GDP data (1940–2010 and 1969–2010) well. Furthermore, the fitting results of the actual GDP show that the US economy peaked in 2008–2009 and subsequently showed signs of decline. There is still a tendency to decline in and after 2011. If there has been a recovery in the economy, it may be weak. Notably, the current GDP often does not account for the damage caused by environmental pressure. However, from the perspective of wealth accumulation, it is more reasonable to include the loss in social economic output caused by environmental pressure.

Table 1. ARC model fitting data

	Data estimated and errors				
Case 1 (1940–2010)	μ_0	σ_0	v	w	q_0
	55.2942031	0.103163597	0.0232751549	0.0022806075	0.001865721743
	ε_1	ε_2	R^2	θ	–
	130.6221750	0.0334648028	0.9937312747	1.412	–
Case 2 (1969–2010)	μ_0	σ_0	v	w	q_0
	1443.01545727	36.21127246	0.03529084852	0.0051325656	0.02509416640
	ε_1	ε_2	R^2	θ	–
	152.8612324	0.02420077117	0.9834099989	1.36	–

Notes: (1) In Case 1, the absolute error: $\varepsilon_1 = \sqrt{\sum_{t=1940}^{2010}\left[D(t) - \hat{Y}(t)\right]^2 / (2010-1939)}$, the relative error:

$\varepsilon_2 = (1/\bar{D}) \cdot \sqrt{\sum_{t=1940}^{2010}\left[D(t) - \hat{Y}(t)\right]^2 / (2010-1939)}$, $\bar{D} = \sum_{t=1940}^{2010} D(t) / (2010-1939)$;

In Case 2, the absolute error: $\varepsilon_1 = \sqrt{\sum_{t=1969}^{2010}\left[D(t) - \hat{Y}(t)\right]^2 / (2010-1968)}$, the relative error:

$\varepsilon_2 = (1/\bar{D}) \cdot \sqrt{\sum_{t=1969}^{2010}\left[D(t) - \hat{Y}(t)\right]^2 / (2010-1968)}$, $\bar{D} = \sum_{t=1969}^{2010} D(t) / (2010-1968)$.

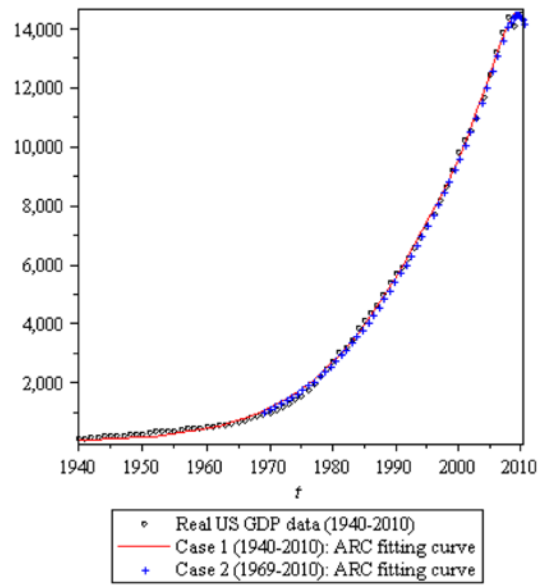
(2) R^2 is the coefficient of determination; $\hat{Y}(t)$ is the $Y_\theta(t)$-based estimated function of US GDP.

(3) Because the difference between the "before" and "after" US GDP data is large, the relative errors better reflect the actual fitting results.

(4) The error ε_2 in Case1 is larger than one in Case2, shows that ARC Model (2) has better fitting results to the GDP data (1969–2010) than the GDP data (1940–2010).

Figure 1. US GDP data and ARC fitting.

Notes: In Figure 1, the units are in billions of US dollars. The ARC fitting curves for the period 1940–2010 (Case 1) and 1969–2010 (Case 2) are illustrated; they can be observed that the ARC Model (2) has better fitting results to the real US GDP data.



6.2. The optimal aggregation of capital

To describe and analyze the aggregation speed and characteristics of capital on a unifying base, the optimal aggregation rate of capital is defined as the ratio of the current optimal number of capital-goods to the initial optimal number of capital-goods. According to Equation 4, the optimal aggregation rate of capital is as follows:

$$R_K^* = \frac{n^*}{n_0^*} \tag{7}$$

where n^* is the current optimal number of capital-goods and n_0^* is the initial optimal number of capital-goods.

In ARC Model (3), we suppose the capital-goods are homogeneous and good $\bar{\mu} = \mu_0 \left(\frac{K}{n}\right)^{\alpha} L^{\beta}$. Based on the data estimated in the section of *fitting the US GDP* (Case 1 and Case 2), we may analyze the optimal aggregation rate of capital using Equation 7. Two cases are presented below:

Case 3: In the case, all the data estimated are come from the Case 1 and the optimal aggregation rate of capital is calculated by using Equation 7.

Case 4: In the case, all the data estimated are come from the Case 2 and the optimal aggregation rate of capital is calculated by using Equation 7.

We observe the aggregation of capital during 2000–2020 for the sake of comparison and contrast, and then both the optimal aggregation rates of capital in Case 3 and Case 4 are depicted in Figure 2.

It is observed from Figure 2 that the optimal aggregation rate of capital-goods manifests a decreasing trend. Because US total capital is increasing during 2000–2020 in general and the optimal aggregation rates of capital decrease in both Case 3 and Case 4, which implies that capital is aggregating with time, coming to the efficient capital-goods or industries. The capital aggregation encourages capital-goods to be in survival of the fittest and capital to be allocated or reallocated in an optimal mode. All of the analysis results support Conclusion 1, Strategy 1, Conclusion 3, and Strategy 3.

According to Strategy 1, Strategy 3, and the computations above, the authorities may make decisions concerning the optimal allocation or reallocation of capital, and these decisions help the economy to achieve efficient actual output.

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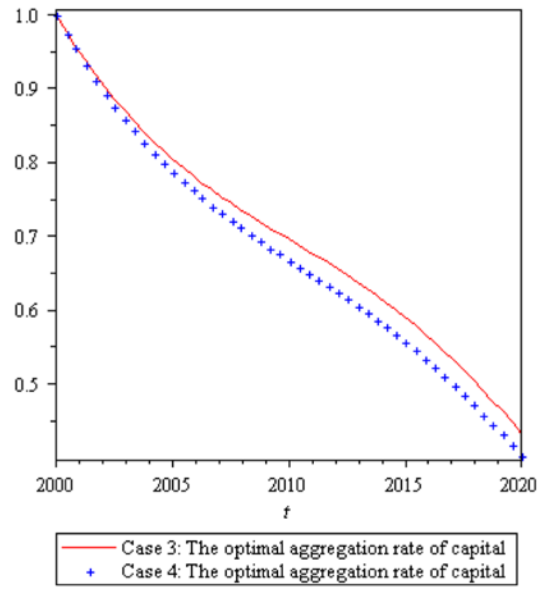


Figure 2. The aggregation of capital.

Notes: In Figure 2, by using Equation 7 and data estimated from the Case 1, the optimal aggregation rates of capital are calculated for Case 3; by using Equation 7 and data estimated from the Case 2, the optimal aggregation rates of capital are calculated for Case 4. The optimal aggregation rates of capital in Case 3 are near to those in Case 4, that means that the optimal aggregation rate of capital is relatively stable in US.

6.3. The optimal aggregation of innovation

To describe and discuss the aggregation speed and characteristics of innovation on a unifying base, the optimal aggregation rate of innovation is defined as the ratio of the current optimal number of innovation-goods to the initial number of innovation-goods. According to Equation 6, the optimal aggregation rate of innovation is as follows:

$$R_I^* = \frac{m^*}{m_0} \tag{8}$$

where m^* is the current optimal number of innovation-goods and m_0 is the initial optimal number of innovation-goods.

In Model (5), we suppose the innovation-goods are homogeneous and good $\bar{q} = \sqrt[m\rho]{q}$. Based on the data estimated in the section of *fitting the US GDP* (Case 1 and Case 2), we may analyze the optimal aggregation rate of innovation using Equation 8. Two cases are presented below:

Case 5: In the case, all the data estimated are come from the Case 1 and the optimal aggregation rate of innovation is calculated by using Equation 8.

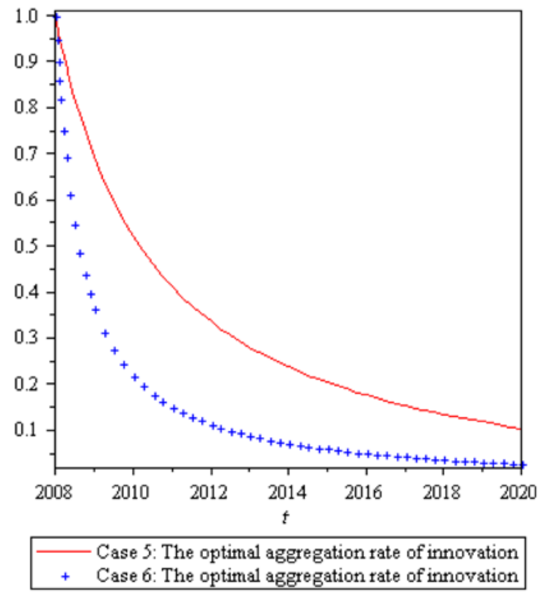
Case 6: In the case, all the data estimated are come from the Case 2 and the optimal aggregation rate of innovation is calculated by using Equation 8.

We observe the aggregation of capital during 2008–2020 for the sake of comparison and contrast, and then both the optimal aggregation rates of innovation in Case 5 and Case 6 are depicted in Figure 3.

It is observed from Figure 3 that the optimal aggregation rate of innovation-goods manifests a decreasing trend. Because US total innovations are increasing during 2008–2020 and the optimal aggregation rates of innovation decrease in both Case 5 and Case 6, which indicates that innovation is aggregating with time, coming to the efficient innovation-goods or emerging industries. The innovation aggregation encourages innovation-goods to be in survival of the fittest and the investment on innovation to be allocated or reallocated in an optimal mode. The analysis supports Conclusion 2, Strategy 2, Conclusion 3, and Strategy 3.

Figure 3. The aggregation of innovation.

Notes: In Figure 3, by using Equation 8 and data estimated from the Case 1, the optimal aggregation rates of innovation are calculated for Case 5; by using Equation 8 and data estimated from the Case 2, the optimal aggregation rates of innovation are calculated for Case 6. The optimal aggregation rate of innovation in Case 6 is much lower than those in Case 5, which means that the aggregation of innovations in recent 40 years is more rapid than ever.



6.4. The comparative analysis

Comparing the optimal aggregation rates in Figure 2 with those in Figure 3, we can observe that innovation is quicker aggregating than capital in optimal mode, which reveals that the characteristics of capital are different from those of innovation. In general, most original capital-goods can be converted to cash in discounted fashion, which will be useful in the future. However, many original innovation-goods may obsolete and will be useless in the future if they are replaced by new or updated goods.

Appendix A indicates that the optimal capital aggregation maximizes the actual output, and Appendix B indicates that the optimal innovation aggregation maximizes the actual output also. Those means that capital and (or) innovation aggregation help to promoting economic growth. As mentioned above, innovation is quicker aggregating than capital, which implies that innovation is more active than capital in promoting economic growth.

7. Conclusions

Based on the Solow growth model and the ARC model, this paper focuses on the optimal aggregation of capital and innovation with environmental pressure and has accomplished the following:

(1) Accounting for the various capital-goods and innovation-goods, this paper builds the economic growth models with environmental pressure, which are suitable to analyze the capital and innovation aggregation, respectively.

(2) This paper presents two computational methods. The first computes the optimal quantity of capital-goods; the second accounts for competition and computes the optimal quantity of innovation-goods.

(3) Given the conservation equation of capital before and after the optimal aggregation, the conservation equation can be fit to innovation. Using the equation, the economic resources can be optimally allocated or reallocated.

(4) The results of empirical research using US GDP data (1940–2010 and 1969–2010) support the methods and conclusions in the paper.

The conclusions in the paper include that capital and innovation aggregation may promote economic growth, and are as follows:

- In social and economic development, capital is aggregating with time. Capital aggregation will promote the capital-goods to be in survival of the fittest and to come to the efficient industries. The same result applies to innovation.
- Because decreases in capital-goods and innovation-goods are related to environmental pressure, which can be controlled by economic policy, the government or authorities are able to allocate and reallocate socioeconomic resources using the formulas in this paper.
- Because the optimal number of capital-goods decreases with innovation growth, innovation growth causes capital to aggregate. Because the optimal number of innovation-goods decreases with capital expansion, capital expansion causes innovations to aggregate.
- Innovation is quicker aggregating than capital in optimal mode, which reveals that the characteristics of capital are different from those of innovation. There is more competition in innovative development than in obtaining capital.

This paper describes and analyzes capital and innovation aggregation, and the analysis for the labor is similar. In fact, the social-economic development is the intertwined process in which capital, labor, and innovations arise and aggregate.

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Notes
1. Differing from the literatures (Dai et al., 2011, 2013), this paper analyzes the aggregating processes for capital and innovation based on Model (2) and reveals the change rules in economic output structure with environmental pressure.
2. Data source: US White House website, http://www.whitehouse.gov.
3. Direct capital average growth rate (1940–2010) is 6.7%, data source: US White House Web, http://www.whitehouse.gov. Average growth rate of employment labor force (1940–2010) is 4.3%, data source: US Bureau of Labor Statistics Web, http://data.bls.gov/pdq/SurveyOutputServlet.
4. The growth rate of utility patent applications in US (1969–2010), data source: US Patent and Trademark Office Web, http://www.uspto.gov/web/offices.
5. In expression (B3), if $\ln \bar{q}(\bar{t}) = 0$, i.e. $\bar{q}(\bar{t}) = 1$, the $t = \bar{t}$ is interpreted as the moment of a new generation of innovation-goods starting to replace the existing ones; and it means that the outdated innovation-goods are dominant when $\ln \bar{q}(t) < 0$.

References
Axelrod, B. S., Kulick, B. J., Plott, C. R., & Roust, K. A. (2009). The design of improved parimutuel-type information aggregation mechanisms: Inaccuracies and the long-shot bias as disequilibrium phenomena. *Journal of Economic Behavior & Organization, 69*, 170–181.
Barro, R. J., & Sala-i-Martin, X. (1995). *Economic growth*. New York, NY: McGraw Hill.
Coyle, B. T. (2007). Aggregation of price risk over commodities: An economic index number approach. *American Journal of Agricultural Economics, 89*, 1085–1097. http://dx.doi.org/10.1111/j.1467-8276.2007.01023.x
Dai, F., Liang, L., & Wu, S. (2013). Money supply and economic growth under environmental pressure: The strategy for re-growth. *International Journal of Monetary Economics and Finance, 6*, 55–80. http://dx.doi.org/10.1504/IJMEF.2013.055717
Dai, F., Qi, J. P., & Liang, L. (2011). Socio-economic development model based on stochastic advance-retreat course. *International Journal of Social Economics, 38*, 416–437. http://dx.doi.org/10.1108/03068291111123129
de la Croix, D., & Michel, P. (2002). *A theory of economic growth*. Cambridge University Press. http://dx.doi.org/10.1017/CBO9780511606434
Fiaschi, D., & Lavezzi, A. M. (2007). Nonlinear economic growth: Some theory and cross-country evidence. *Journal of Development Economics, 84*, 271–290. http://dx.doi.org/10.1016/j.jdeveco.2005.10.007
Jansen, M. (2010). Job auctions and hold-ups. *Labour Economics, 17*, 608–619.
Jones, C. I. (1995). R & D-based models of economic growth. *Journal of Political Economy, 103*, 759–784. http://dx.doi.org/10.1086/jpe.1995.103.issue-4
Jones, C. I. (1998). *Introduction to economic growth*. New York, NY: W. W. Norton.
King, R., Plosser, C., & Rebelo, S. (1988). Production, growth and business cycles. *Journal of Monetary Economics, 21*, 195–232. http://dx.doi.org/10.1016/0304-3932(88)90030-X
Lucas, R. E. (1987). *Models of business cycles*. New York, NY: Basil Blackwell.
Martinet, V., & Rotillon, G. (2007). Invariance in growth theory and sustainable development. *Journal of Economic Dynamics and Control, 31*, 2827–2846. http://dx.doi.org/10.1016/j.jedc.2006.10.001

Metcalfe, J. S. (2010). Technology and economic theory. *Cambridge Journal of Economics, 34*, 153–171. http://dx.doi.org/10.1093/cje/bep075
Mirowski, P. (2007). Markets come to bits: Evolution, computation and markomata in economic science. *Journal of Economic Behavior & Organization, 63*, 209–242.
Plosser, C. (1989). Understanding real business cycles. *Journal of Economic Perspectives, 3*, 51–77. http://dx.doi.org/10.1257/jep.3.3.51
Pouder, R., & John, C. H. (2003). Geographical clusters of firms and innovation. *The Academy of Management Review, 21*, 192–225.
Reed, W. (2001). The Pareto, Zipf and other power laws. *Economics Letters, 74*, 15–19. http://dx.doi.org/10.1016/S0165-1765(01)00524-9
Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy, 94*, 1002–1037. http://dx.doi.org/10.1086/jpe.1986.94.issue-5
Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy, 98*, 71–102. http://dx.doi.org/10.1086/jpe.1990.98.issue-S5
Sanchez, J. R., Gonzalez-Estevez, J., Lopez-Ruiz, R., & Cosenza, M. G. (2007). *A model of coupled maps with Pareto behavior*. http://arxiv.org in its series Quantitative Finance Papers with number nlin/0701016.
Schoenberg, F. P., Peng, R., & Woods, J. (2003). On the distribution of wildfire sizes. *Environmetrics, 14*, 583–592. http://dx.doi.org/10.1002/(ISSN)1099-095X
Solow, R. M. (1956). A contribution to the theory of economic growth. *The Quarterly Journal of Economics, 70*, 65–94. http://dx.doi.org/10.2307/1884513
Solow, R. M. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics, 39*, 312–320. http://dx.doi.org/10.2307/1926047
Solow, R. M. (2000). *Growth theory* (2nd ed.). Oxford: Oxford University Press.
Tsur, Y., & Zemel, A. (2007). Towards endogenous recombinant growth. *Journal of Economic Dynamics and Control, 31*, 3459–3477. http://dx.doi.org/10.1016/j.jedc.2006.12.002

Appendix A

Computing the optimal quantity of capital-goods

If the production capital consists of n capital-goods, that is, the production capital $K=\sum_{i=1}^{n} K_i$, K_i is ith capital-goods. According to Model (2), the basic output on all capital-goods is as follows:

$$\mu = a \cdot c_0 \left(\sum_{i=1}^{n} K_i \right)^{\alpha} L^{\beta} \tag{A1}$$

where K_i is the value of ith capital-goods, $i = 1, \dots, n$.

If the capital-goods are homogeneous, that is, all the K_i are similar in scale, structure, and character, where $K_i = \bar{K}$, the basic output (A1) is expressed as $\mu = n^{\alpha}\bar{\mu} = n^{\alpha}(a \cdot c_0 \bar{K}^{\alpha} L^{\beta})$. Thus, the actual output (2) is expressed as follows:

$$Y = n^{\alpha}\bar{\mu} - n^{\theta\alpha}\bar{\phi} + n^{\alpha}\bar{\sigma} - n^{\theta\alpha}\bar{\kappa} \tag{A2}$$

In Model (A2), let $\frac{dY}{dn}=0$. The optimal number of capital-goods is obtained as follows:

$$n^* = \frac{1}{\sqrt[\alpha]{\bar{\mu}}} \left(\frac{1+q}{v+wq^{\theta}} \right)^{\frac{1}{(\theta-1)\alpha}} \tag{A3}$$

Because $\theta > 1$, $\frac{d^2Y}{dn^2} = (1-\theta)\alpha^2 n^{\alpha-2}(\bar{\mu}+\bar{\sigma}) < 0$ when $\frac{dY}{dn} = 0$. Therefore, the number n^*, determined by formula (A3), maximizes the actual output (A2).

Appendix B

Computing the optimal number of innovation-goods

Suppose innovation consists of m innovation-goods, $q_1, \dots, q_m$; the growth rate of innovation is a linear combination of those of all innovation-goods, that is, $\frac{dq}{q} = \rho_1 \frac{dq_1}{q_1} + \cdots + \rho_m \frac{dq_m}{q_m}$. For convenience, we assume that the integral constant is equal to 1; then, we have the following:

$$q = q_1^{\rho_1} \cdots q_m^{\rho_m} \tag{B1}$$

where q_i is the ith innovation-goods, ρ_i is the competitive coefficient of q_i, $\rho_i > 0$, $\sum_{i=1}^{m} \rho_i = 1$, $i = 1, \dots, m$.

If the competitive coefficient, ρ_i, is larger, the innovation-goods, q_i, have a higher competitive advantage in all the innovation-goods. According to (B1), the actual output (2) is expressed as follows:

$$Y = \mu \cdot \Gamma \tag{B2}$$

where, $\Gamma = 1 + \bar{q}^{m\rho} - \frac{v + w\bar{q}^{m\theta\rho}}{\theta}\mu^{\theta - 1}$, $\bar{q} = (q_1^{\rho_1} \cdots q_m^{\rho_m})^{1/(m\rho)}$ is the geometric average of all innovation-goods, $\rho = \frac{1}{m}\sum_{i=1}^{m}\rho_i$.

If the innovation-goods are homogeneous—that is, all the q_i are similar in character, structure, and competitive advantage, such that $q_i = \bar{q}$ and $\rho_i = \rho$, $i = 1, \ldots, m$—and $\frac{dY}{dm} = 0$, based on model (B2), we obtain the following if $\ln \bar{q} > 0$:

$$m^* = \frac{1}{\rho \ln \bar{q}} \ln \left[\frac{1}{\mu} \left(\frac{1}{w} \right)^{\frac{1}{\theta - 1}} \right] \tag{B3}$$

Because $\theta > 1$, $\frac{d^2Y}{dm^2} = (1 - \theta)(\rho \ln \bar{q})^2 \mu \cdot \bar{q}^{m\rho} < 0$ when and $\ln \bar{q} > 0$. Therefore, the number m^*, determined by formula (B3)[5], maximizes the actual output (B2).

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