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\*Corresponding author: Zouheir Mighri, Laboratoire de Recherche en Economie, Management et Finance Quantitative (LAREMFQ), Institut des Hautes Etudes Commerciales de Sousse, University of Sousse, Tunisia, Route Hzamia Sahloul 3 - B.P. 40, 4054 Sousse, Tunisia  
E-mail: [zmighri@gmail.com](mailto:zmighri@gmail.com)

Reviewing editor:  
David McMillan, University of Stirling, UK

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## REVIEW ARTICLE

# Modeling international stock market contagion using multivariate fractionally integrated APARCH approach

Zouheir Mighri<sup>1\*</sup> and Faysal Mansouri<sup>1</sup>

**Abstract:** The aim of this article is to examine how the dynamics of correlations between two emerging countries (Brazil and Mexico) and the US evolved from January 2003 to December 2013. The main contribution of this study is to explore whether the plunging stock market in the US, in the aftermath of global financial crisis (2007–2009), exerts contagion effects on emerging stock markets. To this end, we rely on a multivariate fractionally integrated asymmetric power autoregressive conditional heteroskedasticity dynamic conditional correlation framework, which accounts for long memory, power effects, leverage terms, and time-varying correlations. The empirical analysis shows a contagion effect for Brazil and Mexico during the early stages of the global financial crisis, indicating signs of “recoupling.” Nevertheless, linkages show a general pattern of “decoupling” after the Lehman Brothers collapse. Furthermore, correlations between Brazil and the US are decreased from early 2009 onwards, implying that their dependence is larger in bearish than in bullish markets.

**Keywords:** FIAPARCH-DCC model, contagion, global financial crisis, decoupling–recoupling, stock markets

**JEL classifications:** C13, C22, C32, C52, C53, G15

## ABOUT THE AUTHORS



Zouheir Mighri is an assistant professor of quantitative methods at the University of Sousse, Tunisia, Institut Supérieur de Gestion de Sousse. He received his PhD in management sciences (since 2011). His current research interests are financial econometrics, linear and non-linear time series econometrics, spatial econometrics, risk management, financial contagion and volatility transmission.

Faysal Mansouri is a professor of quantitative methods at the University of Sousse, Tunisia, Institut des Hautes Etudes Commerciales de Sousse. He received his PhD in industrial economics from the University of Illinois Urbana-Champaign (1992). His research has been published in several journals and, frequently, he participates in conferences and meetings. His major research interests lie in the field of entrepreneurship, microeconomics of banking, economics of innovation, industrial economics, environmental economics, regional economics and risk management.

## PUBLIC INTEREST STATEMENT

The main objective of this research is to investigate the stock market contagion effects of the global financial crisis in a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) dynamic conditional correlation (DCC) framework during the period 2003–2013. We focus on two emerging equity markets, namely Brazil and Mexico, as well as the US during different phases of the global financial crisis. The length and the phases of the crisis are identified based on both an economic approach and a Markov-switching dynamic regression approach. The empirical results confirm a contagion effect for Brazil and Mexico during the early stages of the global financial crisis, indicating signs of “recoupling” phenomenon. However, a general pattern of “decoupling” phenomenon is shown after the Lehman Brothers collapse. Furthermore, correlations between Brazil and the US decrease from early 2009 onwards, implying that their dependence is larger in bearish than in bullish markets.

## 1. Introduction

Unlike past crises, such as the 1997 Asian financial crisis, the 1998 Russian crisis, and the 1999 Brazilian crisis, the recent 2007–2009 global financial crisis originated from the largest and most influential economy, the US market, and was spreading over the other countries' financial markets worldwide. Global financial crisis resulted in sharp declines in asset prices, stock markets, and skyrocketing of risk premiums on interbank loans. It also disrupted country's financial system and threatened real economy with huge contractions. This crisis therefore provides a unique natural experiment for investigating the dynamic interrelationships among international stock markets, which have many implications for international asset pricing and portfolio allocation as well as for policy-makers to develop strategies to insulate economies.

At the onset of crisis, there was talk of “decoupling” in emerging financial markets. However, in addition to its severe recessionary effects inside the US, the crisis penalized emerging markets. In this topic, our interest mainly focuses on how stock market downturn in the US contributed to virulent volatility and contagion in emerging financial markets. Our focus on these markets is motivated by numerous facts. First, the volatility in stock prices is generally considered a gauge of financial stress for different segments of financial markets. Second, over the last decade, volatility in emerging financial markets has become a key factor for determining risk-taking behavior among international investors, mainly the rebalancing of portfolios between different securities. Third, as volatility tends to rise, it encourages write-offs and discourages position taking, influences leverage conditions, and increases risk budgets.

To capture the contagion behavior over time, we use a multivariate fractionally integrated asymmetric power autoregressive conditional heteroskedasticity (FIAPARCH) framework, which provides the tools to understand how financial volatilities move together over time and across markets. Conrad, Karanasos, and Zeng (2011) applied a multivariate FIAPARCH model that combines long memory, power transformations of the conditional variances, and leverage effects with constant conditional correlations (CCCs) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns, and the asymmetric response of volatility to positive and negative shocks are three features that improve the modeling of the volatility process of asset returns. We extend their model by estimating time-varying conditional correlations. We also study the effect of the global financial crisis events on the dynamic conditional correlations (DCCs).

The FIAPARCH model increases the flexibility of the conditional variance specification by allowing an asymmetric response of volatility to positive and negative shocks and long-range volatility dependence. In addition, it allows the data to determine the power of stock returns for which the predictable structure in the volatility pattern is the strongest (Conrad et al., 2011). Although many studies use various multivariate GARCH models in order to estimate DCCs among markets during financial crises (Celik, 2012; Chiang, Jeon, & Li, 2007; Kenourgios, Samitas, & Paltalidis, 2011), the forecasting superiority of FIAPARCH on other GARCH models is supported by Conrad et al. (2011) and Chkili, Aloui, and Nguyen (2012).

To identify the crisis period and its phases, we use both official data sources for all key financial and economic events representing the GFC (Bank for International Settlements [BIS], 2009; Federal Reserve Bank of St. Louis, 2009) and regimes of excess stock market volatility estimated by a Markov-switching dynamic regression (MS-DR) model (see Dimitriou, Kenourgios, & Simos, 2013).

Even though there is a widespread literature on financial contagion phenomenon during several past financial crises (see Kaminsky, Reinhart, & Végh, 2003, for a survey), the research on recent global financial crisis is still raising. As such, some of the existing studies have explored the contagion channels from stock markets in some key developed economies to the emerging stock markets during the 2007–2009 global financial crisis (see Aloui, Aïssa, & Nguyen, 2011; Bekiros, 2014; Chudik & Fratzscher, 2011; Dimitriou et al., 2013; Dooley & Hutchison, 2009; Kenourgios & Padhi, 2012; Morales & Andreosso-O'Callaghan, 2012; Syllignakis & Kouretas, 2011; Wang, 2014).

Witnessed the 2007–2009 global financial crisis, this study aims to explore how contagion transmits from the US stock market to the stock markets of Brazil and Mexico. A special reason for selecting these markets is the substantial interest of international investors in these economies.

The present study makes a number of contributions to the existent literature in the following aspects. First, contrary to other empirical studies, we analyze unconnectedly the contagion effects during different phases of the global financial crisis. Second, our empirical analysis allows testing the “decoupling–recoupling” hypothesis, which implies that some markets show immunity during different phases of a crisis or even the entire crisis period. Third, the time-varying DCCs are captured from a multivariate student-t-AR(1)-FIAPARCH(1,d,1)-DCC model which takes into account long memory behavior, speed of market information, asymmetries, and leverage effects. Fourth, our empirical findings show further evidence of contagion effects on Brazil and Mexico during the early stages of the global financial crisis, which indicates signs of “recoupling” hypothesis. Besides, our results provide evidence on the general “decoupling” hypothesis for both countries after the Lehmann Brothers collapse. Moreover, conditional correlations between Brazil and the US are negative and statistically significant from early 2009 onwards (post-crisis period), implying that the global financial crisis decelerates the integration process of Brazil and its dependence with the US is larger in bearish markets.

The rest of the paper is organized as follows. Section 2 presents the econometric methodology. Section 3 provides the data and a preliminary analysis. Section 4 displays and discusses the empirical findings and their interpretation, while Section 5 provides our conclusions.

## 2. Econometric methodology

### 2.1. Univariate FIAPARCH model

The AR(1) process represents one of the most common models to describe a time series  $r_t$  of stock returns. Its formulation is given as:

$$(1 - \xi L)r_t = c + \varepsilon_t, \quad t \in \mathbb{N} \tag{1}$$

with

$$\varepsilon_t = z_t \sqrt{h_t} \tag{2}$$

where  $|c| \in [0, +\infty]$ ,  $|\xi| < 1$  and  $\{z_t\}$  are independently and identically distributed (i.i.d.) random variables with  $E(z_t) = 0$ . The variance  $h_t$  is positive with probability equal to unity and is a measurable function of  $\Sigma_{t-1}$ , which is the  $\sigma$ -algebra generated by  $\{r_{t-1}, r_{t-2}, \dots\}$ . Therefore,  $h_t$  denotes the conditional variance of the returns  $\{r_t\}$ , that is:

$$E[r_t / \Sigma_{t-1}] = c + \xi r_{t-1} \tag{3}$$

$$\text{Var}[r_t / \Sigma_{t-1}] = h_t \tag{4}$$

Tse (1998) uses a FIAPARCH(1,d,1) model in order to examine the conditional heteroskedasticity of the yen-dollar exchange rate. Its specification is given as:

$$(1 - \beta L) \left( h_t^{\delta/2} - \omega \right) = \left[ (1 - \beta L) - (1 - \phi L) (1 - L)^d \right] (1 + \gamma s_t) |\varepsilon_t|^\delta \tag{5}$$

where  $\omega \in [0, \infty]$ ,  $|\beta| < 1$ ,  $|\phi| < 1$ ,  $0 \leq d \leq 1$ ,  $s_t = 1$  if  $\varepsilon_t < 0$  and 0 otherwise,  $(1 - L)^d$  is the financial differencing operator in terms of a hypergeometric function (see Conrad et al., 2011),  $\gamma$  is the leverage coefficient, and  $\delta$  is the parameter for the power term that takes (finite) positive values.

A sufficient condition for the conditional variance  $h_t$  to be positive almost surely for all  $t$  is that  $\gamma > -1$  and the parameter combination  $(\phi, d, \beta)$  satisfies the inequality constraints provided in Conrad and Haag (2006) and Conrad (2010). When  $\gamma > 0$ , negative shocks have more impact on volatility than positive shocks.

The advantage of this class of models is its flexibility since it includes a large number of alternative GARCH specifications. When  $d = 0$ , the process in Equation 5 reduces to the APARCH(1,1) one of Ding, Granger, and Engle (1993), which nests two major classes of ARCH models. In particular, a Taylor/Schwert type of formulation (Schwert, 1990; Taylor, 1986) is specified when  $\delta = 1$ , and a Bollerslev (1986) type is specified when  $\delta = 2$ . When  $\gamma = 0$  and  $\delta = 2$ , the process in Equation 5 reduces to the FIGARCH(1,d,1) specification (see Baillie, Bollerslev, & Mikkelsen, 1996; Bollerslev & Mikkelsen, 1996) which includes Bollerslev's (1986) GARCH model (when  $d = 0$ ) and the IGARCH specification (when  $d = 1$ ) as special cases.

**2.2. Multivariate FIAPARCH model with DCCs**

In what follow, we introduce the multivariate FIAPARCH process (M-FIAPARCH) taking into account the DCC hypothesis (see Dimitriou et al., 2013) advanced by Engle (2002). This approach generalizes the multivariate CCC FIAPARCH model of Conrad et al. (2011). The multivariate DCC model of Engle (2002) and Tse and Tsui (2002) involves two stages to estimate the conditional covariance matrix  $H_t$ . In the first stage, we fit a univariate FIAPARCH(1,d,1) model in order to obtain the estimations of  $\sqrt{h_{it}}$ . The daily stock returns are assumed to be generated by a multivariate AR(1) process of the following form:

$$Z(L)r_t = \mu_0 + \varepsilon_t \tag{6}$$

where  $\mu_0 = [\mu_{0,i}]_{i=1, \dots, n}$ : the  $N$ -dimensional column vector of constants;  $|\mu_{0,i}| \in [0, \infty]$ ;  $Z(L) = \text{diag}\{\psi(L)\}$ : an  $N \times N$  diagonal matrix;  $\psi(L) = [1 - \psi_i L]_{i=1, \dots, n}$ ;  $|\psi_i| < 1$ ;  $r_t = [r_{i,t}]_{i=1, \dots, N}$ : the  $N$ -dimensional column vector of returns;  $\varepsilon_t = [\varepsilon_{i,t}]_{i=1, \dots, N}$ : the  $N$ -dimensional column vector of residuals.

The residual vector is given by:

$$\varepsilon_t = z_t \odot h_t^{\Lambda/2} \tag{7}$$

where  $\odot$  is the Hadamard product;  $\Lambda$  is the element-wise exponentiation.

$h_t = [h_{it}]_{i=1, \dots, N}$  is  $\Sigma_{t-1}$  measurable and the stochastic vector  $z_t = [z_{it}]_{i=1, \dots, N}$  is independent and identically distributed with mean zero and positive definite covariance matrix  $\rho = [\rho_{ij}]_{i,j=1, \dots, N}$  with  $\rho_{ij} = 1$  for  $i = j$ . Note that  $E(\varepsilon_t / \mathcal{F}_{t-1}) = 0$  and  $H_t = E(\varepsilon_t \varepsilon_t' / \mathcal{F}_{t-1}) = \text{diag}(h_t^{\Lambda/2}) \rho \text{diag}(h_t^{\Lambda/2})$ .  $h_t$  is the vector of conditional variances and  $\rho_{i,j,t} = h_{i,j,t} / \sqrt{h_{i,t} h_{j,t}} \forall i, j = 1, \dots, N$  are the DCCs.

The multivariate FIAPARCH(1,d,1) is given by:

$$B(L) \left( h_t^{\Lambda\delta/2} - \omega \right) = [B(L) - \Delta(L)\Phi(L)] [I_N + \Gamma_t] |\varepsilon_t|^{\Lambda\delta} \tag{8}$$

where  $|\varepsilon_t|$  is the vector  $\varepsilon_t$  with elements stripped of negative values.

Besides,  $B(L) = \text{diag}\{\beta(L)\}$  with  $\beta(L) = [1 - \beta_i L]_{i=1, \dots, N}$  and  $|\beta_i| < 1$ . Moreover,  $\Phi(L) = \text{diag}\{\phi(L)\}$  with  $\phi(L) = [1 - \phi_i L]_{i=1, \dots, N}$  and  $|\phi_i| < 1$ . In addition,  $\omega = [\omega_i]_{i=1, \dots, N}$  with  $\omega_i \in [0, \infty]$  and  $\Delta(L) = \text{diag}\{d(L)\}$  with  $d(L) = [(1 - L)^{d_i}]_{i=1, \dots, N} \forall 0 \leq d_i \leq 1$ . Finally,  $\Gamma_t = \text{diag}\{\gamma \odot s_t\}$  with  $\gamma = [\gamma_i]_{i=1, \dots, N}$  and  $s_t = [s_{it}]_{i=1, \dots, N}$  where  $s_{it} = 1$  if  $\varepsilon_{it} < 0$  and 0 otherwise.

In the second stage, we estimate the conditional correlation using the transformed stock return residuals, which are estimated by their standard deviations from the first stage. The multivariate conditional variance is specified as follows:

$$H_t = D_t R_t D_t \tag{9}$$

where  $D_t = \text{diag} \left( h_{11t}^{1/2}, \dots, h_{NNt}^{1/2} \right)$  denotes the conditional variance derived from the univariate AR(1)-FIAPARCH(1,d,1) model and  $R_t = (1 - \theta_1 - \theta_2)R + \theta_1 \psi_{t-1} + \theta_2 R_{t-1}$  is the conditional correlation matrix.<sup>1</sup>

In addition,  $\theta_1$  and  $\theta_2$  are the non-negative parameters satisfying  $(\theta_1 + \theta_2) < 1$ ,  $R = \{\rho_{ij}\}$  is a time-invariant symmetric  $N \times N$  positive definite parameter matrix with  $\rho_{ii} = 1$ , and  $\psi_{t-1}$  is the  $N \times N$  correlation matrix of  $\varepsilon_\tau$  for  $\tau = t - M, t - M + 1, \dots, t - 1$ . The  $i, j$ -th element of the matrix  $\psi_{t-1}$  is given as follows:

$$\psi_{ij,t-1} = \frac{\sum_{m=1}^M z_{i,t-m} z_{j,t-m}}{\sqrt{\left( \sum_{m=1}^M z_{i,t-m}^2 \right) \left( \sum_{m=1}^M z_{j,t-m}^2 \right)}}, \quad 1 \leq i \leq j \leq N \tag{10}$$

where  $z_{it} = \varepsilon_{it} / \sqrt{h_{it}}$  is the transformed stock return residuals by their estimated standard deviations taken from the univariate AR(1)-FIAPARCH(1,d,1) model.

The matrix  $\psi_{t-1}$  could be expressed as follows:

$$\psi_{t-1} = B_{t-1}^{-1} L_{t-1} L'_{t-1} B_{t-1}^{-1} \tag{11}$$

where  $B_{t-1}$  is a  $N \times N$  diagonal matrix with  $i$ -th diagonal element given by  $\left( \sum_{m=1}^M z_{i,t-m}^2 \right)$  and  $L_{t-1} = (z_{t-1}, \dots, z_{t-M})$  is a  $N \times N$  matrix, with  $z_t = (z_{1t}, \dots, z_{Nt})'$ .

To ensure the positivity of  $\psi_{t-1}$  and therefore of  $R_t$ , a necessary condition is that  $M \leq N$ . If  $R_t$  itself is a correlation matrix, then  $R_{t-1}$  is also a correlation matrix. The correlation coefficient in a bivariate case is given as:

$$\rho_{12,t} = (1 - \theta_1 - \theta_2)\rho_{12} + \theta_2 \rho_{12,t} + \theta_1 \frac{\sum_{m=1}^M z_{1,t-m} z_{2,t-m}}{\sqrt{\left( \sum_{m=1}^M z_{1,t-m}^2 \right) \left( \sum_{m=1}^M z_{2,t-m}^2 \right)}} \tag{12}$$

### 2.3. Crisis period specification

The recent global financial crisis has some unique features, such as the length, breadth, and crisis sources. Numerous studies use major economic and financial events in order to determine the crisis length and source ad hoc (see Chiang et al., 2007; Forbes & Rigobon, 2002, among others). Nevertheless, other studies implement Markov regime switching processes to identify the crisis period endogenously (see Boyer, Kumagai, & Yuan, 2006, among others).

In order to define correctly the crisis period, studies on financial contagion are in some degree arbitrary. Some studies avoid discretion in the definition of the crisis period by using discretion in the choice of the econometric model to estimate the location of the crisis period in time. Baur (2012) uses both key financial and economic events and estimates of excess volatility to identify the crisis period, and investigates the transmission of the global financial crisis from the financial sector to real economy.

In this study, we specify the length of the global financial crisis and its phases following both an economic approach and a statistical approach. First, we define a relatively long crisis period based on all major international financial and economic news events representing the global financial

crisis. We use the official timelines provided by Federal Reserve Board of St. Louis (2009) and the BIS (2009), among others, in order to choose the crisis period. According to these studies, the timeline of the global financial crisis is separated in four phases. Phase 1 described as “initial financial turmoil” spans from 1 August 2007 to 15 September 2008. Phase 2 is defined as “sharp financial market deterioration” and spans from 16 September 2008 to 31 December 2008. Phase 3 described as “macroeconomic deterioration” spans from 1 January 2009 to 31 March 2009. Phase 4 described as a phase of “stabilization and tentative signs of recovery” (post-crisis period), including a financial market rally, spans from 1 April 2009 to the end of the sample period.

For that reason, the crisis can be defined from August 2007 to March 2009 covering the first three phases. Second, we identify regimes of excess stock market conditional volatility ( $h_{it}$ ) via a MS-DR<sup>2</sup> model, which takes into account endogenous structural breaks and thus allows the data to determine the beginning and end of each phase of the crisis. In our estimation, MS-DR model assumes the existence of two regimes (“stable” and “volatile”), where the regime 0 (“stable” regime) defines the lower values of  $h_{it}$  and the regime 1 (“volatile regime”) the higher values.

Stock market’s conditional volatilities are obtained from the AR(1)-FIAPARCH(1,d,1) model during the full sample period. The smoothed regime probabilities of  $h_{it}$  depicted in Figure 1 reveal that all the “volatile”/crisis regimes for each examined market are located within the crisis period based on economic and financial news events described above. Nevertheless, the phases, as identified by the economic approach, are treated as distinct and independent and thus the analysis of contagion does not take into account the lagged impact of the crisis on each of the Brazilian and Mexican stock markets. To overcome this problem, the estimated regimes of excess volatility are used to identify the start date and the end date of the crisis, as well as the different phases of the crisis for each stock market which are treated as overlapping. In other words, this statistical approach indicates the crisis period and the phases endogenously when each market is hit hardest by the global financial crisis.

Table 1 presents the start and the end dates of the crisis periods for each market based on regimes of excess volatility during the period from August 2007 to the end of the sample (10 December 2013). Regimes  $1\alpha \rightarrow 1j$  represent the crisis periods when each market exhibits high persistence of excess volatility (smoothed regime probabilities near one), while all other periods are identified as stable periods with no excess volatility (Regime 0).

### 3. Data and preliminary analyses

In this paper, we use daily stock price indices database.<sup>3</sup> The sample period for all data is from 2 January 2003 to 10 December 2013, leading to a sample size of 2,854 observations. The indices used are S&P500 for the US, IBOVESPA for Brazil and IPC for Mexico. The daily stock index returns are defined as logarithmic differences of stock price indices and thus computed as  $r_t = \ln(x_t/x_{t-1})$  for  $t = 1, 2, \dots, T$  where  $T$ ,  $r_t$ ,  $x_t$  and  $x_{t-1}$  are the total number of observations, the return at time  $t$ , the current stock price index, and the lagged day’s stock price index, respectively. The reason for multiplying the expression  $\ln(x_t/x_{t-1})$  by 100 is due to numerical problems in the estimation part. This will not affect the structure of the model since it is just a linear scaling.

Table 2 (Panel A) displays summary statistics for the stock return series. The IBOVESPA index is the most volatile, as measured by the standard deviation of 1.7619%, while the S&P500 index is the least volatile with a standard deviation of 1.2498%. The measure for skewness shows that Brazilian and Mexican stock returns are positively skewed and thus skewed to the right. The positive skewness indicates that large positive stock returns are more common than large negative returns. From the measure for excess kurtosis, the leptokurtic behavior is apparent in all series with more pronounced fat tails in S&P500 return. This implies that large shocks of either sign are more likely to be present and that the stock-return series may not be normally distributed. In order to accommodate the presence of leptokurtosis, we assume student-t distributed innovations. Furthermore, the Jarque-Bera statistics indicate that the assumption of normality is rejected decisively at the 1% level for all

**Figure 1. Regime classification of stock markets' conditional volatilities ( $h_t$ ).**

Notes: Regime 0, in light blue, corresponds to periods of stable and low volatility. Regime 1, in gray, denotes periods of rising and persistent volatility returns. The red columns indicate the smoothed regime probabilities, while the gray-shaded spaces are the regimes of excess volatilities according to MS-DR model.

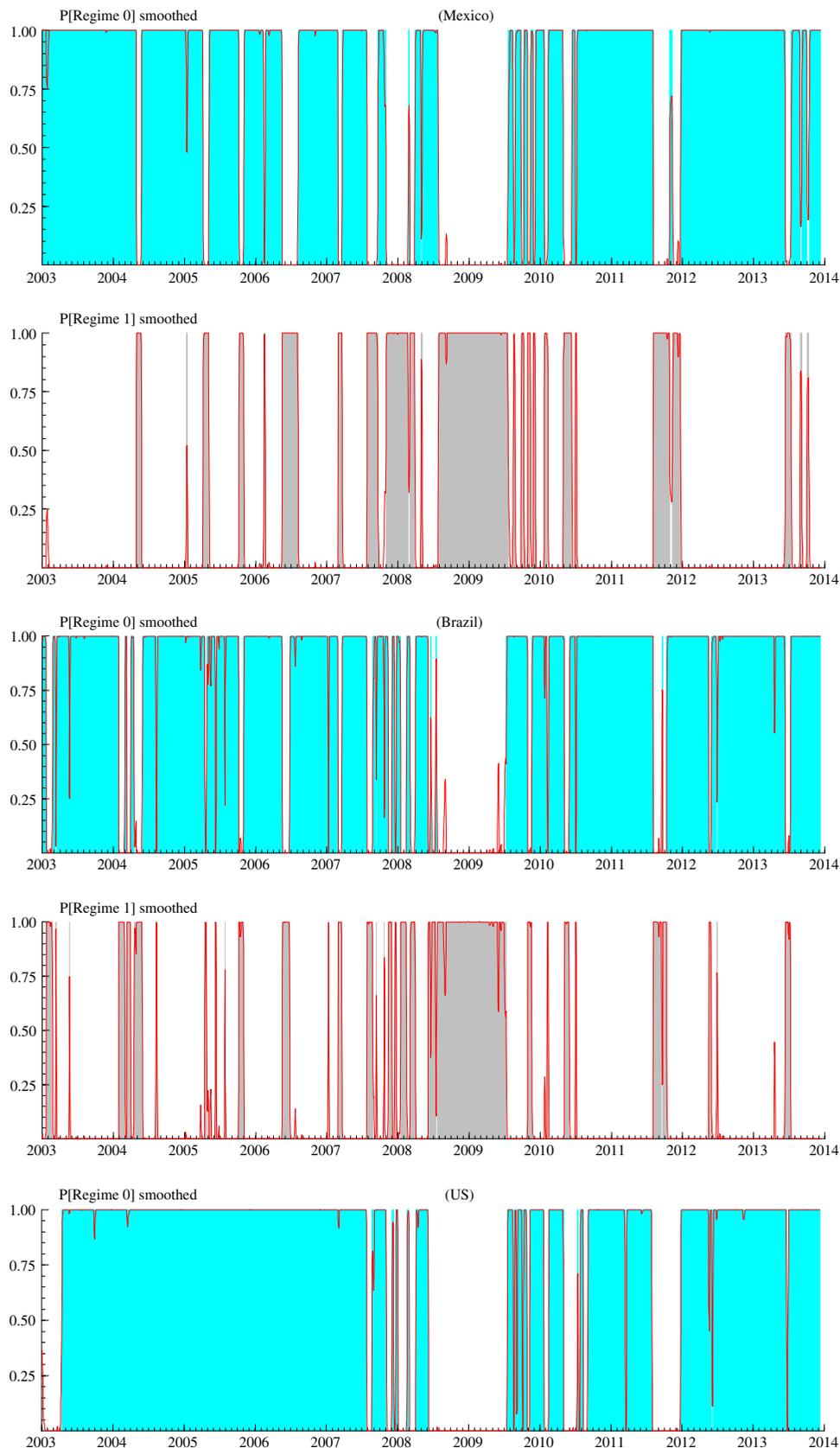
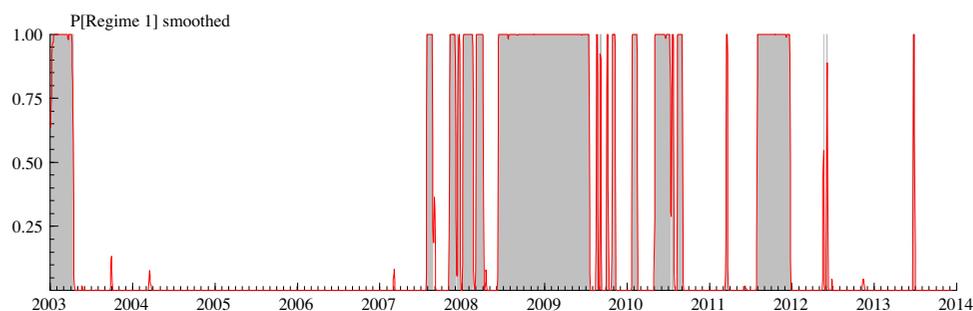


Figure 1. (Continued).



**Table 1. Global financial crisis periods based on regimes of excess volatility**

	Crisis/volatile regime 1a		Crisis/volatile regime 1b	
	Starting date	Ending date	Starting date	Ending date
Mexico	27 July 2007	20 September 2007	01 November 2007	22 February 2008
Brazil	13 November 2007	30 November 2007	15 January 2008	13 February 2008
	Crisis/volatile regime 1c		Crisis/volatile regime 1d	
	Starting date	Ending date	Starting date	Ending date
Mexico	03 March 2008	27 March 2008	28 July 2008	20 July 2009
Brazil	04 March 2008	31 March 2008	20 June 2008	11 July 2008
	Crisis/volatile regime 1e		Crisis/volatile regime 1f	
	Starting date	Ending date	Starting date	Ending date
Mexico	28 October 2009	11 November 2009	22 January 2010	09 February 2010
Brazil	21 July 2008	09 July 2009	29 October 2009	18 November 2009
	Crisis/volatile regime 1g		Crisis/volatile regime 1h	
	Starting date	Ending date	Starting date	Ending date
Mexico	29 April 2010	11 June 2010	05 August 2011	27 October 2011
Brazil	06 May 2010	28 May 2010	05 August 2011	16 September 2011
	Crisis/volatile regime 1i		Crisis/volatile regime 1j	
	Starting date	Ending date	Starting date	Ending date
Mexico	10 November 2011	22 December 2011	12 June 2013	11 July 2013
Brazil	23 September 2011	12 October 2011	12 June 2013	09 July 2013

Notes: Smoothed probabilities of regimes are estimated via a MS-DR model during the full sample period. The identification of the crisis periods for each market (regimes 1a → 1j) is based on high persistence of excess volatility (smoothed regime probabilities near one) during the period from August 2007 to the end of the sample (10 December 2013). Regimes with low persistence of excess volatility (i.e. below one week) are ignored.

indices. The non-normality is apparent from the fatter tails from the normal distribution and mild negative and positive skewness. Moreover, all return series are stationary,  $I(0)$ , suitable for long memory tests, and exhibit strong ARCH effects.

In addition, the Ljung–Box test of serial correlation on the standardized residuals shows that all stock return series exhibit significant serial correlation. Besides, the significance of the Ljung–Box test statistics on the squared standardized residuals tells us that ARCH effects are still there. The existence of such serial correlation may be explained by the non-synchronous trading of the stocks or to some form of market inefficiency, producing a partial adjustment process. The statistical significance of the ARCH-Fisher test statistics confirms the existence of ARCH in the stock return and squared return series.

**Table 2. Summary statistics and long memory test's results**

	US	Brazil	Mexico
<i>Panel A: Descriptive statistics</i>			
Mean	.0329	.0684	.0761
Maximum	11.5800	14.6560	11.0050
Minimum	-9.0350	-11.3930	-7.0080
Std. Deviation	1.2498	1.7619	1.2862
Skewness	-.0559 (.2227)	.1216*** (.0080)	.2148*** (.0000)
Excess kurtosis	11.36*** (.0000)	6.0199*** (.0000)	6.6992*** (.0000)
Jarque-Bera	15347.17*** (.0000)	4316.466*** (.0000)	5358.901*** (.0000)
Unconditional correlations (Brazil and Mexico vs. US)	-	.6709*** (.0000)	.6987*** (.0000)
<i>Panel B: Serial correlation and LM-ARCH tests</i>			
LB(20)	111.056*** (.0000)	63.621*** (.0000)	53.4724*** (.0001)
LB <sup>2</sup> (20)	4261.26*** (.0000)	3333.12*** (.0000)	2282.81*** (.0000)
ARCH 1-10	108.77*** (.0000)	99.383*** (.0000)	56.707*** (.0000)
<i>Panel C: Long memory tests (GPH test—d estimates)</i>			
<i>Squared returns r<sup>2</sup></i>			
$m = T^5$	.7369*** [8.8159]	.5021*** [8.9072]	.5086*** [6.9794]
$m = T^6$	.6794*** [12.4014]	.6501*** [12.0452]	.6595*** [11.6176]
<i>Absolute returns  r </i>			
$m = T^5$	.7308*** [7.4180]	.5027*** [6.6427]	.5532*** [4.8396]
$m = T^6$	.6646*** [10.8991]	.5312*** [9.4658]	.5172*** [8.0716]

Notes: Stock returns are daily frequency.  $r^2$  and  $|r|$  are squared log return and absolute log return, respectively.  $m$  denotes the bandwidth for the Geweke and Porter-Hudak's (1983) test. Observations for all series in the whole sample period are 2,854. The numbers in brackets are t-statistics and numbers in parentheses are p-values. LB(20) and LB<sup>2</sup>(20) are the 20th order Ljung-Box tests for serial correlation in the standardized and squared standardized residuals, respectively.

\*\*\*Statistical significance at 1% level.

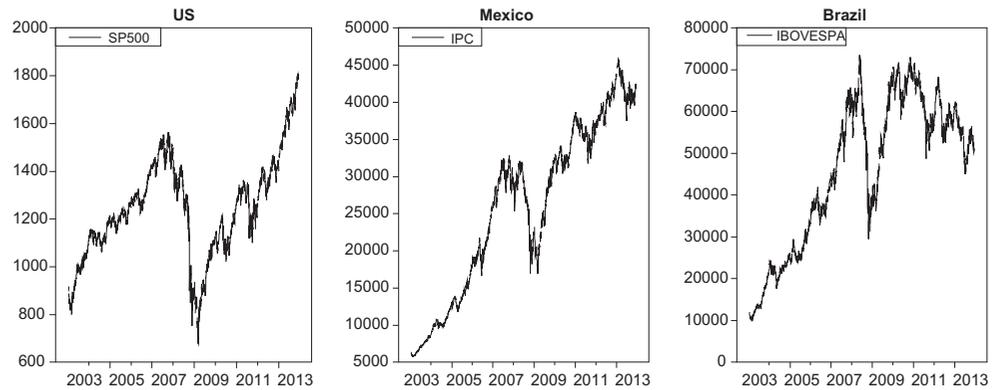
\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

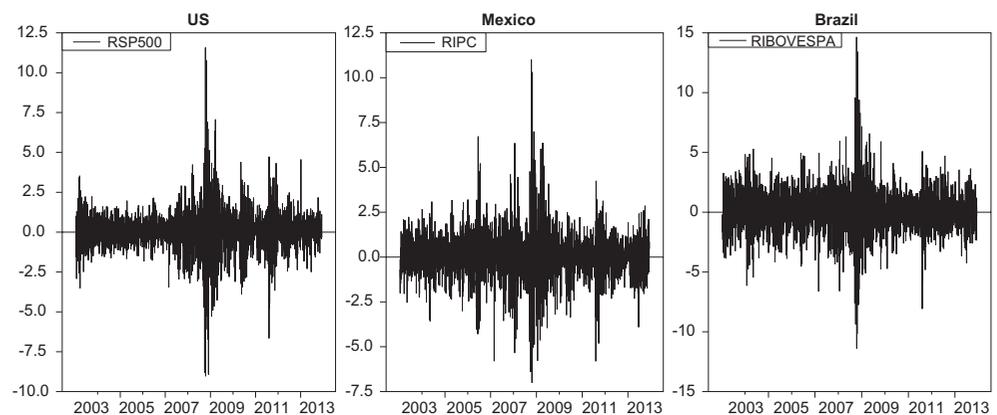
Finally, all unconditional correlations are statistically significant at the 1% level, with the higher value (.6987) between Mexico and the US and the lower one (.6709) between Brazil and the US.

In order to capture long memory phenomenon in our time series, we use the Geweke and Porter-Hudak's (1983) test on two proxies of volatility, namely squared returns and absolute returns. Based on the results displayed in Table 2 (Panel C), we conclude that there is a long-range memory process for all indices, while all volatility proxies seem to be governed by fractionally integrated process. Overall, these findings support that FIAPARCH seems to be an appropriate specification to capture volatility clustering, asymmetries, and long-range memory features.

**Figure 2. Stock price index behavior over time.**



**Figure 3. Stock return behavior over time.**



To provide more insights into stock market interactive linkages during the period under study, we depict in Figure 2 the evolution of the US, Brazilian, and Mexican stock price indices during the period from 2 January 2003 to 10 December 2013. The first impression is that the stock prices and returns almost follow a similar movement. Besides, the figure shows strong co-movements among Brazil, Mexico, and the US and significant declines in the levels during 2008, especially at the time of Lehman Brothers collapse (15 September 2008).

Besides, we depict in Figure 3 the evolution of stock market returns over time. The figure shows that all stock markets trembled since 2008. Moreover, the plot shows a clustering of larger return volatility around and after 2008. This means that the stock markets are characterized by volatility clustering, i.e. large (small) volatility tends to be followed by large (small) volatility, revealing the presence of heteroskedasticity. This market phenomenon has been widely recognized and successfully captured by ARCH/GARCH family models to adequately describe stock returns dynamics. This is important because the econometric model will be based on the interdependence of the stock markets in the form of second moments by modeling the time-varying variance-covariance matrix for the sample.

#### 4. Empirical results

##### 4.1. The univariate AR(1)-FIAPARCH(1,d,1) estimates

###### 4.1.1. Estimation results

In order to take into account the serial correlation and the GARCH effects observed in our time series data, and to detect the potential long memory in volatility, we estimate the

student-t-AR(1)-FIAPARCH(1,d,1) model<sup>4</sup> defined by Equations 1 and 5. Table 3 reports the estimation results of the student-t-AR(1)-FIAPARCH(1,d,1) model for each stock index return series of our sample.

The estimates of the constants in the mean are statistically significant at 1% level or better for all the series except for IBOVESPA index return. Besides, the constants in the variance are all significant. With the exception of IBOVESPA index, the AR coefficient is highly significant. The estimate for the  $\phi$  parameter is insignificant only for IBOVESPA index. For all indices, the estimates of the leverage term ( $\gamma$ ) are statistically significant, confirming the assumption that there is negative correlation between returns and volatility. In addition, the estimates of the power term ( $\delta$ ) and the fractional differencing parameter ( $d$ ) are highly significant for all indices. Interestingly, the highest power terms are obtained for the Mexican index, while the US and Mexican ones are characterized by the highest degree of persistence. In all cases, the estimated degrees of freedom parameter ( $\nu$ ) is highly significant and leads to an estimate of the kurtosis which is equal to  $3(\nu - 2)/(\nu - 4)$  and is also different from three.

In all cases, the ARCH parameters satisfy the set of conditions which guarantee the positivity of the conditional variance. According to the values of the Ljung–Box tests for serial correlation in the standardized and squared standardized residuals, there is no statistically significant evidence, at the 1% level, of misspecification in almost all cases.

#### 4.1.2. Tests of fractional differencing and power term parameters

Numerous studies have documented the persistence of volatility in stock returns (see Ding & Granger, 1996; Ding et al., 1993, among others). The majority of these studies have shown that the volatility process is well approximated by an IGARCH process. Nevertheless, from the FIAPARCH estimates reported in Table 3, it appears that the long-run dynamics are better modeled by the fractional differencing parameter.

**Table 3. Univariate AR(1)-FIAPARCH(1,d,1) models (maximum likelihood estimates, MLE)**

	SP500		IBOVESPA		IPC	
	Coefficient	t-prob	Coefficient	t-prob	Coefficient	t-prob
<i>Estimates</i>						
<i>c</i>	.0446**	.0008	.0481	.0860	.0820**	.0000
$\xi$	-.0615**	.0002	.0002	.9910	.0603**	.0013
$\omega$	1.6294**	.0000	3.5316**	.0000	2.1275**	.0000
<i>d</i>	.4113**	.0000	.3309**	.0000	.4328**	.0000
$\phi$	.1955**	.0000	.1266	.1974	.2177**	.0000
$\beta$	.5652**	.0000	.4040**	.0002	.5892**	.0000
$\gamma$	.9987**	.0000	.6476**	.0001	.6135**	.0000
$\delta$	1.2443**	.0000	1.3748**	.0000	1.4705**	.0000
$\nu$	6.6468**	.0000	9.5754**	.0000	7.3277**	.0000
<i>Diagnostics</i>						
LB(30)	27.9627	.5199	38.4798	.1120	20.5570	.8748
LB <sup>2</sup> (30)	33.0086	.2354	29.4580	.3896	42.0978*	.0424

Notes: For each of the five indices, Table 3 reports the MLE for the student-t-AR(1)-FIAPARCH(1,d,1) model. LB(30) and LB<sup>2</sup>(30) indicate the Ljung–Box tests for serial correlation in the standardized and squared standardized residuals, respectively.

\*\*Statistical significance at 1% level.

\*Statistical significance at 5% level.

**Table 4. Wald test of power term parameters**

	SP500		IPC		IBOVESPA	
	$\chi^2(1)$	p-value	$\chi^2(1)$	p-value	$\chi^2(1)$	p-value
$H_0: \delta = 1$	37.3941**	.0000	7.0697**	.0078	18.1588**	.0000
$H_0: \delta = 2$	347.033**	.0000	36.5475**	.0000	57.6471**	.0000

\*\*Statistical significance at 1% level.

\*Statistical significance at 5% level.

To test for the persistence of the conditional heteroskedasticity models, we examine the likelihood ratio (LR) statistics for the linear constraints  $d = 0$  (APARCH(1,1) model) and  $d \neq 0$  (FIAPARCH(1,d,1) model).<sup>5</sup> We construct a series of LR tests in which the restricted case is the APARCH(1,1) model ( $d = 0$ ) of Ding et al. (1993). Let  $l_0$  be the log-likelihood value under the null hypothesis that the true model is APARCH(1,1) and  $l$  the log-likelihood value under the alternative that the true model is FIAPARCH(1,d,1). Then, the LR test,  $2(l - l_0)$ , has a  $\chi^2$  distribution with 1 degree of freedom when the null hypothesis is true.

The outcome of the LR tests provides a clear rejection of the APARCH(1,1) model against the FIAPARCH(1,d,1) one for all indices. When the restricted case is the APARCH(1,1) model, the LR test yields a  $\chi^2$  value of 73.1 for the US, 44.036 for Brazil, 6.308 for Hong Kong, 50.368 for Mexico, and 52.594 for India. Thus, purely from the perspective of searching for a model that best describes the volatility in the stock price series, the FIAPARCH(1,d,1) model appears to be the most satisfactory representation. This finding is important since the time series behavior of volatility could affect asset prices through the risk premium (see Christensen & Nielsen, 2007; Christensen, Nielsen, & Zhu, 2010; Conrad et al., 2011).

With the aim of checking for the robustness of the LR testing results discussed above, we apply the Akaike (AIC), Schwarz (SIC), Shibata (SHIC), or Hannan–Quinn (HQIC) information criteria to rank the ARCH-type models. According to these criteria, the optimal specification (i.e. APARCH or FIAPARCH) for all indices is the FIAPARCH one. The two common values of the power term ( $\delta$ ) imposed throughout much of the GARCH literature are  $\delta = 2$  (Bollerslev’s model) and  $\delta = 1$  (the Taylor/Schwert specification). According to Brooks, Faff, and McKenzie (2000), the invalid imposition of a particular value for the power term may lead to sub-optimal modeling and forecasting performance. For that reason, we test whether the estimated power terms are significantly different from unity or two using Wald tests. Table 4 illustrates the Wald test of power term parameters.

From Table 4, we find that all the three estimated power coefficients are significantly different from unity and two at the 1% level or better. On the basis of these results, support is found for the (asymmetric) power fractionally integrated model, which allows an optimal power transformation term to be estimated.

**4.2. The bivariate AR(1)-FIAPARCH(1,d,1)-DCC estimates**

The analysis above suggests that the FIAPARCH specification describes the conditional variances of the five stock indices well. Nevertheless, stock market volatilities move together across assets and markets. According to Bauwens and Laurent (2005), Bauwens, Laurent, and Rombouts (2006), and Silvennoinen and Teräsvirta (2007), among others, recognizing this feature through a multivariate modeling structure could lead to obvious gains in efficiency compared to working with separate univariate specifications. Therefore, the multivariate FIAPARCH model seems to be essential for enhancing our understanding of the relationships between the (co)volatilities of economic and financial time series.

In this section, within the framework of the multivariate DCC model, we analyze the dynamic adjustments of the variances for the five indices. Overall, we estimate two bivariate specifications for our analysis, namely IBOVESPA-SP500 and IPC-SP500. Table 5 (Panels A and B) reports the

estimation results of the bivariate student-t-AR(1)-FIAPARCH(1,d,1)-DCC model. The constant term in the mean equation ( $\mu_0$ ) is only statistically significant for the Mexican stock market. The  $\mu_1$  parameter is significantly negative for the US, while it is significantly positive for Mexico. Antoniou,

**Table 5. Estimation results from the bivariate AR(1)-DCC-FIAPARCH(1,d,1) model**

Panel A: Estimation results	IBOVESPA-SP500				IPC-SP500			
	IBOBESPA		SP500		IPC		SP500	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
<i>Mean equation</i>								
$\mu_0$	.0412	.1415	.0283*	.0008	.0592*	.0024	.0283	.0008
$\mu_1$	-.0031	.9016	-.0371*	.0002	.0666*	.0077	-.0371	.0002
$\mu_2$	.0217	.5534	-	-	.0060	.8121	-	-
<i>Variance equation</i>								
$\omega_0$	.2600**	.0383	.0684*	.0000	.0779*	.0026	.0684	.0000
d-FIARCH	.2289*	.0004	.3886*	.0000	.4033*	.0000	.3886	.0000
$\phi_i$	.0402	.8397	.2082*	.0000	.2388*	.0003	.2082	.0000
$\beta_i$	.2226	.3517	.5621*	.0000	.5680*	.0000	.5621	.0000
$\gamma_{1i}$	.63*	.0005	.9999*	.0000	.6618*	.0000	.9999	.0000
$\delta$	1.6432*	.0000	1.2117*	.0000	1.4015*	.0000	1.2117	.0000
<i>Multivariate DCC equation</i>								
$\rho_{i,US}$			.6163*				.6101*	
(p-value)			(.0000)				(.0000)	
$a$			.0235*				.0098**	
(p-value)			(.0004)				(.0213)	
$b$			.9712*				.9887*	
(p-value)			(.0000)				(.0000)	
$v$			8.4207*				7.6455*	
(p-value)			(.0000)				(.0000)	
<i>Panel B: Diagnostic tests</i>								
LB(16)	18.2328	.3104	23.7090	.0961	10.4316	.8431	19.6461	.2366
LB <sup>2</sup> (16)	12.1883	.7309	9.6443	.8845	11.6268	.7692	11.6831	.7655
Hosking(16)	81.4911				68.4555			
(p-value)	(0.0585)				(0.2975)			
Hosking <sup>2</sup> (16)	56.1509				48.5313			
(p-value)	(0.6852)				(0.8942)			
Li-McLeod(16)	81.4531				68.4815			
(p-value)	(0.0589)				(0.2967)			
Li-McLeod <sup>2</sup> (16)	56.1887				48.5725			
(p-value)	(0.6839)				(0.8934)			

Notes: The p-values are in parentheses.  $v$  indicates the student's distribution's degrees of freedom.  $\rho_{i,US}$  is the average DCC between the US and each of stock markets (Brazil and Mexico). LB(16) and LB<sup>2</sup>(16) denote the Ljung-Box tests of serial correlation on both standardized and squared standardized residuals. Hosking(16) and Hosking<sup>2</sup>(16) denote the Hosking's multivariate portmanteau statistics on both standardized and squared standardized residuals. Li-McLeod(16) and Li-McLeod<sup>2</sup>(16) indicate the Li and McLeod's multivariate portmanteau statistics on both standardized and squared standardized residuals.

\*\*Statistical significance at 5% level.

\*Statistical significance at 1% level.

Koutmos, and Pericli (2005) argue that the negativity of the AR(1) term in the mean equation is due to the existence of positive feedback trading in markets, while the positivity of the AR(1) term is due to price friction or partial adjustment. Moreover, the  $\mu_2$  coefficient is insignificant for the Brazilian and Mexican markets. The effect of the US stock returns on the returns of Brazil and Mexico is insignificant. This proves the absence of a potential effect of the US stock market on those emerging stock markets. The constant term in the variance equation ( $\omega_{i0}$ ) is statistically significant for all stock markets.

The ARCH and GARCH effects are captured by, respectively, the parameters  $\phi_i$  and  $\beta_i$ . The coefficients for the lagged variance ( $\beta_i$ ) are positive and statistically significant for all stock markets, except the Brazilian case. Besides, the parameters  $\phi_i$  in the variance equation are positive and significantly different from zero for all stock returns, except the IBOVSPA one. This justifies the suitability of the FIAPARCH(1,d,1) specification as the best fitting of the time-varying volatility. In addition, the estimation results provide evidence that the index returns exhibit fractional dynamics. The estimated  $d$ -value is statistically significant, greater than zero, and indicates the presence of positive persistence phenomenon in the index series volatility. Thus, volatility displays the long memory or long range dependence property.

Besides, the power term  $\delta$  of stock returns for the predictable structure in the volatility pattern is positive and statistically significant for all stock markets. In addition, the estimates for the leverage parameter  $\gamma$  are positive and statistically significant for all stock market returns, confirming the assumption of the leverage effect. Indeed, a positive value of  $\gamma$  means that past negative shocks have a severe impact on current conditional volatility than past positive shocks. That is, negative shocks give rise to higher volatility than positive shocks. Moreover, the  $\gamma$  parameter accounts for volatility asymmetry in which positive and negative returns of the same magnitude do not produce an equal degree of volatility.

In Table 5, we also report the estimates of the bivariate DCC(1,1) model. The parameters  $a$  and  $b$  of the DCC(1,1) model, respectively, capture the effects of standardized lagged shocks and the lagged DCCs effects on current DCC. The statistical significance of these coefficients in each pair of stock markets indicates the existence of time-varying correlations. When  $a = 0$  and  $b = 0$ , we obtain the Bollerslev's (1990) CCC model. As shown in Table 5, the estimated coefficients  $a$  and  $b$  are significantly positive and satisfy the inequality  $a + b < 1$  in each of the pairs of stock markets.

The significativity of the DCC parameters ( $a$  and  $b$ ) reveals a considerable time-varying co-movement and thus a high persistence of the conditional correlation. The sum of these parameters is close to unity and ranges between .9947 (US–Brazil) and .9985 (US–Mexico). This implies that the volatility displays a highly persistent fashion. Since  $a + b < 1$ , the dynamic correlations revolve around a constant level and the dynamic process appears to be mean reverting. We also note the existence of two unconditional correlations pairs of the standardized innovations from the estimated univariate FIAPARCH models. The values of these correlations vary between .6101 (US–Mexico) and .6163 (US–Brazil).

The multivariate DCC-FIAPARCH model has some advantages. First, it captures the long range dependence property. Second, it allows obtaining all possible pair-wise conditional correlation coefficients for the index returns in the sample. Third, it's possible to investigate their behavior during periods of particular interest, such as periods of the global financial crisis. Fourth, the model allows looking at possible financial contagion effects between the US and some international stock markets which have been affected by the recent financial crisis.

Finally, it is crucial to check whether the selected index time series display evidence of multivariate long memory ARCH effects and to test the ability of the multivariate FIAPARCH specification to capture the volatility linkages between stock markets. Kroner and Ng (1998) have confirmed the fact that only few diagnostic tests are kept to the multivariate GARCH-class models compared to the diverse

**Table 6. Constant correlation tests**

	$\rho_{\text{Mexico, US}}$		$\rho_{\text{Brazil, US}}$	
	Coefficient	p-value	Coefficient	p-value
LMC test statistic	31.4463***	.0000	35.8925***	.0000
ES test statistics				
E-S test(5)	16.8687***	.0098	22.8231***	.0008
E-S test(10)	43.2182***	.00001	51.4514***	.0000

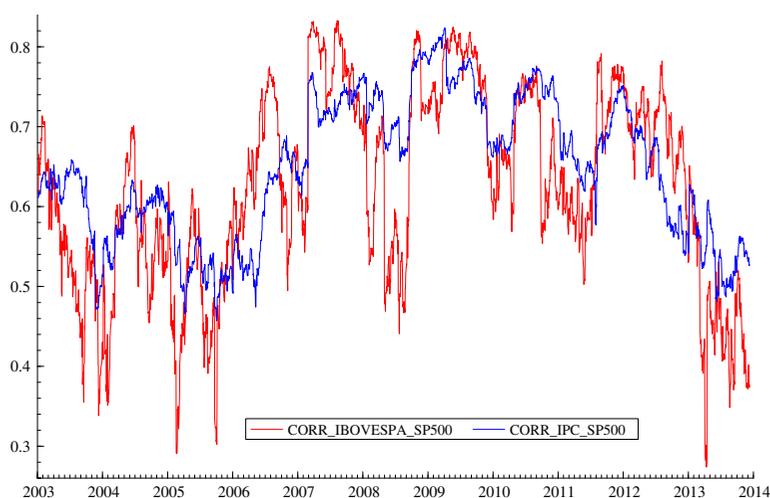
Note: LMC  $\sim \chi^2(N(N - 1)/2)$  under  $H_0$ : CCC model. ES test(j)  $\sim \chi^2(j + 1)$  under  $H_0$ : CCC model.

\*\*\*Statistical significance at 1% level.

\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

**Figure 4. The DCC behavior over time.**



diagnostic tests devoted to univariate counterparts. Furthermore, Bauwens et al. (2006) have noted that the existing literature on multivariate diagnostics is sparse compared to the univariate case. In our study, we refer to the most broadly used diagnostic tests, namely the univariate Ljung–Box tests of serial correlation on the standardized and squared standardized residuals as well as the Hosking’s and Li and McLeod’s multivariate portmanteau statistics on both standardized and squared standardized residuals. According to the Hosking (1980), Li and McLeod (1981), and McLeod and Li (1983) autocorrelation test results reported in Table 5 (Panel B), the multivariate diagnostic tests allow accepting the null hypothesis of no serial correlation on the standardized and squared standardized residuals, and thus there is no evidence of statistical misspecification.

In order to further examine whether the conditional correlations changed over time, we use the LM Test for constant correlation of Tse (2000) and the Engle and Sheppard (2001) test for dynamic correlation. Table 6 reports the tests results, which show a statistically significant rejection of the CCC hypothesis for both pair-wise correlations.

Figure 4 illustrates the evolution of the estimated DCCs dynamics among the US, Brazil, and Mexico. A common characteristic is the increasing trend of both DCCs with different intensities over time. The above findings motivate a more extensive analysis of DCCs, in order to capture contagion dynamics during different phases of the global financial crisis.

### 4.3. Statistical analysis of DCC coefficients

In what follows, we examine the DCCs shifts behavior and provide further findings on the contagion effects during different phases of the global financial crisis. In order to identify which of the sub-periods exhibit contagion effect for the Brazilian and Mexican stock markets, we create dummy

variables, which are equal to unity for the corresponding phase of the crisis and zero otherwise. In order to describe the behavior of the DCCs over time (see Chiang et al., 2007; Engle, 2002, among others), the dummies are created to the following mean equation:

$$\rho_{ij,t} = \omega_{ij} + \sum_{p=1}^P \varphi_p \rho_{ij,t-p} + \sum_{k=1}^{\lambda} \beta_k \text{dummy}_{k,t} + e_{ij,t} \tag{13}$$

where  $\omega_{ij}$  is a constant term,  $\rho_{ij,t}$  is the pair-wise conditional correlation between the stock returns  $i$  (Brazil and Mexico) and the stock returns  $j$  (US), and  $k = 1, \dots, \lambda$  are the number of dummy variables corresponding to the different phases of the global financial crisis, which are identified based on an economic and a statistical approach.

Based on the economic approach,  $\text{dummy}_{k,t}$  ( $k = 1, 2, 3$ ) corresponds to the first three phases of the global financial crisis, while the fourth dummy ( $k = 4$ ) to the phase of “stabilization and recovery” (post-crisis period). Following the statistical approach, the first two dummies ( $k = 1, 2$ ) correspond to the 10 crisis regimes ( $1a \rightarrow 1j$ ) presented in Table 1, while the 11 dummy to the post-crisis regime (from the end of crisis regime 1j onwards).

Next, we examine whether the conditional variance equation of the DCCs series exhibits symmetries or asymmetries behavior following Engle and Ng (1993). Table 7 reports the stochastic properties of the DCCs. From this table, we conclude that the pair-wise DCCs exhibit significant departure from the normal distribution assumption. Besides, there is a strong ARCH effect for both DCCs series.

Engle and Ng (1993) propose a set of tests for asymmetry in volatility, known as sign and size bias tests. The Engle and Ng tests should thus be used to determine whether an asymmetric

**Table 7. Statistical properties of the multivariate FIAPARCH-DCCs**

	$\rho_{\text{Brazil, US}}$	$\rho_{\text{Mexico, US}}$
Mean	.6248	.6470
Maximum	.8328	.8236
Minimum	.2739	.4571
Std. Deviation	.1219	.0882
Skewness	-.3169*** (.0000)	-.1138** (.0130)
Excess kurtosis	-.7089*** (.0000)	-1.0674*** (.0000)
Jarque-Bera	107.5338*** (.0000)	141.6318*** (.0000)
<i>LM-ARCH(10) test</i>		
ARCH(10)	16063*** (.0000)	49298*** (.0000)
<i>Serial correlation test</i>		
LB(10)	25460.4*** (.0000)	27518.6*** (.0000)
LB <sup>2</sup> (10)	25834.6*** (.0000)	27575*** (.0000)
<i>Unit root test</i>		
ADF test statistic	-.9239	-.4801
	I(0)	I(0)

Notes: The asymptotic critical values of ADF unit root test are -2.5658 (1%), -1.9409 (5%), and -1.6166 (10%). The numbers in parentheses are  $p$ -values. LB(10) and LB<sup>2</sup>(10) denote the Ljung-Box tests of serial correlation on both standardized and squared standardized residuals.

\*\*\*Statistical significance at 1% level.

\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

model is required for a given series, or whether the symmetric GARCH model can be deemed adequate. In practice, the Engle–Ng tests are usually applied to the residuals of a GARCH fit to the returns data.

Define  $S_{t-1}^-$  as an indicator dummy variable such as:

$$S_{t-1}^- = \begin{cases} 1 & \text{if } \hat{z}_{t-1} < 0 \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

The test for sign bias is based on the significance or otherwise of  $\phi_1$  in the following regression:

$$\hat{z}_t^2 = \phi_0 + \phi_1 S_{t-1}^- + v_t \quad (15)$$

where  $v_t$  is an independent and identically distributed error term. If positive and negative shocks to  $\hat{z}_{t-1}$  impact differently upon the conditional variance, then  $\phi_1$  will be statistically significant.

It could also be the case that the magnitude or size of the shock will affect whether the response of volatility to shocks is symmetric or not. In this case, a negative size bias test would be conducted, based on a regression where  $S_{t-1}^-$  is used as a slope dummy variable. Negative size bias is argued to be present if  $\phi_1$  is statistically significant in the following regression:

$$\hat{z}_t^2 = \phi_0 + \phi_1 S_{t-1}^- z_{t-1} + v_t \quad (16)$$

Finally, we define  $S_{t-1}^+ = 1 - S_{t-1}^-$ , so that  $S_{t-1}^+$  picks out the observations with positive innovations. Engle and Ng (1993) propose a joint test for sign and size bias based on the following regression:

$$\hat{z}_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \phi_2 S_{t-1}^- z_{t-1} + \phi_3 S_{t-1}^+ z_{t-1} + v_t \quad (17)$$

Statistical significance of  $\phi_1$  indicates the presence of sign bias, where positive and negative shocks have differing impacts upon future volatility, compared with the symmetric response required by the standard GARCH formulation. However, the significance of  $\phi_2$  or  $\phi_3$  would suggest the presence of size bias, where not only the sign but also the magnitude of the shock is important. A joint test statistic is formulated in the standard fashion by calculating  $TR^2$  from regression (17), which will asymptotically follow a  $\chi^2$  distribution with 3 degrees of freedom under the null hypothesis of no asymmetric effects.

Table 8 reports the results of Engle–Ng tests. First, the individual regression results show that the residuals of the symmetric GARCH model for the  $\rho_{\text{Brazil, US}}$  series do suffer from sign bias and/or positive size bias, but they do exhibit negative size bias. Second, for the  $\rho_{\text{Mexico, US}}$  series, the individual regression results show that the residuals of the symmetric GARCH model do not suffer from sign bias, negative size bias, and/or positive size bias. Finally, the  $\chi^2(3)$  joint test statistics have  $p$ -values

**Table 8. Tests for sign and size bias for DCCs**

Variable	$\rho_{\text{Brazil, US}}$			$\rho_{\text{Mexico, US}}$		
	Coefficient	Std. Error	p-value	Coefficient	Std. Error	p-value
$\phi_0$	1.1704***	.0962	.0000	1.0587***	.1607	.0000
$\phi_1$	-.3128**	.1349	.0206	-.2009	.2353	.3933
$\phi_2$	-.156	.0949	.1003	-.1468	.1723	.3942
$\phi_3$	-.1936**	.0964	.0448	-.0630	.1607	.6950
$\chi^2(3)$	6.8914*	-	.0754	.8807	-	.8301

\*\*\*Statistical significance at 1% level.

\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

of .0754 and .8301, respectively, demonstrating a very acceptance of the null of no asymmetries for the  $\rho_{\text{Mexico, US}}$  series and a rejection for the null hypothesis for the  $\rho_{\text{Brazil, US}}$  series. The results overall would thus suggest motivation for estimating a symmetric GARCH and asymmetric GARCH volatility models, respectively, for these particular series.

Furthermore, the conditional variance equation of the  $\rho_{\text{Brazil, US}}$  series is assumed to follow an asymmetric GARCH (1,1) specification under a student distributed innovations. In our analysis, we choose the student-t-GJR-GARCH(1,1) model (Glosten, Jagannathan, & Runkle, 1993) including the dummy variables identified by the two approaches:

$$h_{ij,t} = \alpha_0 + \alpha_1 h_{ij,t-1} + \sum_{k=1}^{\lambda} \xi_k \text{dummy}_{k,t} + v_1 e_{ij,t-1}^2 + \alpha_2 e_{ij,t-1}^2 I(e_{ij,t-1} < 0) \quad (18)$$

Moreover, the conditional variance equation of the  $\rho_{\text{Mexico, US}}$  series is assumed to follow a symmetric student-t-GARCH (1,1) specification:

$$h_{ij,t} = \alpha_0 + \alpha_1 h_{ij,t-1} + v_1 e_{ij,t-1}^2 + \sum_{k=1}^{\lambda} \xi_k \text{dummy}_{k,t} \quad (19)$$

The statistical significance of the estimated dummy coefficients indicates structural changes in mean and/or variance shifts of the correlation coefficients due to external shocks during the different periods of the global financial crisis. Specifically, a positive and statistically significant dummy coefficient in the mean equation indicates that the correlation during a specific period of the crisis is significantly different from that of the previous phase, supporting a financial contagion effect.

#### 4.3.1. Economic and statistical approaches' results

Table 9 reports the estimates of the mean and variance equations, setting a dummy variable for each phase of the global financial crisis according to the economic approach. For the mean equation, dummy coefficient  $\beta_{1,t}$  is positive and not significantly different from that of the pre-crisis period (stable) for both the Brazilian and Mexican markets during the first phase of the crisis ("initial financial turmoil"), implying a "decoupling" from the global financial crisis and thus supporting the absence of financial contagion effects. At the next phase of "sharp financial market deterioration," Brazilian's and Mexican's dummy coefficient  $\beta_{2,t}$  is statistically significant, implying a "recoupling" from the global financial crisis and thus the presence of contagion phenomenon. During the third phase of "macroeconomic deterioration," "recoupling" is present for the Brazilian and Mexican markets (i.e. significant  $\beta_{3,t}$  coefficient), indicating the occurrence of contagion effect. Finally, Brazilian's dummy coefficient ( $\beta_{4,t}$ ) is negative and statistically significant at the fourth phase of "stabilization and signs of recovery" (post-crisis period), supporting their decreasing dependence with the US. Nevertheless, "decoupling" is present only for the Mexican market (i.e. insignificant  $\beta_{4,t}$  coefficient).

Table 10 reports the estimating results of the statistical approach using 10 dummy variables for the periods identified by regimes of excess volatility. The first major difference from the results of Table 9 is observed on the initial stages of the global financial crisis, since contagion effects in Brazil and Mexico started to appear before the collapse of Lehmann Brothers (15 September 2008). This finding indicates that the impact of the crisis on each stock market has not been lagged. Specifically, a contagion effect exists from 28 July 2008 to 20 July 2009 (crisis regime 1d) and from 10 November 2011 to 22 December 2011 (crisis regime 1i) for Mexico. Besides, the Brazilian market seems to be affected by a contagion effect from 21 July 2008 to 09 July 2009 (crisis regime 1e).

A second difference from the results of Table 9 is observed for Brazil and Mexico. In contrary to the results based on the economic approach, which report signs of contagion after the Lehmann Brothers collapse (phase 1 of initial turmoil) for Mexico, the statistical approach does not show a crisis regime during this period. In addition, Mexico exhibits contagion during the period from 10 November 2011 to 22 December 2011 (crisis regime 1i), while the results from the economic

**Table 9. Tests of changes in dynamic correlations between the US stock market returns and those of Brazil and Mexico during the phases of global financial crisis (economic approach)**

Mean equation	US–Mexico			US–Brazil		
	Coefficient	Std. Error	p-value	Coefficient	Std. Error	p-value
$\omega_{ij}$	.0014***	.000005	.0000	.0033***	.000028	.0000
$\varphi_1$	.9978***	.000134	.0000	.9942***	.000067	.0000
$\beta_1$	.0001	.000171	.6996	.0003	.000228	.1963
$\beta_2$	.0003***	.000034	.0000	.0006***	.000213	.0097
$\beta_3$	.0004*	.000231	.0920	.0003***	.000106	.0014
$\beta_4$	-.0001	.000119	.3094	-.0001***	.000005	.0000
<i>Variance equation</i>						
$\alpha_0$	.0005***	.000021	.0000	.0006***	.000006	.0000
$\nu_1$	-.0564***	.009662	.0000	-.0202***	.000210	.0000
$\alpha_1$	.9741***	.019580	.0000	.3446***	.001110	.0000
$\alpha_2$	-		-	.5994***	.015893	.0000
$\xi_1$	-.0002***	.000010	.0000	-.0001***	.000015	.0000
$\xi_2$	-.0003***	.000012	.0000	-.0005***	.000006	.0000
$\xi_3$	-.0004***	.000017	.0000	.0091***	.000073	.0000
$\xi_4$	.0001***	.000002	.0000	-.0002***	.000005	.0000
$\nu$	2.0006***	.000504	.0000	2.0868***	.003902	.0000
<i>Diagnostics</i>						
LB(20)	21.341	-	.3773	19.812	-	.4697
LB <sup>2</sup> (20)	2.081	-	1.0000	12.909	-	.8813

Notes: Estimates are based on mean Equation 13 and variance Equations 18 and 19 in the text.  $\varphi_p$  is the coefficient of the pair-wise conditional correlation ( $\rho_{t-1}$  with 1 lag among the US and Brazil and among the US and Mexico. The lag length is determined by the SIC criteria (Box–Jenkins procedure).  $\beta_{k,t}$  and  $\xi_{k,t}$ , where  $k = 1, 2, 3, 4$ , are the dummy variable coefficients for the three phases of the global financial crisis and the fourth phase of “stabilization and recovery” (based on financial/economic events),  $\alpha_1$  is the coefficient of  $h_{t-1}$ , and  $\alpha_2$  is the asymmetric (GJR) term. LB(20) and LB<sup>2</sup>(20) denote the Ljung–Box tests of serial correlation on both standardized and squared standardized residuals.

\*\*\*Statistical significance at 1% level.

\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

approach support its immunity for the last months of 2011 (phase 4 of stabilization and tentative signs of recovery). This finding may be due to the fact that the statistical approach treats the second, third, and fourth phases from the economic approach as an overlapping phase. Therefore, each phase affects the other, and until the crisis has passed, no phase seems to have a clear end point (see Dimitriou et al., 2013). On the other hand, the dependence among Brazil and the US is significantly decreased during the post-crisis regime, which is in line with results based on the economic approach.

As a final point, a higher volatility of the correlation structure implies that the stability of the correlation is less reliable for the execution of investment strategies. Nevertheless, the dummy variables in variance equation reported in Tables 9 and 10 are either negative or positive and highly significant for both markets during the phases of the crisis identified by either the economic or the statistical approach. This result indicates that correlation coefficient volatility is either increased or decreased, helping investors to use it as a guide for portfolio decisions.

**Table 10. Tests of changes in dynamic correlations between the US stock market returns and those of Brazil and Mexico during the phases of global financial crisis (statistical approach)**

Mean equation	US–Mexico			US–Brazil		
	Coefficient	Std. Error	p-value	Coefficient	Std. Error	p-value
$\omega_{ij}$	.0018297***	.0005175	.0004	.0039439***	.0000840	.0000
$\varphi_1$	.9969096***	.0007978	.0000	.9934736***	.0001415	.0000
$\beta_1$	.0006923	.0004559	.1289	.0009575	.0010938	.3814
$\beta_2$	.0003897	.0002725	.1526	-.0017901	.0019882	.3679
$\beta_3$	.0003108	.0006529	.6340	.0016421	.0011886	.1671
$\beta_4$	.0005148***	.0001843	.0052	-.0019489	.0048885	.6901
$\beta_5$	.0010945	.0010204	.2835	.0006007*	.0003508	.0869
$\beta_6$	-.0014265	.0009415	.1297	.0046007	.0028223	.1031
$\beta_7$	.0002480	.0006444	.7004	.0007957	.0022801	.7271
$\beta_8$	.0005296	.0003768	.1598	.0005322	.0012495	.6702
$\beta_9$	.0007900**	.0003828	.0391	.0010221	.0021770	.6387
$\beta_{10}$	.0004484	.0008410	.5939	.0021588	.0031634	.4950
$\beta_{11}$	-.0000373	.0003503	.9152	-.0003508	.0004508	.4365

*Variance equation*

C	.0023348***	.0001131	.0000	.0000712***	.0000006	.0000
$\nu_1$	-.4982828***	.0395981	.0000	-.0077506***	.0000885	.0000
$\alpha_1$	.9518713***	.0246623	.0000	.7884086***	.0013429	.0000
$\alpha_2$	-	-	-	.2459675***	.0039656	.0000
$\xi_1$	.0003219***	.0000369	.0000	.0001000***	.0000052	.0000
$\xi_2$	-.0013627***	.0000581	.0000	.0001500***	.0000039	.0000
$\xi_3$	.0019627***	.0002541	.0000	-.0000087***	.0000025	.0004
$\xi_4$	-.0012153***	.0000393	.0000	.0009058***	.0000459	.0000
$\xi_5$	.0003018***	.0000463	.0000	-.0000407***	.0000021	.0000
$\xi_6$	.0006279*	.0003337	.0599	.0006086***	.0000103	.0000
$\xi_7$	.0027808***	.0001317	.0000	.0002193***	.0000033	.0000
$\xi_8$	-.0004424***	.0000603	.0000	-.0000096	.0000163	.5541
$\xi_9$	-.0012275***	.0000555	.0000	.0000213***	.0000043	.0000
$\xi_{10}$	.0013483***	.0000598	.0000	.0000711***	.0000046	.0000
$\xi_{11}$	.0009626***	.0000507	.0000	.0000019*	.00000098	.0570
$\nu$	2.0002459***	.0001425	.0000	2.1906783***	.0178383	.0000

*Diagnostics*

LB(16)	15.371	-	.4977	13.272	-	.6528
LB <sup>2</sup> (16)	1.779	-	1.0000	3.819	-	.9992

Notes: Estimates are based on mean Equation 13 and variance Equations 18 and 19 in the text.  $\varphi_0$  is the coefficient of the pair-wise conditional correlation ( $\rho_{t-1}$  with 1 lag among the US and Brazil and among the US and Mexico). The lag length is determined by the SIC criteria (Box–Jenkins procedure).  $\beta_{k,t}$  and  $\xi_{k,t}$ , where  $k = 1, 2, 3, 4$ , are the dummy variable coefficients for the three phases of the global financial crisis and the fourth phase of “stabilization and recovery” (based on financial/economic events),  $\alpha$  is the coefficient of  $h_{t-1}$ , and  $\alpha_2$  is the asymmetric (GJR) term. LB(16) and LB<sup>2</sup>(16) denote the Ljung–Box tests of serial correlation on both standardized and squared standardized residuals.

\*\*\*Statistical significance at 1% level.

\*\*Statistical significance at 5% level.

\*Statistical significance at 10% level.

#### 4.3.2. Economic and policy implications

The conditional correlation estimates following both crisis identification approaches show a general pattern of the so-called “decoupling–recoupling” phenomenon for Brazil and Mexico during the early stages of the crisis and the existence of contagion effects before and after the Lehman Brothers collapse. Some recent studies found support for the “decoupling” view (see Bekiros, 2014; Berglöf, Korniyenko, Plekhanov, & Zettelmeyer, 2010; Bianconi, Yoshino, & Machado de Sousa, 2013; Dooley & Hutchinson, 2009; Dimitriou et al., 2013, among others). This is based on the assumption that the emerging markets are nowadays the major drivers of world economic growth as opposed to the US economy.

These results could be explained as follows. After a decade of growth, the Brazilian and Mexican economies have built up strong consumer demand accumulated high levels of foreign exchange reserves and significant budget surpluses. Another potential clarification for the “decoupling” phenomenon at the beginning of the crisis may be that investors considered the news of the US subprime crisis as a single-country case and the crisis signal has not fully recognized. Nonetheless, the severity of the global financial crisis proved insufficient to preserve immunity after 15 September 2008. This may be caused by shifts in investors’ common but changing appetite for risk. As the global financial crisis become deep, investors scramble to sell their assets and move into cash producing higher correlations and leading to contagion effects.

It appears that investors’ appetite for risk falls after the Lehmann Brothers collapse. Therefore, they immediately reduce their exposure to stock markets, which they consider risky markets. This behavior consequently leads the Brazilian, Mexican, and US stock markets to fall in value together. This result is consistent with the DCC paths shown in Figure 4. Finally, Brazil and Mexico do not exhibit increasing co-movements with the US from early 2009 onwards (post-crisis period).

The contagion results in Brazil and Mexico during different periods of the global financial crisis seem to be unrelated to their trade and financial characteristics. Table 11 reports the trade and financial profiles of Brazil and Mexico during the period 2007–2012. The trade characteristics of Mexico reveal more sensitive revenues from exports of manufactured products (finished-product export-oriented markets), while the economic performance of Brazil depends greatly on exports of commodity products (commodity price-dependent markets).

Furthermore, it is worth noting that Mexico has a high degree of economic openness, measured by trade to GDP ratios, while Brazil has the lowest degree. Our evidence based on both economic and statistical crisis identification approaches have shown a pattern of contagion for the Brazilian and Mexican markets that could be attributed to their common trade characteristics. Indeed, both markets are decoupled at the beginning of the global financial crisis (economic approach), while contagion appears faster in Mexico than in Brazil (statistical approach), and the crisis seems to affect the two commodity price-dependent markets during different crisis periods.

Finally, the financial profiles of the Brazilian and Mexican markets are also different. Brazil has the highest ratio of market capitalization to GDP and Mexico the lowest one. In addition, the most liquid market (measured by turnover to market capitalization) is Brazil, while Mexico is the less liquid one. A similar pattern of contagion for deeper and more liquid stock markets would be expected, since these financial characteristics may serve as channels of financial contagion. Nevertheless, the economic and financial variables seem to be irrelevant to the correlation dynamics of Brazil and Mexico during the global financial crisis. Therefore, the connection of economic and financial structure to the contagion dynamics needs further analysis in terms of quantifying the impact of macroeconomic and financial variables on stock markets’ behavior.

#### 5. Conclusion

The present study contributes to the literature on contagion phenomenon between emerging and the US financial markets during the recent global financial crisis. It uses a multivariate student-t-AR(1)-FIAPARCH(1,d,1)-DCC approach to investigate the transmission of the global financial crisis

**Table 11. Trade and financial profiles of Brazil and Mexico**

	2007	2008	2009	2010	2011	2012
<i>Brazil</i>						
Agricultural raw materials exports (% of merchandise exports)	3.78	3.53	3.77	3.94	3.53	3.79
Food exports (% of merchandise exports)	26.35	27.58	34.20	31.08	30.50	32.19
Fuel exports (% of merchandise exports)	8.29	9.46	9.00	10.14	10.54	11.03
Ores and metal exports (% of merchandise exports)	11.05	12.13	11.71	17.79	19.30	15.59
Exports of commodities-% of merchandise exports	49.47	52.70	58.68	62.94	63.88	62.61
Manufactures exports (% of merchandise exports)	47.85	44.85	39.47	37.06	34.12	35.04
Trade (% of GDP)	25.21	27.14	22.12	22.77	24.51	26.62
Market capitalization of listed companies (% of GDP)	100.26	35.64	72.05	72.12	49.62	54.69
Turnover ratio (%)	56.21	74.27	73.91	66.43	69.29	67.88
<i>Mexico</i>						
Agricultural raw materials exports (% of merchandise exports)	.36	.36	.35	.36	.38	.39
Food exports (% of merchandise exports)	5.36	5.55	6.99	6.06	6.33	5.88
Fuel exports (% of merchandise exports)	15.77	17.39	13.51	14.04	16.30	14.37
Ores and metal exports	2.76	2.66	2.54	2.99	3.99	3.81
Exports of commodities (% of merchandise exports)	24.24	25.96	23.39	23.45	26.99	24.45
Manufactures exports (% of merchandise exports)	72.14	73.57	76.03	76.02	72.32	74.34
Trade (% of GDP)	57.06	58.07	56.03	60.95	63.75	66.39
Market capitalization of listed companies (% of GDP)	38.12	21.16	38.04	43.20	34.93	44.25
Turnover ratio (%)	30.99	34.33	26.89	27.31	25.95	25.31

Notes: This table presents the trade profiles and some financial characteristics of Brazil and Mexico markets during the period 2007–2012. The total exports of commodity products are equal to the sum of the exports of agricultural raw materials, food, fuel, ores, and metal products. Trade to GDP ratio is the sum of exports and imports of goods and services measured as a share of gross domestic product. Market capitalization of listed companies is in percentage of GDP. Turnover ratio corresponds to the value of shares traded during a year divided by the average market capitalization for a year. Data are obtained from the World Development Indicators database.

from the US to the Brazilian and Mexican stock markets. A preliminary analysis shows that this model is appropriate to capture long memory feature, asymmetries, and leverage effects of the selected stock markets.

The length and the phases of the crisis are identified based on both economic and statistical approaches, respectively. The first method is based on all key international financial and economic events that represent the global financial crisis, while the second one is based on a MS-DR that explores the regimes of excess volatility for each emerging stock market. The economic approach during the four phases of the crisis supports a general pattern of “recoupling” for Brazil and Mexico after the failure of Lehman Brothers. However, the statistical analysis of conditional correlation coefficients during the several phases of the crisis shows a general pattern of “decoupling” for the Brazilian and Mexican stock markets. In addition, a “recoupling” phenomenon is observed for both markets at the early stages of the global financial crisis. Moreover, the “recoupling” pattern is only shown for the Mexican market during the period spanning from 10 November 2011 to 22 December 2011 (crisis regime 1). These findings show a general pattern of contagion for some crisis periods that could be attributed to their common trade and financial characteristics. Based on the economic analysis, the results indicate a decreasing co-movement among the US and Brazil during the post-crisis period (from early 2009 onwards), implying that dependence is larger in bearish than in bullish markets.

From policy-makers' perspective, this study offers important information about the directions for potential undertaking measures in order to protect emerging markets from contagion during future crises. They should cautiously examine and uncover possible "decoupling" strategies that may protect emerging markets from contagion. In particular, when a decline in investors' appetite for risk happened after a period of high-risk appetite, markets will have a high tendency to react adversely to events that might not otherwise warrant major reaction (Dimitriou et al., 2013). Therefore, it may be appropriate to provide funding expeditiously to stabilize sentiment and have lighter conditionality (Kumar & Persaud, 2001). Increasing liquidity rapidly could substantially decrease the magnitude of a crisis and potential contagion effects. Finally, future research could attempt to model the contagion phenomenon by quantifying the impact of macroeconomic and financial factors on stock markets' behavior.

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**Author details**

Zouheir Mighri<sup>1</sup>  
 E-mail: [zmighri@gmail.com](mailto:zmighri@gmail.com)  
 Faysal Mansouri<sup>1</sup>  
 E-mail: [faysal.mansouri@fsegt.rnu.tn](mailto:faysal.mansouri@fsegt.rnu.tn)

<sup>1</sup> Laboratoire de Recherche en Economie, Management et Finance Quantitative (LAREMFQ), Institut des Hautes Etudes Commerciales de Sousse, University of Sousse, Route Hzamia Sahloul 3 - B.P. 40, 4054 Sousse, Tunisia.

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**Notes**

- Engle (2002) derives a different form of DCC model. The evolution of the correlation in DCC is given by:  $Q_t = (1 - \alpha - \beta)Q + \alpha z_{t-1} + \beta Q_{t-1}$ , where  $Q = (q_{ij})$  is the  $N \times N$  time-varying covariance matrix of  $z_t$ ,  $Q = E[z_t z_t']$  denotes the  $n \times n$  unconditional variance matrix of  $z_t$ , while  $\alpha$  and  $\beta$  are non-negative parameters satisfying  $(\alpha + \beta) < 1$ . Since  $Q_t$  does not generally have units on the diagonal, the conditional correlation matrix  $R_t$  is derived by scaling  $Q_t$  as follows:  $R_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2}$ .
- In MS-DR model, the lags of the dependent variable are added in the same way as other regressors. An example is:  

$$y_t = v(s_t) + \alpha y_{t-1} + X_t' \beta + \varepsilon_t$$
 where  $\varepsilon_t \rightarrow N(0, \sigma^2)$   
 $s_t$  is the random variable denoting the regime. If there are two regimes, we could also write:
  - Regime 0:  $y_t = v(0) + \alpha y_{t-1} + X_t' \beta + \varepsilon_t$
  - Regime 1:  $y_t = v(1) + \alpha y_{t-1} + X_t' \beta + \varepsilon_t$
 which shows the regime dependent intercept more clearly.
- The database is downloaded from the web site <http://www.econstats.com>. When database is not available due to national holidays, bank holidays, or any other reasons, stock price indices are assumed to be the same as those of the previous trading day.
- The  $z_t$  random variable is assumed to follow a student distribution (see Bollerslev, 1987) with  $\nu > 2$  degrees of freedom and with a density given by:

$$D(z_t, \nu) = \frac{\Gamma\left(\nu + \frac{1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right) \sqrt{\pi(\nu-2)}} \left(1 + \frac{z_t^2}{\nu-2}\right)^{\frac{1}{2}-\nu}$$

where  $\Gamma(\nu)$  is the gamma function and  $\nu$  is the parameter that describes the thickness of the distribution tails. The Student distribution is symmetric around zero and, for  $\nu > 4$ , the conditional kurtosis equals  $3(\nu-2)/(\nu-4)$ , which exceeds the normal value of three. For large values of  $\nu$ , its density converges to that of the standard normal.

For a Student-t distribution, the log-likelihood is given as:

$$L_{\text{Student}} = T \left\{ \log \Gamma\left(\frac{\nu+1}{2}\right) - \log \Gamma\left(\frac{\nu}{2}\right) - \frac{1}{2} \log[\pi(\nu-2)] \right\} - \frac{1}{2} \sum_{t=1}^T \left[ \log(h_t) + (1+\nu) \log\left(1 + \frac{z_t^2}{\nu-2}\right) \right]$$

where  $T$  is the number of observations,  $\nu$  is the degrees of freedom,  $2 < \nu \leq \infty$  and  $\Gamma(\cdot)$  is the gamma function.

5. The test results are available from the authors upon request.

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