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## RESEARCH ARTICLE

# What do you do when the binomial cannot value real options? The LSM model<sup>1</sup>

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**Abstract:** The Least-Squares Monte Carlo model (LSM model) has emerged as the derivative valuation technique with the greatest impact in current practice. As with other options valuation models, the LSM algorithm was initially posited in the field of financial derivatives and its extension to the realm of real options requires considering certain questions which might hinder understanding of the algorithm and which the present paper seeks to address. The implementation of the LSM model combines Monte Carlo simulation, dynamic programming and statistical regression in a flexible procedure suitable for application to valuing nearly all types of corporate investments. The goal of this paper is to show how the LSM algorithm is applied in the context of a corporate investment, thus contributing to the understanding of the principles of its operation.

**Keywords:** G31—capital budgeting|investment policy, G0—general, A—general economics and teaching, G3—corporate finance and governance

### 1. Introduction

Since Myers proposed the application of the options theory to the valuation of business investment in 1977, the number of theoretical and empirical works which have taken up the idea has grown substantially. Few people would today question the importance of so-called “non-monetary” or strategic outcomes in business investment or the use of the options theory for valuation thereof. Brand image, customer fidelity, technical knowledge or operational flexibility are all clear examples of strategic outcomes to emerge from investments and which are of vital importance to firms. These results yield value to the firm in that they provide something which would be impossible without them, in other words, offer new opportunities, decision-making possibilities or “real options”. As

### ABOUT THE AUTHORS

Authors of this paper can be considered specialists in Real Options. Among others topics, their recent research has focused on applying Real Options in corporate investment valuation, pricing electrical companies and evaluating the way investors consider growth options when pricing equity. With this paper, their research aims the extension of Real Options models in the practical arena showing practitioners of finance how the *Least-Squares Monte Carlo* algorithm is implemented in valuing real options.

### PUBLIC INTEREST STATEMENT

The search for flexibility in valuation models is justified whenever the benefits in terms of greater accuracy and operational simplicity of the valuation, make up for the costs in terms of abstraction and implementation of the new proposals. This paper aims to assess the possible flexibility of real options valuation through the use of the *Least-Squares Monte Carlo* (henceforth LSM) algorithm for valuing derivatives. This technique merges Monte Carlo simulation, dynamic programming and statistical regression in a valuation tool which is flexible enough to be adapted to virtually any business investment opportunity regardless of the nature and number of real options and sources of uncertainty.

Myers pointed out, these are akin to call and put options, and therefore, open to valuation using the same arguments of replication and arbitrage.

Financial directors are forced to accept the idea that “the only certainty about the future is its uncertainty” (Aggarwal, 1993). Nevertheless, it is equally true that firms will respond to whatever, a priori, uncertain events occur in the future, and that even if these events cannot be anticipated, the decisions taken in response to them can. When faced with this uncertain future, identifying and correctly evaluating these possibilities or business response options (real options) thus proves vital to firms.

The real options approach not only shows us how to value these real options correctly but also enables us to reconcile numerous strategic business decisions with the financial principle of value creation (Copeland & Tufano, 2004). In much the same vein, Myers states in a later work (Myers, 1996) that many managers who have never heard of Black and Scholes behave and adopt decisions as if they were acting in line with the precepts of the real options approach when, for example, embarking on a research and development investment project with a negative NPV or when launching the firm into new, unexplored markets which offer seemingly little return, merely for their strategic value.

Indeed, the real options approach is today far more widely known than used. Many managers are aware of the usefulness of the approach and its basic tenets yet only a few actually apply the analytical models and numerical options valuation techniques (Verbeeten, 2006). Amongst the reasons given to account for the scant application of the approach, Newton and Pearson (1994) or Mathews, Datar, and Johnson (2007) point to the operational complexity of the models, Block (2007) and Baker, Dutta, and Saadi (2011) pinpoint misunderstanding of the models, and, finally, Lander and Pinches (1998) refer to a failure of mathematical tools to fulfil certain requirements thereof. An even greater problem is added to these difficulties, namely the somewhat contradictory lack of flexibility in the real options approach, reflected in the absence of any general model—however complex it may prove to understand or use—which may be used to value, if not all, at least some of the most common real options.

Whereas discounted cash flow formulas may be directly applied to virtually all investment opportunities, the real options model lacks any similar general formula, by contrast comprising an amalgam of analytical formulas and numerical valuation models, each of which is suited to the valuation of a specific decision right on a specific underlying asset. Not even the binomial method, arguably the most flexible of all conventional options valuation models, may be directly applied to certain stochastic processes other than continuous Brownian-type motions, multiple consecutive American style options or numerous sources of uncertainty.

The present paper specifically aims to assess the possible flexibility of real options valuation through the use of the Least-Squares Monte Carlo (henceforth LSM) algorithm for valuing derivatives, proposed by Longstaff and Schwartz (2001). This technique merges Monte Carlo simulation, dynamic programming and statistical regression in a valuation tool which is flexible enough to be adapted to virtually any business investment opportunity regardless of the nature and number of real options and sources of uncertainty.<sup>2</sup>

The purpose of our paper is thus twofold. First, we aim to show how the LSM algorithm is implemented in valuing real options when the particular nature of the underlying advocates moving the application of replication and arbitrage to the latter sources of uncertainty on which flows depend. If applying LSM is already “complicated/confusing/complex” in the field of financial derivatives—where the underlying is a listed asset—it is even more so when the ambiguity and complexity involved in the business investment hinder any direct estimation of the underlying and the uncertain variables on which future cash flows depend need to be modelled.

Secondly, we aim to show how LSM proves superior to other numerical procedures which are widespread in real options valuation, applying a common assumption in business investment, namely considering the multidimensional nature of cash flows. Taking account of more than one uncertain variable complicates valuation enormously and prevents it from becoming common practice amongst financial professionals. Not even the well-established Cox, Ross and Rubinstein binomial model (1979), felt to be one of the most flexible options valuation models is able to embrace with ease the multidimensional nature of real options, given that the number of nodes making up the tree grows exponentially with the number of uncertain variables.<sup>3</sup>

According to Amram and Kulatilaka (1999), the choice of the real options valuation model should aim to strike a trade off between its simplicity and transparency, and the benefits in terms of greater accuracy derived from its use. In comparison to the binomial model (Cox et al., 1979), LSM algorithm widens the range of real options which may be valued to include such frequent cases as American style options dependent on multiple state variables or stochastic processes other than the common geometric Brownian. This is the result of the many advantages that the various techniques which make up the LSM algorithm are able to contribute to it. Monte Carlo simulation allows the number of uncertain variables as well as the range of stochastic processes considered when describing it to be extended substantially. For its part, dynamic programming, also present in the binomial model, allows for the possibilities of early exercise in valuing American type options by summing up the sequence of all the possible exercise decisions in two elements: the value of immediate exercise and a *continuation value*, which in turn reflects the consequences of all the following decisions, beginning with the position resulting from the immediate decision. Finally, statistical regression enables the calculation problem involved in valuing options dependent on multiple sources of uncertainty to be solved.

On the other hand, and continuing with the comparison to the binomial model, LSM lacks the intuitive and simple nature which is so much a feature of the binomial model. Thus, whereas the binomial model has the advantage of graphically representing the development of the value of the underlying through decision trees which are easily understood by the user, LSM requires a combination of complex techniques which are applied automatically through a series of computer packages which remain “obscure” to the user.

In an attempt to shed light on this obscurity and aid the spread of LSM amongst firms’ financial managers, the present paper shows a step-by-step application of the LSM algorithm based on a straightforward numerical example of a business investment. Specifically, we posit a fictitious valuation case in which the underlying asset is first estimated based on a state variable, followed subsequently by the inclusion of a second stochastic factor. It thus proves possible to show how the underlying asset should be estimated based on the simulated values of the uncertain variables and how the remaining LSM techniques should be implemented. Although using examples is commonplace to illustrate the application of LSM in the domain of financial derivatives (Cortazar, Gravet, & Urzua, 2008; Longstaff & Schwartz, 2001; Moreno & Navas, 2003a, 2003b; Stentoft, 2004), to the best of our knowledge, this issue remains unexplored in the field of real options. Moreover, the nature of the investment is established so as to facilitate application of the binomial model and enable a comparison between the two techniques. The subsequent extension in the number of state variables helps to illustrate the usefulness of the LSM model.

## 2. A numerical example

The characteristics of the case we posit to illustrate the LSM algorithm ensure a certain balance between realism and an easy understanding of how the LSM model works, compatible with the initial application of the binomial model, necessary to allow a comparison between the two approaches. The case in hand is an investment project of infinite life span which after year five generates a constant and perpetual cash flow. It specifically involves the setting-up of a manufacturing

**Table 1. Characterisation of the case**

Initial value of the demand	$S_0$	100 units
Annual expected growth rate of demand	$\alpha$	15%
Demand volatility	$\sigma$	30%
Net margin per product unit	$m$	One monetary unit
Capital cost of the business	$\mu$	15%
Risk free interest rate	$r$	5%
Duration of the business	$T$	Indefinite
Market share	$c$	50%
Analysis time	$t$	Annual

facility to meet demand of a certain good,  $S_t$ , which we assume to constitute the only source of uncertainty (state variable). We initially assume its development follows the usual geometric Brownian process:

$$dS_t = \alpha S_t dt + \sigma S_t dz_t \tag{1}$$

where the expected growth rate,  $\alpha$ , is 15% and represents the capital gain calculated on the total expected return corresponding to a financial asset which is perfectly correlated with the state variable,  $\mu$  and the dividend return or *convenience yield*,  $\delta$ ,  $\alpha = (\mu - \delta)$ ; and with a volatility,  $\sigma$ , of 30%.

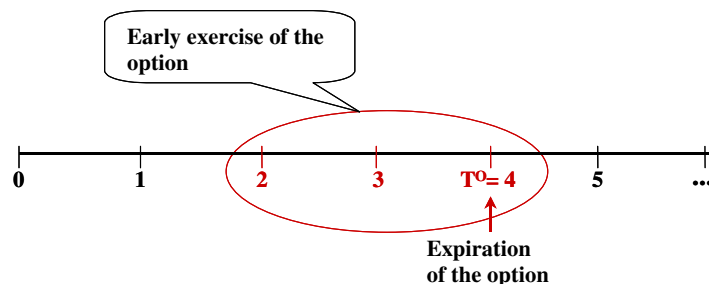
The initial demand value,  $S_0$ , is set at 100 physical units and the project’s market share,  $c$ , is 50%. The cash flow is determined by a margin,  $m$ , which is known and is constant and equal to one monetary unit per product unit sold. We also assume the existence of complete capital markets, a risk free rate of return,  $r$ , of 5% and an investment opportunity cost,  $\mu$ , of 15%, which remain constant during the period valued. Table 1 shows the main parameters of the case.

In addition to the stream of cash flows defined by the above parameters, the investment project offers the possibility of its abandon for an amount of 1,000 monetary units at various points throughout the lifespan of the project. Specifically, the decision to abandon may be taken at the end of the second, third and fourth year. This decision right allows control over possible losses caused by an unfavourable growth in demand and, following the real options approach, may be likened to a Bermuda put option with an exercise price equal to the liquidation value. The timeline of the valuation problem for this option is shown in Figure 1.

**2.1. Valuation of the case using the binomial model**

Valuation of the business and the possibility of its abandonment may be carried out simply based on the binomial model. Considering an annual analysis subinterval, the upward and downward movements of the state variable may be calculated based on the multipliers  $u = e^{\sigma\sqrt{\Delta t}} = 1.35$  and  $d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u} = .74$ , with the result that the risk neutral likelihoods take values  $q = \frac{e^{r\Delta t} + \delta - d}{u - d} = .51$  and  $1 - q = \frac{u - e^{r\Delta t} - \delta}{u - d} = .49$  respectively. Applying these likelihoods to the successive discount of the optimal

**Figure 1. Timeline of the valuation problem.**



exercise value of the option, we obtain a value for the option equal to 124.2 monetary units. This value results from linking the extended value of the project—with the option to abandon—and the value of the project without options.

The first tree in Table 2 shows the valuation of the stream of cash flows generated by the investment project without the possibility of abandonment. Each node shows, in this order, the certainty equivalent value of the state variable,  $S_{i,t}^*$ , in the state of the nature  $i$  at moment  $t$ , the corresponding certainty equivalent cash flow,  $F_{i,t}^*$ , and the updated value of future cash flows,  $V_{i,t}$ . The certainty equivalent cash flows of the project are obtained at each node by simply multiplying the certainty equivalent of demand,  $S_{i,t}^*$ , by market share,  $c$ , and the net margin by each unit sold,  $m$ . In other words:

$$F_{i,t}^* (S_{i,t}^*) = S_{i,t}^* \times c \times m \tag{2}$$

The value of the business at the end of year five,  $t=5$ , is obtained under the assumption of perpetual constant cash flow, and is equal to the certainty equivalent cash flow generated at year five divided by the risk free interest rate:

$$V_{i,5} (S_{i,5}^*) = F_{i,5}^* / r \quad \text{for each node } i \text{ of the tree in } t=5$$

To obtain the value of the business in previous years, we discount the expected cash flows in the following yearly period and the value of the project at the end of that year. That is, the value of the business in the state of the nature  $i$  at moment  $t=1, \dots, 4$ ,  $V_{i,t}$ , is given by:

$$V_{i,t} = \frac{(F_{i+1,t+1}^* + V_{i+1,t+1}) \cdot q + (F_{i,t+1}^* + V_{i,t+1}) \cdot (1-q)}{1+r} \tag{3}$$

from which the value of the business without options is 1,251.56 monetary units at the initial date,  $t=0$ .

The second tree of Table 2 shows the joint valuation of the business and the option to abandon which is exercisable at the end of the second, third and fourth year. The recursive optimisation process required by estimation of the exercise policy commences at the option expiry date ( $t=4$ ) and stretches backwards in time to the first possible moment of exercise ( $t=2$ ). This process aims to maximise the value of the extended business at each node, the optimal decision value being the maximum between the immediate abandonment value of the business and that corresponding to its continuation. When estimating the latter, it is necessary to consider the possibility of optimal abandonment at a subsequent node which is included in the exercise policy calculated for the following periods. By applying this process, the value of the extended business with the abandonment option is equal to 1,375.76 monetary units which, when coupled with the 1,251.56 monetary units of the business without options, implies an abandonment option value of 124.2 monetary units.

The first tree shows the value of the business without options. For each node of the tree, the reported values are: (1) the certainty equivalent value of the state variable ( $S^*$ ); (2) the certainty equivalent cash flow ( $F^*$ ); and (3) the value of the discounted future cash flows ( $V$ ). The present value of the business without options comes to 1,251.56 monetary units. The second tree reflects the joint value of the business and the abandonment option. For the nodes at which the possible exercise of the option is valued, the following are shown: (1) the immediate exercise value; (2) the continuation value derived from the optimum exercise policy; and (3) the value of the optimal decision. The present value of the extended business with abandonment option is 1,375.76 monetary units, the value attributable to the option to abandon thus being 124.2 monetary units.

**Table 2. Valuation of the investment project based on the binomial model**

Decision trees for the investment project without options				Decision trees for the investment project with options			
S			448.17	Immediate exercise value			
F			224.33	Continuation value			4,486.7
V			4,486.7	Optimal decision value			
S				Immediate exercise value			1,000
F			332.01	Continuation value			3,489.76
V			3,489.76	Optimal decision value			<b>3,489.76</b>
S				Immediate exercise value			
F			245.96	Continuation value		1,000	
V			2,462.3	Optimal decision value			2,462.3
S				Immediate exercise value			
F			123.11	Continuation value		2,708.25	
V			2,708.25	Optimal decision value			<b>2,708.25</b>
S				Immediate exercise value			
F			182.21	Continuation value		1,000	
V			1,915.22	Optimal decision value			<b>1,915.22</b>
S				Immediate exercise value			
F			134.99	Continuation value		1,000	
V			67.57	Optimal decision value			<b>1,644.9</b>
S				Immediate exercise value			
F			134.99	Continuation value		1,486.32	
V			1,351.34	Optimal decision value			<b>1,486.32</b>
S				Immediate exercise value			
F			100	Continuation value		1,000	
V			1,051.09	Optimal decision value			<b>1,051.09</b>
S				Immediate exercise value			
F			50.05	Continuation value		1,244.11	
V			1,151.08	Optimal decision value			<b>1,244.11</b>
S				Immediate exercise value			
F			74.08	Continuation value		1,000	
V			37.08	Optimal decision value			<b>1,107.43</b>
S				Immediate exercise value			
F			74.08	Continuation value		1,014.11	
V			815.71	Optimal decision value			<b>1,014.11</b>
S				Immediate exercise value			
F			54.88	Continuation Value		1,000	
V			27.47	Optimal decision value			<b>1,000</b>
S				Immediate exercise value			
F			54.88	Continuation Value		986.62	
V			631.73	Optimal decision value			<b>1,000</b>

(Continued)

**Table 2. (Continued)**

Decision trees for the investment project without options			Decision trees for the investment project with options											
	t=0	t=1	t=2	t=3	t=4	t=5		t=0	t=1	t=2	t=3	t=4	t=5	
S				40.66			40.66	Immediate exercise value				1,000		
F				20.35			20.35	Continuation value				972.68		407.02
V				447.67			407.02	Optimal decision value				<b>1,000</b>		
S					30.12			Immediate exercise value					1,000	
F					15.08			Continuation value					316.58	
V					316.58			Optimal decision value					<b>1,000</b>	
S							22.31	Immediate exercise value						
F							11.17	Continuation value						223.37
V							223.37	Optimal decision value						
	t=0	t=1	t=2	t=3	t=4	t=5			t=0	t=1	t=2	t=3	t=4	t=5

### 2.2. Case valuation using the LSM model

Valuation of the proposed investment project based on the LSM algorithm entails starting simulation of the future evolution of the state variable in accordance with the assumed stochastic process. Simulation of the uncertain variable is performed in each of the first five years of the business's lifespan,<sup>4</sup> and after year five the cash flow is assumed to remain constant. The expression for the discrete approximation of the evolution of the state variable is achieved from the logarithmic transformation of the process,  $x_t = \ln(S_t)$ , and the application of Itos' Lemma to the corresponding differential stochastic equation. The resulting expression takes the following form:

$$S_t = S_0 \exp \left[ (\alpha - 0.5\sigma^2) \Delta t + \sigma Z \sqrt{\Delta t} \right] \tag{4}$$

where  $Z$  is the standard normal random variable linked to the continuous diffusion process of the variable.

Assuming complete markets allows transformation of the previous expression into its certainty equivalent based on the risk free interest rate,  $r$ , and the  $\delta$  parameter which, as previously pointed out, represents the expected return per dividend, or *convenience yield*:

$$S_t^* = S_0 \exp \left[ (r - \delta - 0.5\sigma^2) \Delta t + \sigma Z \sqrt{\Delta t} \right] \tag{5}$$

where the asterisk indicates that these are risk adjusted values.

Based on this expression, together with a series of sample values from the standard normal distribution, certainty equivalent values for the state variable—the demand— $S_t^*$ , are simulated at each future moment  $t$ .<sup>5</sup>

By way of an example, Table 3 shows the certainty equivalent values simulated for the Monte Carlo method for the state variable at 10 different trajectories throughout the five first years of the business.

Based on these simulated values, the certainty equivalent cash flows of the project without options,  $F_t^*$ , are estimated, as pointed out, by multiplying the demand met by market share and the unit margin. The estimated cash flows for each of the trajectories are shown in Table 4. The final column reflects the value of the business at the end of year five,  $t=5$ , on the assumption that after that point a constant and perpetual cash flow equal to the last one recorded will be generated. The final row of the table reflects the mean value of each of the cash flows derived from the ten simulated values.

**Table 3. Simulated trajectories for the certainty equivalent transformation of the state variable**

Path	$S_0$	$S_1^*$	$S_2^*$	$S_3^*$	$S_4^*$	$S_5^*$
1	100	122.60	81.39	127.93	122.15	83.08
2	100	85.52	99.14	79.68	63.55	53.31
3	100	104.21	103.30	116.66	154.64	132.62
4	100	89.15	110.47	140.61	162.78	238.93
5	100	132.50	151.20	111.76	99.09	178.03
6	100	161.76	139.33	134.99	208.25	224.53
7	100	86.05	63.05	64.64	89.11	66.59
8	100	119.72	117.56	91.93	90.92	72.21
9	100	103.16	118.40	116.07	132.45	124.05
10	100	127.02	137.93	141.47	161.51	144.73



**Table 4. Simulated certainty equivalent cash flows and the value of the business at year five**

Path	$F_1^*$	$F_2^*$	$F_3^*$	$F_4^*$	$F_5^*$	$V_5$
1	61.368	40.740	64.036	61.140	41.584	830.742
2	42.807	49.623	39.885	31.811	26.683	533.063
3	52.160	51.708	58.394	77.404	66.383	1,326.180
4	44.626	55.293	70.382	81.482	119.594	2,389.204
5	66.321	75.683	55.942	49.601	89.113	1,780.253
6	80.971	69.739	67.572	104.239	112.390	2,245.285
7	43.073	31.561	32.356	44.606	33.331	665.867
8	59.924	58.843	46.015	45.508	36.143	722.049
9	51.635	59.265	58.097	66.297	62.095	1,240.513
10	63.582	69.040	70.811	80.842	72.446	1,447.286
Average	56.647	56.149	56.349	64.293	65.976	1,318.044

Taking the mean values of the ten simulations performed at each period as an approximation of the expected certainty equivalent cash flows and updating to the risk free interest rate, the current value of the business is equal to 1.29086 monetary units.<sup>6</sup>

$$V_0 = \frac{54.647}{1+0.05} + \frac{56.149}{(1+0.05)^2} + \frac{56.349}{(1+0.05)^3} + \frac{64.293}{(1+0.05)^4} + \frac{65.976}{(1+0.05)^5} + \frac{1,318.044}{(1+0.05)^5} = 1,290.86$$

As with the binomial model, pinpointing the optimal exercise policy for the abandonment option entails applying a recursive optimisation process commencing at the option expiry date ( $t=4$ ) and concluding at the first exercise date ( $t=2$ ). At the option expiry date, the strategy which maximises the extended value of the investment involves exercising the option for the trajectories in the money, as if it were a European derivative. In the abandonment option, the trajectories in the money will obviously be those for which the liquidation value of the business is greater than the continuation value. The liquidation value of the business coincides with the exercise price of the option, 1,000 monetary units. The continuation value (value of the business when the option is not exercised) is the result of discounting the cash flows generated from the expiry date, shown in Table 4:

$$V_{4,i}^{\text{continuation}}(S_{4,i}) = \frac{F_{5,i}^* + V_{5,i}^{\text{continuation}}}{(1+r)} \tag{6}$$

$$\text{being } V_{5,i}^{\text{continuation}}(S_{5,i}) = \frac{F_{5,i}^*}{r} \tag{7}$$

where subscript  $i$  refers to the  $i$ th simulated trajectory.

The calculations corresponding to each of the trajectories simulated are shown in Table 5. The outcomes to emerge from the simulations in the example reveal that four of these trajectories are in the money when the option expires, the strategy which maximises the extended value of the investment being that which derives from the abandonment of the project for these trajectories.

As can be seen, analysing the exercise strategy at expiry using the LSM method is carried out by simply considering the derivative as if it were European. After doing this, and taking the initial series of 10 simulated trajectories, estimation of the optimal frontier at the moments of possible exercise prior to expiry can commence.

This is where the main differences between the LSM and binomial models emerge. Monte Carlo simulation generates the approximation corresponding to a trajectory at a given moment, assuming

**Table 5. Trajectories in the money at the option expiry date**

Path	$V_4^{\text{exercise}}$	$V_4^{\text{continuation}}$	In the money	Optimal decisión
1	1,000	830.786	Yes	Exercise
2	1,000	533.092	Yes	Exercise
3	1,000	1,326.250	No	No exercise
4	1,000	2,389.331	No	No exercise
5	1,000	1,780.348	No	No exercise
6	1,000	2,245.405	No	No exercise
7	1,000	665.903	Yes	Exercise
8	1,000	722.088	Yes	Exercise
9	1,000	1,240.579	No	No exercise
10	1,000	1,447.363	No	No exercise

that its value at the previous moment is known. Taken independently, each trajectory thus offers a perfect foresight solution, simulation of a large number of trajectories proving necessary if a good approximation to the state variable is to be achieved. By contrast, at a given moment in the binomial model each value of the variable generates at least two possible scenarios in the following period.

In order to know what to do when faced with a specific trajectory at a specific point, procedures which use simulation in American options valuation require some kind of mechanism offering an *expected* exercise frontier. We should note that estimating the most suitable exercise strategy for each trajectory is not recommendable since the decision to exercise the option at a given point is taken assuming that the information from future moments is known, which yields an upwards biased value for the option.

In the LSM model, the *expected* exercise frontier is performed by statistical regression. Thus, at each point at which early exercise of the option is allowed, the continuation value or value of the investment when the option is going to be maintained until next period (dependent variable) is regressed on some transformations of the simulated value of the state variable (independent variables). Moreover, this regression is only posited for trajectories which are in the money at a specific point, that is those for which the value of the business if the option is exercised is greater than the value if it is not, as if expiry of the right were to occur at that point since, a priori, they are the only ones for which continuing the business is worth considering.

Below, we show how the successive regressions of the investment in hand would be formulated. At  $t=3$ , the option holder must decide whether to exercise the option immediately, in other words, abandon the project, yielding a value of  $V_{3,i}^{\text{exercise}} = 1,000$  monetary units, or to carry on to the next point,  $t=4$ , and opt for the wisest choice. The value of the latter possibility is obtained by applying the optimal exercise policy from  $t=4$  which, as it coincides with expiry, involves comparing the values of the underlying investment if either exercising the option or not. The value of keeping the option alive at  $t=3$  is thus given by:

$$V_{3,i}^{\text{continuation}}(S_{3,i}^*) = \frac{F_{4,i}^*}{1+r} + \begin{cases} \frac{V_{4,i}^{\text{exercise}}}{1+r} & \text{if } V_{4,i}^{\text{exercise}} \geq \frac{F_{5,i}^* + V_{5,i}^*}{(1+r)} \\ \frac{F_{5,i}^* + V_{5,i}^*}{(1+r)^2} & \text{if } V_{4,i}^{\text{exercise}} < \frac{F_{5,i}^* + V_{5,i}^*}{(1+r)} \end{cases} \quad (8)$$

where the subscript  $i$  represents the set of trajectories in the money at point  $t=3$ , out of the total of ten simulations performed for each interval.

The continuation value thus obtained is the dependent regression variable,  $Y_{3,i}$ . The independent regression variables are based on the simulated values of the state variable. For the purpose of

**Table 6. Variables involved in regression at t=3**

Path	$V_{3, \text{CONT.}} = Y_3$	$S_3$	$(S_3)^2$	$\hat{Y}_3$	Optimal decision t=3
1	1,010.609	127.93	16,366.085	1,011.169	Cont.
2	982.677	79.68	6,348.902	989.595	Exercise
3		116.66	27,775.555	1,001.915	
4		140.61	19,771.172	1,024.648	
5		111.76	12,490.298	998.693	
6		134.99	18,222.300	1,018.279	
7	994.862	64.64	4,178.329	992.495	Exercise
8	995.722	91.93	8,451.125	990.612	Exercise
9		116.07	13,472.245	1,001.498	
10		141.47	20,013.761	1,025.677	

illustration, we use a degree two polynomial regression,<sup>7</sup> the equation to be estimated at this point therefore being:

$$Y_{3,i^*}(S_{3,i^*}^*) = a_3 + \beta_3 (S_{3,i^*}^*)^2 + \omega_3 S_{3,i^*}^* \text{ where } Y_{3,i^*}(S_{3,i^*}^*) = V_{3,i^*}^{\text{continuation}}(S_{3,i^*}^*)$$

Table 6 shows the variables involved in the regression at t=3 for trajectories in the money at this point, which are trajectories 1, 2, 7 and 8. After analysing the optimal decision corresponding to these trajectories one period later, at t=4, the second column shows the  $Y_{3,i}$  continuation values obtained from the future cash flow discount to emerge from optimal abandonment/non-abandonment. The following approximation coefficients are estimated from the vectors corresponding to the independent regression variables—columns three and four,

$$\hat{Y}_3 = 1057.046 - 1.652S_3^* + 0.010 (S_3^*)^2$$

Finally, columns five and six show the result of regression for trajectories in the money at t=3. The optimal decision reached for each trajectory at t=3 is obtained simply by comparing the continuation value derived from regression,  $\hat{Y}_{3,i^*}$  and the liquidation value of the business, in other words 1,000 monetary units. As can be seen, the optimal decision at t=3 is immediate abandonment of the business for trajectories 2, 7 and 8, whereas for trajectory 1 the optimal decision is to continue with the business until at least t=4.

The function of the expected continuation value for point t=2 is estimated in the same way as for t=3. Determining the exercise policy entails regressing the value of the project in the event of continuation over the constant and simulated values of the only source of uncertainty considered. The continuation values for trajectories in the money at t=2 are calculated applying the optimal decisions calculated for the following moments, t=3 and t=4, in such a way that exercising the option is considered as soon as the *critical frontier* is reached:

$$V_{2,i^*}^{\text{continuation}}(S_{2,i^*}^*) = \frac{F_{3,i^*}^*}{1+r} + \begin{cases} \frac{V_{3,i^*}^{\text{exercise}}}{1+r} & \text{if } V_{3,i^*}^{\text{exercise}} \geq \hat{Y}_{3,i^*} \\ \frac{F_{4,i^*}^*}{(1+r)^2} + \frac{V_{4,i^*}^{\text{exercise}}}{(1+r)^2} & \text{if } V_{4,i^*}^{\text{exercise}} \geq \frac{F_{5,i^*}^* + V_{5,i^*}}{1+r} \\ \frac{F_{4,i^*}^*}{(1+r)^2} + \frac{F_{5,i^*}^* + V_{5,i^*}}{(1+r)^3} & \text{if } \begin{cases} V_{3,i^*}^{\text{exercise}} < \hat{Y}_{3,i^*} \\ V_{4,i^*}^{\text{exercise}} < \frac{F_{5,i^*}^* + V_{5,i^*}}{1+r} \end{cases} \end{cases} \quad (9)$$

where once again the subscript  $i^*$  represents the trajectories in the money at point t=2, of the ten initial simulations. In t=2 trajectories 1, 2, 7 and 8 remain in the money.

**Table 7. Variables involved in regression at t = 2**

Path	$V_{2, \text{CONT.}} = Y_2$	$S_2$	$(S_2)^2$	$\hat{Y}_2$	Optimal decision t = 2
1	1,023.472	81.39	6,624.409	1,006.556	Cont.
2	990.366	99.14	9,828.319	1,007.246	Cont.
3					
4					
5					
6					
7	983.196	63.05	3,975.609	988.717	Exercise
8	996.205	117.56	13,819.486	990.721	Exercise
9					
10					

As can be seen, the regression dependent variable,  $Y_{2,i} = V_{2,i}^{\text{continuation}}$ , is calculated from the “real” values of the flows and not from the expected continuation value which is derived from the previously estimated regression. Approximation of the expected function of the value of keeping the option alive at t = 2 is given by the following expression:

$$\hat{Y}_2 = 794.556 + 4.711S_2 - 0.026(S_2)^2$$

For point t = 2, Table 7 shows the “real” values derived from continuing with the business at least up to the following period and the continuation values estimated from regression.

The optimal exercise strategy at t = 2 is shown in the final column, abandonment of the business for trajectories 7 and 8 and continuation thereof for at least one more period for trajectories 1 and 2 proving the optimal course of action.

By applying the abandonment rule identified at each of the exercise opportunities evaluated, abandoning the business proves optimal for trajectories 7 and 8 at point t = 2, for trajectory 2 at t = 3, and for trajectory 1 at the expiry date t = 4. For the remaining trajectories, at no point is considering

**Table 8. Estimation of the extended value of the investment**

Path	Optimal decision			Extended value
	t = 2	t = 3	t = 4	
1	Continue	Continue	Exercise	1,023.717
2	Continue	Exercise	Exercise	984.070
3	No exercise	No exercise	No exercise	1,252.134
4	No exercise	No exercise	No exercise	2,143.694
5	No exercise	No exercise	No exercise	1,622.475
6	No exercise	No exercise	No exercise	2,054.685
7	Exercise	Exercise	Exercise	948.051
8	Exercise	Exercise	Exercise	964.100
9	No exercise	No exercise	No exercise	1,179.112
10	No exercise	No exercise	No exercise	1,381.048
			Extended present value	<b>1,355.308</b>
			Convencional present value	<b>1,290.86</b>
			Option to abandon	<b>64.449</b>

the possibility of abandoning the business felt to be a recommendable course of action. By thus specifying the optimal exercise strategy, determining the cash flows to emerge from the most recommendable decisions for each case proves fairly straightforward. Table 8 shows the extended present value of the underlying investment project derived from the optimal decision in each case.

After averaging the outcomes obtained for each trajectory, we see how the value derived from active management of the underlying investment, the Extended Value, comes to 1,355.31 monetary units compared to the 1,290.86 monetary units which the traditional “passive” cash-flow discount of the project yields. The value of the option to abandon is therefore 64.45 monetary units.

Estimation of such a small number of trajectories is clearly insufficient to obtain a good approximation of the option value and sparks the difference between the outcome of the valuation from the binomial model and that obtained through the LSM algorithm. However, the Monte Carlo method allows us to enhance the quality of the approximation by increasing the number of simulations, as inversely the standard error of its estimation depends on the number of experiments performed, whatever the temporal dimension of the problem.

### **2.3. Comparing the binomial model and the LSM model**

As can be seen, valuing the investment case posited proves more straightforward using the binomial model, since the initial hypotheses fit the assumptions on which the model is based very well. In fact, the assumption that the sources of uncertainty involved in the business may be summed up in a single variable—demand—stochastic evolution of which is approached using geometric Brownian movement, is established so as to facilitate the estimation of the upward and downward variations of the discretisation process. Moreover, simplification of the abandonment possibilities to a small number of dates allows implementation of the dynamic programming recursive procedure of the binomial model using a simple spreadsheet.

The LSM procedure merges information from the discretisation of the variable not only with dynamic programming but also statistical regression. This is because each trajectory generated using the Monte Carlo method constitutes a perfect foresight solution, making necessary both simulation of a large number of trajectories as well as use of some technique to provide an expected stopping at the correct approximation of the value option.

From the above, it can be deduced that implementing the LSM procedure proves a priori less intuitive and more complex than the binomial model, as a result of which its use depends on the possibilities of applying the latter. Common factors such as an increase in the number of exercise dates, the existence of multiple sources of uncertainty or adjusting its future evolution to patterns other than Brownian, complicate application of the binomial model, when indeed not rendering it impossible.

One example is that including a second stochastic factor entails a substantial alteration of the tree shape representation which the procedure adopts. In addition to the joint density of the two variables needing to follow a bivariate lognormal distribution, the discrete approximation of the two variables needs to be constructed based on a five node scheme, giving rise to a three dimensional tree in the shape of an inverted pyramid. Extending this representation to the subintervals into which the valuation period is divided means exponentially increasing the possible states of the nature, complicating implementation of the recursive process required to estimate the optimal stopping rule.

For its part, in the LSM method, including a second stochastic factor merely involves doubling the effort required for initial simulation of the state variables and extending the number of independent variables involved in regression. However, it does not entail any change in the method applied to estimate the exercise frontier linked to possible abandonment. For example, if we assume the unit exploitation margin to follow a geometric Brownian process<sup>8</sup> with parameters  $\alpha_M = 6\%$ ,  $\sigma_M = 15\%$  and  $\mu_M = 6\%$ , not correlated<sup>9</sup> with the evolution of the first stochastic factor, we would only need to generate the sample representations corresponding to the second variable, shown in Table 9, and

**Table 9. Second factor stochastic simulations throughout the lifespan of the investment**

Path	$M_0$	$M_1$	$M_2$	$M_3$	$M_4$	$M_5$
1	1	1.288	1.423	1.458	1.569	1.930
2	1	1.131	1.023	1.139	1.459	1.670
3	1	.982	1.448	1.769	1.645	1.851
4	1	.767	.816	.958	1.001	1.043
5	1	.766	1.059	1.196	1.622	1.623
6	1	1.090	1.011	.994	1.212	1.288
7	1	1.150	1.685	1.704	1.727	1.340
8	1	1.157	1.128	1.058	1.132	1.295
9	1	.971	.998	.906	.706	.708
10	1	1.119	1.578	1.546	1.505	1.350

**Table 10. Estimated flows throughout the lifespan of the investment**

Path	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$V_5$
1	79.066	57.980	93.341	95.898	80.245	1,603.105
2	48.406	50.785	45.425	46.427	44.572	890.434
3	51.221	74.863	103.306	127.343	122.895	2,455.140
4	34.245	45.115	67.457	81.549	124.680	2,490.810
5	50.813	80.167	66.887	80.469	144.613	2,889.024
6	88.220	70.490	67.189	126.339	144.734	2,891.432
7	49.517	53.187	55.151	77.015	44.651	892.021
8	69.356	66.379	48.666	51.510	46.796	934.8634
9	50.131	59.146	52.626	46.813	43.955	878.118
10	71.164	108.955	109.479	121.669	97.829	1,954.392
Average	59.214	66.707	70.953	85.503	89.497	1,787.934

obtain, together with those corresponding to the first stochastic factor, the net cash flows of the project as shown in Table 10.

Based on the values of the cash-flows, implementing the LSM method to estimate the Extended Present Value of the business involves the same operations routine as in the case of a stochastic factor. We simply need to include the terms of the second stochastic factor together with the crossed product of the sources of uncertainty in the series of independent variables in regression. In this case, the abandonment option comes to 40.71 monetary units.

### 3. Conclusion

New valuation proposals based on Monte Carlo simulation, dynamic programming and statistical regression are destined to revolutionise business valuation and business investments. These flexible models are able to value different sources of value in any kind of investment regardless of the nature of its options and sources of uncertainty. In this vein and in just a short space of time, the algorithm proposed by Longstaff and Schwartz (2001) has emerged as the procedure for valuing American, financial or real derivatives to have the greatest practical impact. The growing number of papers, prominent amongst which are Yang and Zhigiang (2013), Kitapbayev, Moriarty, Mancarella, and Bloöchle (2013), Laude and Jonen (2011), Moreno, Navas, and Todeschini (2009), Schwartz and Moon (2000, 2001), Miltersen and Schwart (2004), Schwartz (2004), León and Piñeiro (2004), Rubio and Lamothe (2006), Abadie and Chamorro (2006), and Alonso, Azofra, and de la Fuente (2009) to

name but a few, bear witness to its possibilities for development and adaptation to business investment and options.

The LSM algorithm is the result of merging three different techniques, each of which contributes its own advantages to the valuation procedure. Firstly, Monte Carlo simulation greatly extends the range of possible stochastic processes to be considered when characterising sources of uncertainty. Secondly, dynamic programming allows the possibility of early exercise in the valuation of American style options to be included, and thirdly, statistical regression allows the calculation problem involved in valuing options dependent on multiple sources of uncertainty to be solved. Overall, the LSM algorithm provides a flexible and powerful tool capable of valuing virtually any kind of option using up only a limited amount of resources.

The main drawback of this procedure lies in the fact that its application requires a high degree of calculation, which is only assumable through an automated software package. The use of these valuation packages by those in charge of financial management may entail a certain loss in transparency as far as the calculation process is concerned. For this reason, it is interesting to shed some light on the “black box” of this kind of optimisation tool and to facilitate understanding of the principles governing its functioning.

Nevertheless, the search for flexibility in valuation models is justified whenever the benefits in terms of greater accuracy and operational simplicity of the valuation, make up for the costs in terms of abstraction and implementation of the new proposals. Specifically, when the option valued may be exercised at various points prior to the expiry date, its value depends on multiple sources of uncertainty or if the latter follow stochastic processes other than Brownian, the LSM model which we have explained here emerges as the most efficient of the existing alternatives.

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#### Notes

1. An earlier version of this article appeared in the Working Paper Series of FUNCAS (Fundación de las Cajas de Ahorros, Spain), available at <http://www.funcas.es/publicaciones/Sumario.aspx?IdRef=7-05729>

2. See Cerrato and Cheung (2007) for a review of the accuracy and efficiency of some recent Monte Carlo estimators under different market scenarios in financial derivatives.
3. This problem is known as the *curse of dimensionality* (Bellman, 1961) and leads to the model's inoperability due to the increased amount of resources consumed when solving the problem.
4. Only, when direct simulation of the underlying asset is possible, the Monte Carlo method is extended to the expiry of the option, as occurs with financial derivatives and certain real options applications. When direct simulation of the underlying is not advisable, approximation of the sources of uncertainty is required up to the expiry of the cash flows underlying the option exercise. As a result, for the example in hand, simulation is continued up to the fifth period.
5. Since an exact discretisation formula is available for the geometric Brownian process, small time increases,  $\Delta t$ , are not necessary to achieve a good approximation. Other processes for which an exact approximation is available are square root diffusion processes (Cox, Ingersol, & Ross, 1985) as well as certain jump processes. When exact simulation is not possible some kind of discretisation method needs to be used, the most prominent being the Euler scheme due to its ease of use and virtually universal application. In this case, minimising discretisation errors does require simulation of a “large” number of intermediate steps at each date, with the subsequent increase in the resources needed to use it.
6. Using only ten simulations entail an error in approximation which can be reduced by increasing the number of trajectories. For a simulation of 200,000 trajectories, the current value of the business comes to 1,259.29 monetary units.



7. As regards the basic functions to be used in regression, Longstaff and Schwartz (2001) and Moreno and Navas (2003b) propose using up to a two degree polynomial function as a good approximation to the exercise frontier.
8. In order to simplify the explanation, we assume the same process used for modelling the first stochastic factor. Thus, we can use the same expressions presented previously to the margin simulation, with simple replacement of the values of their parameters.
9. See Alonso (2009) for simulation of random correlated variables.

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