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RESEARCH ARTICLE

Is a night better than a day: Empirical evidence

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Abstract: In this study, we analyze the portfolio allocation based on time asymmetry of stock characteristics. In particular, we analyzed the empirical data of changes in financial stock prices during the day period and during the night period and have found that characteristics such as mean and variance are different for changes during the day and changes during the night. Also, the portfolio characteristics, such as covariance between stocks, differ on whether we take into account day changes or night changes in prices. That greatly affects the allocation of fund to the portfolio for an investor who trades frequently. The portfolio should be re-balanced every day in order to achieve optimality and much higher return. At the same level of risk the returns on this new portfolio may by several times larger than the returns on a portfolio without everyday re-balancing. We computed numerically the allocation of funds for the stocks from the finance industry and showed that the increase in returns is substantial.

Keywords: stock returns, symmetry, covariance, portfolio analysis

1. Introduction

The standard theory of portfolio allocation, the one we teach students in classrooms, assumes that stock characteristics do not change frequently. In this theory, the expected stock returns, as well as their variances and covariances, are assumed to be constant through time, i.e. independent of the

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RESEARCH GROUP OVERVIEW

The authors are interested in investments, frequent trading, asymmetry of return distribution, and market microstructure.

PUBLIC INTEREST STATEMENT

The stock prices experience changes to their value differently during non-trading hours compared to changes during trading hours. The different dynamics of changes allows informed investor to increase an annualized return of optimal portfolio by tens of percent if the investor is willing to trade frequently (twice a day). We showed that the optimal portfolio should be re-balanced every day at the opening and at the close of the market. At the same level of risk, an investor has much higher expected returns such an optimal portfolio compared to a portfolio without everyday re-balancing. We showed how one could allocate the funds to create such a portfolio.

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time of day. In general, this might not be true. Indeed, the flow of information depending on the time of day may be changing and, therefore, might affect stock characteristics.

Starting from classical works by Markowitz (1952, 1991a) and Merton (1969) to more elaborated theories (see, e.g. Campbell, Lo, & MacKinley, 1998 and references therein) stock characteristics are assumed to be constant. This assumption is reasonable for one-period cases, as seen in Markowitz (1952), but, generally, models with constant characteristics may be erroneous. This may either cost a fortune in diminished returns or cause investors to accept higher risk. One of the possible ways to account for this problem is to anticipate structural shifts (see Ang & Bekaert, 2002; Ang & Chen, 2002; Bai, Lumsdaine, & Stock, 1998; Bai & Perron, 2001; Campbell, 2000; Hashem Pesaran, Pettenuzzo, & Timmermann, 2006; Pástor & Stambaugh, 2001; Wachter & Warusawitharana, 2009; Wright, 1996 and references therein). Unfortunately, computing stock characteristics, such as expected returns, during short-time intervals is not a simple procedure; therefore, both investors and academics generally assume that these characteristics do not change in weeks, if not months. Hundreds of articles are published every year regarding computation and approximation of stock characteristics in an environment of structural breaks, but there is yet to be a universal consensus on an accurate computational model for said changes.

In this paper, we want to pursue a different approach. Instead of looking for structural changes, we assume that securities are in different states and switch from one state to another at periodic intervals. Although one can consider as many states for a security as one deems necessary, we assume that each stock can only be in one of two different states for simplicity. The stock switches from one state to another one periodically and frequently. Contrary to the common belief of smooth and slow changes in stock characteristics on the scale of months, we postulate that abrupt changes exist caused by the natural flow of information. Information is generated differently during trading and non-trading hours, thus the stock characteristics will change from work hours to non-work hours and vice versa. We define the first state based on stock behavior during NYSE trading hours, which we will call the "DAY" state. The second state is characterized by stock behavior during the hours when NYSE is closed, which we will call the "NIGHT" state. We assume that during any state the characteristics of the stock behavior stay constant, i.e. there are no structural breaks during the day or the night. However, it is important to note that we do not make the assumption that "DAY" characteristics, such as expected return, covariance matrix, etc. are equal to "NIGHT" characteristics. From this point of view, every stock alternates between the "DAY" and "NIGHT" state, each state having its "Daily" and "Nightly" characteristics, respectively. For simplicity, we allow stock trading either when the exchange opens or just before the exchange closes, which means there is no intraday trading in our model.

We propose the following trading strategy. As the exchange opens, we buy the best portfolio based on "Daily" characteristics of stocks. At market closing, we re-balance the portfolio using the "Nightly" values. Next morning, we sell our "Nightly" portfolio and buy a new one, based on "Daily" characteristics again, and continue this process for the desired period of time. We claim that such a strategy allows us to realize much higher returns compared to the common Buy-and-Hold strategy.

This paper is organized as follows. In the second section, we discuss our model and compute the important stock characteristics. The third section is devoted to the empirical analysis of the financial industry, creation of the optimal portfolio, and choice of correct allocation of funds. This section also includes efficient portfolios plots and results of our study. The last section concludes the paper.

2. Model

In our model, each real stock is in one of two states, with each state considered to be a different security. One of the securities, the "Daily security" is the stock traded during NYSE trading hours. We can invest in it for one period by buying a stock at the opening of NYSE and selling it at the close of NYSE. Conversely, we can invest in the "Nightly security" for one period by buying stock at

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Figure 1. Comparison of DAY and NIGHT returns.

Notes: Along the horizontal axis we plotted the return on DAY (circle) or NIGHT (star) security (Bank of America stock), while the vertical axes shows the frequency of observation of given returns. It is evident that the DAY return variance is larger than the NIGHT return variance.

Histogram of return distribution
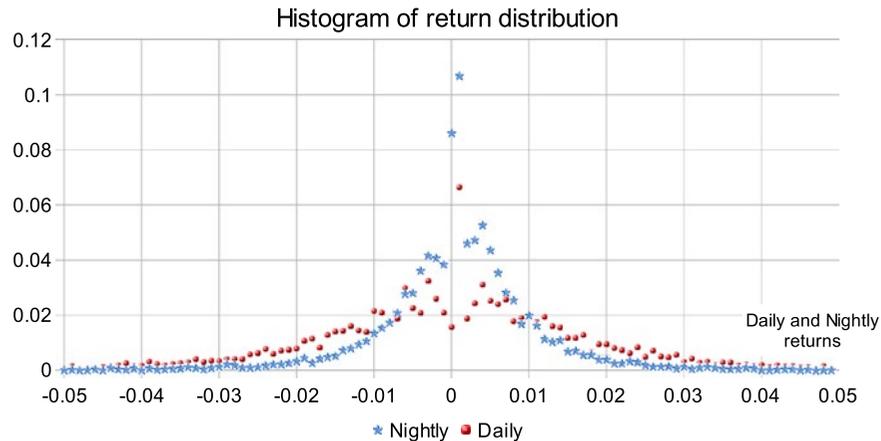
the close of NYSE and selling it next trading day at the opening of NYSE. Buying and keeping the stock from the opening of the market to the opening of the market next day (or from closing to closing) would correspond to investing equally in both securities, "Daily" and "Nightly", for one period. Generally, even if the underlying company for a stock is the same, the two securities are separate: they have difference characteristics, e.g. different expected returns and different standard deviations.

We compute the one period "Daily" return as the difference between the logarithms of the closing price and opening price on a particular day. The one period "Nightly" return is the difference between the logarithm of the opening price on a particular day and the logarithm of the closing price on the previous trading day. We find the "Daily" and "Nightly" average returns, as well as the standard deviations and covariances of the one period returns, over the course of several years. Due to different levels of information flow about economic, financial, and political activities during the "Day" and "Night," we cannot expect the above characteristics to be necessarily the same.

Below, we have provided an example of the differences between stock characteristics in the two states. Consider the returns on the Bank of America stock. The average historical return on "Daily BAC" from January 2000 to January 2012 is −.00107, and the standard deviation is .028. For "Nightly BAC" during the same period, the average daily return is .00071 and standard deviation is .019. The two securities are statistically different with respect to both average returns (p-value p_{return}=.0039) and variance (p-value $p_{variance}$<10⁻¹⁰). As seen below, these stock characteristics, such as average return or variance, are statistically different not only for Bank of America stock, but also for almost all other stocks. With few exceptions, the variance of returns for the "Nightly" securities is statistically lower (p_{var}<10⁻¹⁰) than the variance for the corresponding "Daily" securities, which means buying one period overnight stock is less risky than keeping the security for one period during trading hours (Figure 1 and Table 1).

Because each stock effectively consists of two securities, "Daily" and "Nightly" securities, we can form 2*N* securities out of *N* stocks. Each security has its own characteristics which are assumed to be constant for an observable period of time, i.e. the "Daily" security has its own set of characteristics which are the same during any trading day, while "Nightly" security has its own values for characteristics, which are constant for any non-trading period and may be different from "Daily" values. One can compute these characteristics, such as expected returns, variances, or covariances between securities. Then one can apply the classical portfolio analysis as shown in Markowitz (1952, 1959, 1991b), Ruppert (2006) for 2*N* securities and find the optimal portfolio (see Appendix for more details).

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Table 1. The DAY and NIGHT Returns and Standard Deviations for Selected Stocks with Corresponding *p*-values.

Stock symbol	DAY return	NIGHT return	*p*-value	DAY std	NIGHT std	*p*-value
af	.000096	.000059	.935	.0223	.0110	<10⁻⁶
aig	−.002982	.001732	<10⁻⁶	.0391	.0339	<10⁻⁶
asi	**.000617**	**−.000137**	**.106**	**.0234**	**.0103**	**<10**⁻⁶
bac	−.000994	.000757	.005	.0280	.0193	<10⁻⁶
bap	**.000889**	**.000070**	**.046**	**.0189**	**.0122**	**<10**⁻⁶
bbt	.000129	.000054	.868	.0220	.0119	<10⁻⁶
bbx	−.003517	.002614	<10⁻⁶	.0499	.0288	<10⁻⁶
boh	.000969	−.000393	<10⁻³	.0189	.0073	<10⁻⁶
c	−.002732	.001900	<10⁻⁶	.0300	.0239	<10⁻⁶
cib	**.000362**	**.000903**	**.351**	**.0277**	**.0158**	**<10**⁻⁶
cma	−.000159	.000142	.544	.0239	.0128	<10⁻⁶
cnb	.000999	−.000605	.003	.0224	.0190	<10⁻⁶
cpf	−.001119	.000248	.088	.0392	.0198	<10⁻⁶
ddr	−.000450	.000635	.117	.0351	.0143	<10⁻⁶
gs	.000133	−.000044	.714	.0224	.0140	<10⁻⁶
hbc	**.000653**	**−.000585**	**.001**	*.0111*	*.0174*	**<10**⁻⁶
ire	**−.001983**	**.000709**	**.002**	*.0303*	*.0378*	**<10**⁻⁶
jll	.001243	−.000748	<10⁻³	.0265	.0112	<10⁻⁶
lm	−.000349	.000314	.223	.0265	.0137	<10⁻⁶
mig	−.000467	.000780	.035	.0286	.0154	<10⁻⁶
nbg	**−.000725**	**−.000142**	**.388**	*.0255*	*.0268*	**.005**
sfe	−.000034	−.001073	.267	.0455	.0238	<10⁻⁶
sfi	−.001217	.001119	.013	.0463	.0225	<10⁻⁶
spg	.000707	.000091	.174	.0229	.0095	<10⁻⁶
vly	.000227	−.000069	.443	.0191	.0093	<10⁻⁶
vno	.000225	.000231	.990	.0224	.0097	<10⁻⁶
vtr	.001190	.000039	.019	.0241	.0119	<10⁻⁶
wdr	−.000386	.000447	.097	.0252	.0111	<10⁻⁶
y	.000498	−.000219	.026	.0163	.0069	<10⁻⁶

Notes: From 215 stocks, 59 out of 187 US stocks and 12 out of 28 international stocks have statistically different DAY and NIGHT average returns at $\alpha = .05$. International stocks are denoted by bold font. Very few stocks have higher NIGHT return standard deviation than DAY return standard deviation (denoted by italic) and all of them are international stocks (10 out of 28).

3. Analysis and Computations

We analyzed 394 stocks from the financial industry, considering only those stocks whose prices are available to us for the 1 January 2000 to 31 December 2012 period. There are 215 stocks satisfying this condition: 187 of them are US stocks, while 28 are international stocks. Stock tickers were taken from www.nyse.com/about/listed/lc_all_industry.html, and daily prices for each stock, including opening and closing prices, were downloaded from finance.yahoo.com.

Because we are interested in one period returns, i.e. the changes in price during a predefined one period, we need to check that prices are available for these predefined periods for all stocks under consideration. We require one period returns for all stocks on a given day to compute the covariance matrix and, later, the optimal portfolio; therefore, we discard one period returns for all stocks on a particular day or night if the return for one or several stocks was missing for that time period. For example, if one stock had no available prices for Wednesday, one period returns for all stock on that

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Figure 2. The scatter plot of the relationship between average one-period DAY (horizontal axis) and average one-period NIGHT returns (vertical axes) of a stock.

Notes: For many stocks, DAY and NIGHT average returns have opposite signs. Although the majority of stocks exhibit average DAY+NIGHT returns of less than .001 (corresponding to less than 30% annualized return), several stocks have average one period DAY or NIGHT returns of the order .0015–.0035 which correspond to about 50% to above 100% annualized returns.
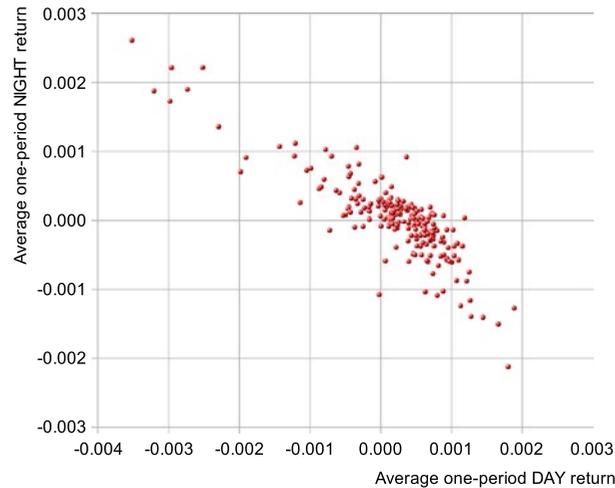
Wednesday were discarded. As we do not know the opening and closing prices for a stock on Wednesday, we cannot compute the "Daily" one period return for Wednesday, or the "Nightly" one-period returns for Tuesday–Wednesday (we need to know the opening price on Wednesday) and for Wednesday–Thursday (we need to know the closing price on Wednesday). Therefore, returns for Tuesday–Wednesday "Night" and Wednesday–Thursday "Night" stocks were discarded, as well.

Before we discuss the allocation of funds for the optimal portfolio, we would like to present the general characteristics of "Daily" and "Nightly" securities. First, we considered the return characteristics for 215 stocks (187 domestic stocks and 28 international stocks). At a significance level of $\alpha=.01$, there are 13 domestic stocks and 2 international stocks with higher "Nightly" average returns than "Daily" average returns. There are 17 domestic and 3 international such stocks at $\alpha=.05$. However, the number of stocks with "Daily" above "Nightly" average returns is larger for both domestic and international stocks. At $\alpha=.01$, these numbers jump to 27 for domestic and 7 for international stocks. For $\alpha=.05$, there are 42 domestic stocks and 9 international stocks with higher "Daily" than "Nightly" average returns. For the rest out of 215 stocks, there is no statistically significant difference.

Figure 2 shows the relationship between the average "Daily" and "Nightly" returns for all stocks under consideration. It is obvious that the returns are drastically different for the two states. If the average return in one state is high and positive, the average return on the same stock in the second state is likely to be high and negative as there is a strong negative correlation between average returns on "Daily" and "Nightly" securities. The correlation coefficient is $\rho=-.86$ with the corresponding t-statistic $t=-24.5$ (statistically different from zero with $p<10^{-10}$).

For the majority of stocks, the total daily returns, which is the sum of "Daily" and "Nightly" returns using the terminology in the current paper, are about .001 or less which correspond to less than 30% annualized return. However, we see that some averages of one period returns may deviate considerably from zero, with some returns as high as .002–.035 which translates into 50–100% annualized return. We need to note that an investor can realize such levels of return only if the investor keeps the stock exclusively during the night periods (for some stocks) or during the days (for other stocks) and re-balances the portfolio every morning and every evening.

Analogously, Figure 3 shows the relationship between the "Daily" and "Nightly" standard deviations of returns for all stocks under consideration. The standard deviations of these returns are significantly different at $\alpha=.01$. For the majority of stocks, the p-value is extremely low—below 10^{-6}.

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Figure 3. The scatter plot of the relationship between standard deviations of DAY (horizontal axis) and NIGHT returns (vertical axes).

Notes: Most stocks have a statistically lower standard deviation of NIGHT returns than standard deviation of DAY returns. Stocks with lowest variance of NIGHT returns have standard deviations of the order of .01 or less, which corresponds to about 15% or less if annualized.
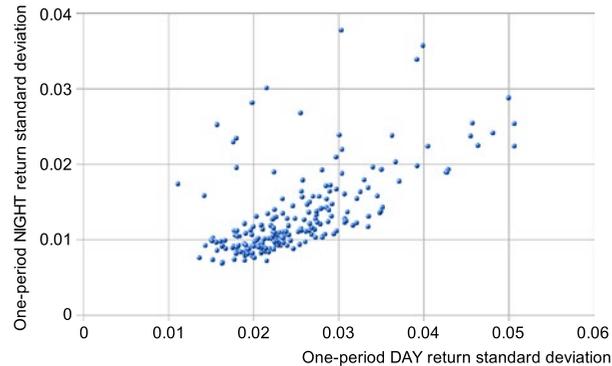
Only 10 stocks have a larger standard deviation of "Nightly" returns than standard deviation of "Daily" returns for the same stock. All such stocks are international stocks. The rest of the stocks (205 out of 215) have higher variances for "Daily" returns with a *p*-value of less than 10^{-6}. It is interesting to note that there is a statistically significant positive correlation $\rho = .63$ (*p*-value is less than 10^{-6}) between "Daily" and "Nightly" standard deviations.

Combining our analysis of both returns and standard deviations, we see that there are 17 US and 2 international stocks with significantly ($\alpha = .05$) higher average "Nightly" returns and lower standard deviations than for "Daily" securities (correspondingly 13 US and 1 international securities for $\alpha = .01$). There are only five international stocks and no domestic stock with higher "Daily" returns and lower variance (at both $\alpha = .05$ and .01). The aforementioned stocks exhibit higher return with lower risk.

The above discussion demonstrated that the "Daily" and "Nightly" characteristics of the same underlying stock are different. As a result, we can treat the two states as different securities. Now, we would like to create a portfolio which consists of "Daily" and "Nightly" securities (see Appendix for more details). Based on the characteristics of the two states, we are able to compute weights, the fraction of total wealth that an investor should contribute to each security in a portfolio. Exactly half of these weights corresponds to the allocation of funds during the day trading hours, e.g. to the "DAY" portfolio, while the other half of weights corresponds to the allocation of funds the investor should use for buying the "NIGHT" portfolio. An investor should buy a portfolio based on "Daily" weights at the open of the exchange and re-balance the portfolio at the close of the market according to the "Nightly" weights. As the market opens the following day, the investor should re-balance the portfolio again using the "Daily" weights of securities and so on. The analysis below shows the advantages of such re-balancing. As commonly done, all computations below assume ideal and frictionless markets, such as absence of bid-ask spread, trading costs, tax considerations, unlimited and free short sales, and enough liquidity to buy or sell any stock.

For easier interpretation, we report this portfolio analysis graphically. We plot the efficient frontier for four different scenarios (see Figure 4) assuming there is no market friction associated with re-balancing a portfolio. The first scenario, buy and hold, is a strategy in which an investor buys a stock and keeps it day and night. Using the terminology of this present paper, the weights of "Daily" and "Nightly" ecurities for the particular stock are equal. The second scenario is a strategy in which an investor buys a portfolio in the morning, keeps it only during the day and sells it at market close. At night, the investor has zero exposure to the market. In this scenario, the weights for all "Nightly" securities are zeros. The third scenario is the case in which an investor buys a portfolio at the close of the market, keeps the portfolio overnight and sells it in the morning. The investor has no stock in possession and does not trade during the day at all. The fourth scenario is a strategy in which an investor re-balances the portfolio every morning and every evening according to the "Daily" and "Nightly" weights.

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Figure 4. The plot of optimal portfolios (efficient frontiers) for the four strategies under consideration.

Notes: The first strategy, the Buy-and-Hold, is denoted by small triangles. The second strategy, the DAY scenario, is denoted by points. The third strategy, the NIGHT scenario, is denoted by points and dashed. And the best strategy, rebalancing the portfolio at every open and close of the market, is denoted by solid line. One can easily see that the last strategy delivers much higher expected return for a given level of standard deviation of the portfolio.
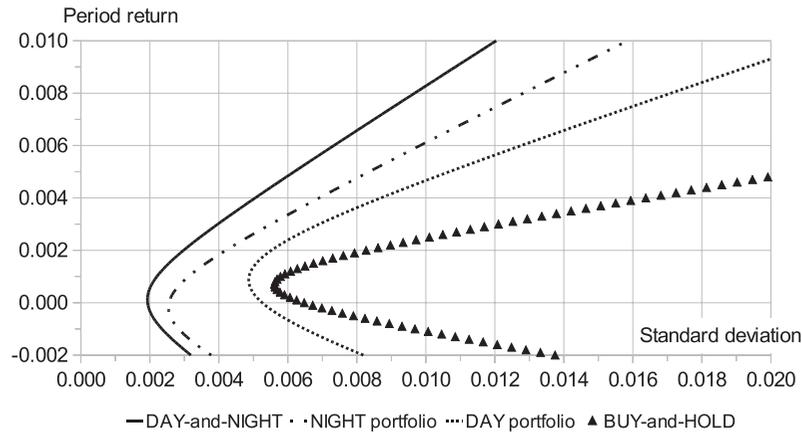
From Figure 4, it is evident that the first scenario (buying a portfolio and keeping it through the day and night) is the worst possible strategy. For the same level of risk, which is measured by standard deviation, the expected return on the efficient portfolio is the lowest one out of the four scenarios. The second scenario (keeping the portfolio only during the day) is mildly better, but the increase in expected return is not very large. For the same level of risk (chosen to be approximately .01–.02, corresponding to about 15–30% annual standard deviation), the increase in expected return is about .002–.004 which translates into an increase of 60–170% in annual return. For the third scenario (keeping portfolio only overnight), the increase in expected return for a given level of risk is even larger and of the order of .0035–.008 (about 140–600%, if annualized). However, the last scenario is the best strategy for an investor to use. For the same level of risk, the increase in expected return is drastic—about .006–.012 (350% to above 1000% annually).

The above analysis cannot be generalized to the whole stock market without any further research which includes stocks from other industries. However, that does not mean that the investor cannot increase the portfolio return significantly without such a research. If subset of stocks allows for the extremely high returns, the investor can use only that subset, ignoring everything else, and receive the high return on "restricted" portfolio. In other words, the option to add securities from other industries may not damage the risk-return characteristics of the portfolio, as there is always the choice to add some securities with zero weight.

Unfortunately, this analysis has a few disadvantages: we have too many assumptions which might not hold in the real world. For example, one cannot neglect trading costs with such frequent trading. In practical world, a trading twice per day gives rise to large trading costs which will eat into profits. If an investor is interested in low-price stocks, the discussed strategies may not be advantageous as trading costs, especially bid-ask spread, are very high compared to stock prices. Conversely, for high-price stocks, the bid-ask spread is significantly lower in percentage terms relative to the price; therefore, costs are relatively lower and the strategy can be more profitable.

Although the expected return of a portfolio will depend strongly on the choice of stocks included, we roughly estimate the profit of a hypothesized portfolio. For this hypothesized portfolio, we buy about 100–500 shares of each stock, prices of which range from $50 to $100, and re-balance the portfolio every morning and evening. The trading costs are estimated to be about $.1–.15 per share and will lower the daily return by about .003–.005. As these trading costs will eliminate any increase in returns, the second strategy is no longer viable and is not better than the "buy and hold" strategy.

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Although the same reasoning is applicable to the third strategy, the third scenario may still be profitable compared to the "buy and hold" because at higher level of risks, increases in returns are much larger yet the trading costs are roughly the same. For example, at higher risks, such as standard deviation of approximately .02 (about 30% annually), the third strategy may increase the expected return by 50–70% compared to the "buy and hold" strategy, whereas the increase in expected return using the second strategy will be eliminated. Finally, if we follow the fourth strategy and re-balance the portfolio every morning and evening, the expected return increases approximately .001–.007 or about 25% (for lower risk portfolios) to 400% (for higher risk portfolios) annually compared to the "buy and hold" strategy.

4. Conclusions and Implications

We consider a stock be in one of two states: the "DAY" state during trading hours and the "NIGHT" state during non-trading hours. We have shown that the stock characteristics (in "Financials" industry), such as average returns and variances, are statistically different for the same stock in the two states. Therefore, any stock, at least from the financial industry, must be treated as two separate securities: a "DAY" security and a "NIGHT" security. This study has provided empirical analysis based on 215 financial industry stocks in the time interval from January 2000 to December 2012. We have shown that an investor can increase his return on the optimal portfolio up to 400% using new portfolio allocation methods. This study concludes that an investor can optimize a frontier portfolio by re-balancing said portfolio twice a day according to periodically changing weights.

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References
Ang, A., & Bekaert, G. (2002). International asset allocation with regime shifts. *Review of Financial Studies, 15*, 1137–1187. http://dx.doi.org/10.1093/rfs/15.4.1137
Ang, A., & Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of Financial Economics, 63*, 443–494. http://dx.doi.org/10.1016/S0304-405X(02)00068-5
Bai, J., Lumsdaine, R., & Stock, J. (1998). Testing for and dating common breaks in multivariate time series. *Review of Economic Studies, 65*, 394–432.
Bai, J., & Perron, P. (2001). Multiple structural change models: A simulation analysis. In D. Corbae, S. Durlauf, & B. E. Hansen (Eds.), *Econometric essays* (pp. 212–237). Cambridge: Cambridge University Press.
Campbell, J. Y. (2000). Asset pricing at the millennium. *The Journal of Finance, 55*, 1515–1567. http://dx.doi.org/10.1111/jofi.2000.55.issue-4
Campbell, J. Y., Lo, A. W., & MacKinley, A. C. (1998). *The econometrics of financial markets*. Princeton, NJ: Princeton University Press.

Hashem Pesaran, M., Pettenuzzo, D., & Timmermann, A. (2006). Forecasting time series subject to multiple structural breaks. *Review of Economic Studies, 73*, 1057–1084. http://dx.doi.org/10.1111/j.1467-937X.2006.00408.x
Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance, 7*, 77–91.
Markowitz, H. M. (1959). *Portfolio selection: Efficient diversification of investments* (1st ed.). New York, NY: Wiley.
Markowitz, H. M. (1991a). Foundations of portfolio theory. *The Journal of Finance, 46*, 469–477. http://dx.doi.org/10.1111/j.1540-6261.1991.tb02669.x
Markowitz, H. M. (1991b). *Portfolio selection: Efficient diversification of investments* (2nd ed.). Cambridge, MA: Basil Blackwell. ISBN: 978-1-55786-108-5.
Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *The Review of Economics and Statistics, 51*, 247–257. http://dx.doi.org/10.2307/1926560
Pástor, Ľ., & Stambaugh, R. (2001). The equity premium and structural breaks. *The Journal of Finance, 56*, 1207–1239. http://dx.doi.org/10.1111/jofi.2001.56.issue-4
Ruppert, D. (2006). *Statistics and finance: An introduction, Springer texts in statistics* (pp. 148–150). New York, NY: Springer. ISBN: 978-0387202709.
Wachter, J., & Warusawitharana, M. (2009). Predictable returns and asset allocation: Should a skeptical investor time the market? *Journal of Econometrics, 148*, 162–178. http://dx.doi.org/10.1016/j.jeconom.2008.10.009
Wright, J. H. (1996). Structural stability tests in the linear regression model when the regressors have roots local to unity. *Economics Letters, 52*, 257–262. http://dx.doi.org/10.1016/S0165-1765(96)00852-X

Appendix

Markowitz (1952, 1959, 1991b) was the first to discuss portfolio allocation. To find the optimal portfolio, we need to make several assumptions. First, we assume that the stock characteristics are constant. Second, we assume that there are no market frictions, e.g. no trading costs, such as bid-ask spread or commission, associated with buying or selling stock. Next, we assume an investor can buy and sell any security in any quantity. Short selling is allowed in any quantity and does not require a margin. Additionally, we assume that the liquidity is not a problem and the investor always can buy and sell the required quantity at the current market price. Finally, we assume that the investor will contribute all wealth to stock purchase and none to the risk-free asset.

Consider an investor, who is interested in minimizing the risk of his portfolio (the standard deviation of returns on an investor's portfolio) while keeping the expected return at a given value. Mathematically, we need to solve the following minimization problem:

$$\sum_{i=1}^{N} w_i \, \Omega_{ij} \, w_j = \sigma_0^2 - \min$$

$$\sum_{i=1}^{N} w_i \, E(R_i) = R_0 - \text{given value}$$

$$\sum_{i-1}^{N} w_i = 1$$

where Ω is the covariance matrix of returns (Ω_{ij} is the covariance between securities "i" and "j"), w_i is the weight (allocation of funds) to security "i", and $E(R_i)$ is the expected return on security "i".

The solution to the above problem is known and can be found in many books (see, e.g. Ruppert, 2006):

$$w_i = C_1 \, \Omega_{ij}^{-1} \, e_j + C_0 \, \Omega_{ij}^{-1} \, E(R_j) + R_0 \left(C_2 \, \Omega_{ij}^{-1} \, E(R_j) + C_0 \, \Omega_{ij}^{-1} \, e_j \right)$$

where e_i is a vector of ones and constants are

$$C_0 = - \frac{\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} e_j}{\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} E(R_j) \times \sum_{i,j} e_i \, \Omega_{ij}^{-1} e_j - \left(\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} e_j \right)^2}$$

$$C_1 = \frac{\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} E(R_j)}{\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} E(R_j) \times \sum_{i,j} e_i \, \Omega_{ij}^{-1} e_j - \left(\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} e_j \right)^2}$$

$$C_2 = \frac{\sum_{i,j} e_i \, \Omega_{ij}^{-1} e_j}{\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} E(R_j) \times \sum_{i,j} e_i \, \Omega_{ij}^{-1} e_j - \left(\sum_{i,j} E(R_i) \, \Omega_{ij}^{-1} e_j \right)^2}$$

Knowing the expected returns and the covariance matrix, an investor can find the C_0, C_1, and C_2 values and, therefore, can calculate the securities' weights, w_i. For such weights, the portfolio will have required return, R_0, with lowest possible standard deviation (risk), σ_0.

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