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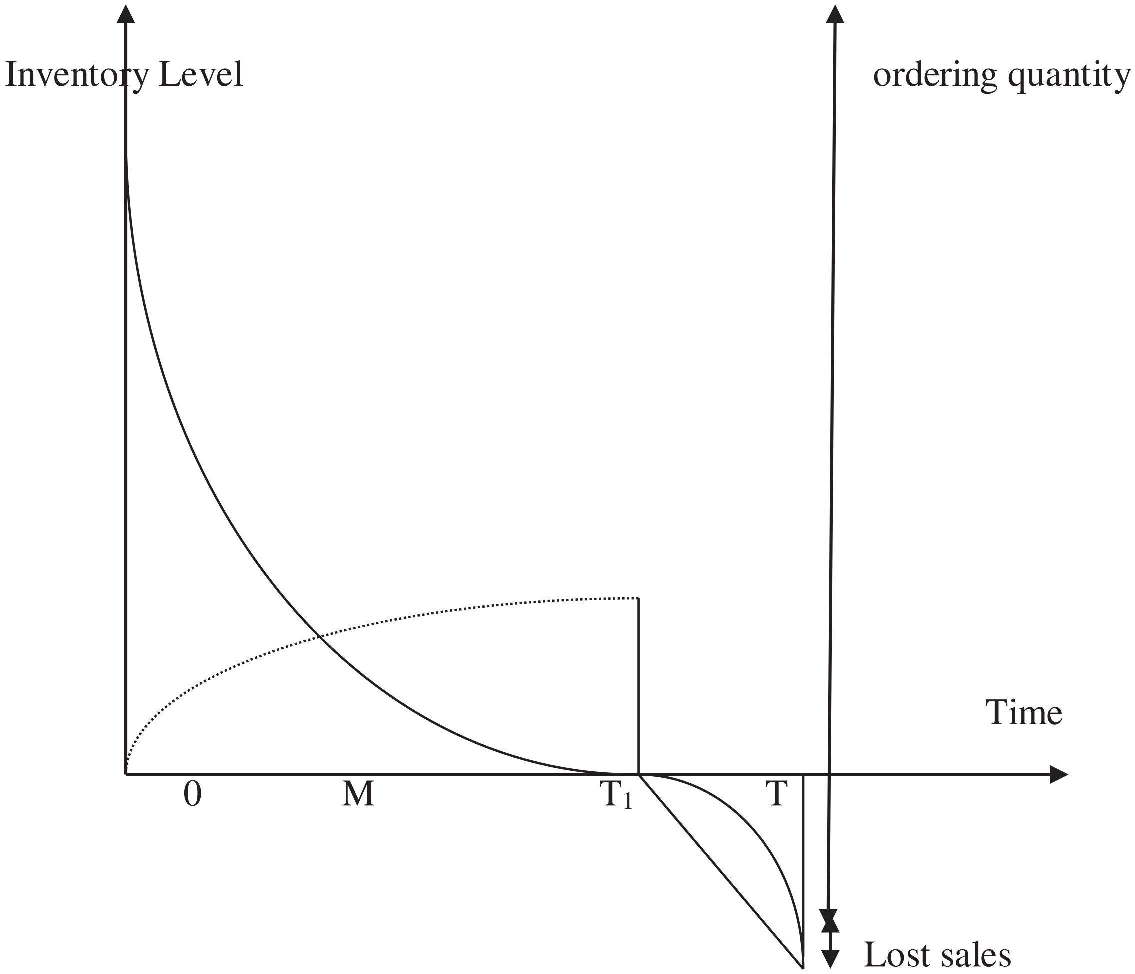

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An inventory model for deteriorating items under inflation and permissible delay in payments by genetic algorithm

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Abstract: Inventory models play a leading role in analyzing a lot of realistic situations arising at places like, food and vegetable markets, market yards, oil exploration industries, etc. In the present article, we developed an inventory model for deteriorating items with permissible delay in payment under inflation. In the given model, demand rate is considered as stock-dependent and deterioration rate of each item follows Weibull distribution. The model is developed under two different circumstances depending on whether the credit period is (1) less than the cycle time (2) greater than the cycle time. Also, a new algorithm is developed under these scenarios to obtain the EOQ. Finally results are analyzed and demonstrated with illustrative examples by Genetic Algorithm.

Subjects: Applied Mathematics; Mathematical Modeling; Non-Linear Systems

Keywords: Weibull distribution; inflation; stock-dependent demand; permissible delay; genetic algorithm

1. Introduction

Deterioration of items is a frequent and natural phenomenon which cannot be ignored. In realistic scenario, the life cycle of seasonal product, fruits, electric component, volatile liquid, food, etc. are short and finite usually can undergo deterioration. Thus, the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers. The present article investigates inventory model for deteriorating items with stock-dependent demand rate. The deterioration of inventory in stock during the storage period constitutes

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PUBLIC INTEREST STATEMENT

The aim of this paper is to widen the concept of demand and supply by model building. This work assumed that demand rate is stock-dependent. Demand is also related to shortage under the conditions of permissible delay in payment. The optimal cycle time is determined to minimize total inventory cost. It will add benefits to the society and individuals who suffers from problems related to partial backlogging. In addition, some managerial insights on the basis of numerical examples are also concluded for maximizing profits and cost.

an important factor which has attracted the attention of researchers. Dye (2002) developed an inventory model with stock-dependent demand and partial backlogging. Chakrabarty, Giri, and Chaudhuri (1998) extended the Philip's model (1974). Skouri and Papachristos (2003) investigate optimal time of an EOQ model with deteriorating items and time-dependent partial backlogging. Deterioration can't be ignored in business scenarios. Rau, Wu, and Wee (2004) developed an integrated inventory model to determine economic ordering policies of deteriorating items in a supply chain management system. Teng and Chang (2005) developed economic production quantity in an inventory model for deteriorating items. Deterioration refers to decay, damage, or spoilage like foods, drugs, chemicals, electronic components and radio-active substances, deterioration may happen during normal period of storage and the loss is to be taken into account where we analyze inventory systems. Dave and Patel (1983) investigate an inventory model for deteriorating items with time proportional demand. Roychowdhury and Chaudhuri (1983) introduced an order level inventory model considering a finite rate of replenishment and allowing shortages. In this direction their model Misra (1975), Deb and Chaudhuri (1986) considered that deterioration rate is time dependent. Berrotoni (1962) investigates some difficulties of fitting empirical data to mathematical distribution. It can be said that the rate of deterioration increases with age. It may be inferred that the work of Berrotoni (1962) inspired Covert and Philip (1973) to design an inventory model for deteriorating items with Weibull distribution by two parameters. Mandal and Phaujdar (1989) developed a production inventory model for deteriorating items with uniform rate of production and stock-dependent demand. In this direction, some precious works in this area were also done by Padmanabhan and Vrat (1995), Ray and Chaudhuri (1997), Mandal and Maiti (1999).

In all the above models, the time value of money and inflation were not considered because of the belief that the time value of money and inflation would not affect significantly the decisions regarding inventory management. But in real life, the impact of time value of money and inflation cannot be ignored while deciding the optimal inventory policies. Today, inflation has become a permanent feature of the economy. Many researchers have shown the inflationary effect on inventory policy. Bierman and Thomas (1977), Buzacott (1975), Chandra and Bahner (1988), Jesse, Mitra, and Cox (1983), Misra (1979) developed their inventory models assuming a constant inflation rate. An inventory model with deteriorating items under inflation when a delay in payment is permissible is calculated by Liao, Tsai, and Su (2000). Bhahmbhatt (1982) introduced an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages. Both in deterministic and probabilistic inventory models of classical type it is observed that payment is made to the supplier for goods just after getting the consignment. But actually current scenario a supplier grants some credit period to the retailer to increase the demand. In this respect Goyal (1985) just formulated an EOQ model under some conditions of permissible delay in payment. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). Chung, Huang, and Huang (2002) and Chung and Huang (2003) developed an optimal replenishment policy for EOQ models under permissible delay in payments. In recent times to make the real inventory systems more practical and realistic, Aggarwal and Jaggi (1995) extended the model with a constant deterioration rate. Hwang and Shinn (1997) developed lot-sizing policy for exponential demand when delay in payment is permissible. Shah and Shah (1998) presented a probabilistic inventory model with a cost in case delay in payment is permissible. Subsequently Jamal, Sarker, and Wang (1997) developed further following the lines of Aggarwal and Jaggi's (1995) model to take into consideration for shortage and make it more practical and acceptable in real situation. Some of the recent works in this area may relate to Datta, Paul, and Pal (1998), Dye (2002), Chung (2003), Zhou and Yang (2005), Goyal and Chang (2009), and Yang, Teng, and Chern (2010), Kumar, Singh, and Kumari (2013), developed an inventory model with stock-dependent demand rate for deterioration items. Kumar, Singh, and Kumari (2014), Tayal, Singh, and Sharma (2015), developed an inventory model for deteriorating items with seasonal products and an option of an alternative market. Kumar and Kumar (2016a), developed an inventory model with stock-dependent demand rate for deterioration items. Recently Kumar and Kumar (2016b) presented an inventory model for deteriorating items stock-dependent demand and partial backlogging.

This study is developing an inventory model with stock-dependent demand rate with permissible delay in payment in real life situations. It will help the retailers to manage the business. The effect of preservation technique is used to reduce the deterioration and the time value of the money cannot be ignored in determining the optimal inventory decision. The concept of the inflation should be considered especially for long-term investment and forecasting. The model shows the effect due to changes in various parameters by taking suitable numerical examples and sensitivity analysis.

2. Notations and assumptions

2.1. Notations

The following notations are used throughout this paper:

$q(t)$	Inventory level at time t
$S = q(0)$	Stock level at the beginning of each cycle after fulfilling backorders
H	Length of the planning horizon
K	Constant rate of inflation ($/$/unit time)
$C(t)$	Unit purchase cost for an item bought at time t, i.e. $C(t) = C_0.e^{KT}$, where C_0 is the unit Purchase cost at time zero
h	Holding cost ($/unit/year) excluding interest charges
C_0	Unit purchase cost
C_2	Shortage cost ($/unit/time)
C_3	The ordering cost/cycle
i_e	Interest earned ($/time)
i_p	Interest charged ($/time)
M	Permissible delay in settling the accounts
T_1	Time at which shortages start ($0 \le T_1 \le T$)
T	Length of a cycle
TCU (T_1, T)	The Total cost function per unit time
$\text{TCU}_1\ (T_1, T)$	The Total cost function per unit time for $T_1 > M$ (Case I)
$\text{TCU}_2\ (T_1, T)$	The Total cost function per unit time for $T_1 \le M$ (Case II)

2.2. Assumptions

To develop the mathematical model, the following assumptions are being made:

(i) The inventory system involves only one item.

(ii) The rate of replenishment is instantaneous.

(iii) A fraction $z(t)$ of the on hand inventory deteriorates per unit time where $z(t) = \alpha\beta t^{\beta-1}$, $0 < \alpha < 1$, $t > 0$, $\beta > 1$.

(iv) Shortages are allowed and the backlog rate is defined to be $R(t)/1 + \delta(T-t)$ when inventory is negative. The backlogging parameter δ is a positive constant.

(v) The demand rate $R(t)$ at time t is $R(t) = \begin{cases} a + b\,q(t), & 0 \le t \le T_1 \\ a, & T_1 \le t \le T \end{cases}$ where a and b are non-negative constraints.

3. Formulation and solution of the model

Based on the above description, during the time interval $[0, T_1]$, the inventory level at time t will satisfy the following differential representing the inventory status is given by:

$$\frac{dq(t)}{dt} + \alpha\beta t^{\beta-1}q(t) = -\left(a + bq(t)\right) \quad 0 \le t \le T_1 \tag{1}$$

With the boundary condition $q\ (T_1) = 0$, the solution of Equation (1) is:

$$q(t) = a(\alpha\beta T_1^{\beta} + b)\left[e^{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - t^{\beta+1}) + b(T_1 - t)} - 1\right] \tag{2}$$

Again in the second time interval [T_1, T' the instantaneous inventory will satisfy. Thus, the differential equation below represents the inventory status:

$$\frac{dq(t)}{dt} = -\frac{a}{1+\delta(T-t)}, \quad T_1 \le t \le T \tag{3}$$

With the condition q (T_1) = 0, we get the solution of equation (3) is:

$$q(t) = \frac{a}{\delta}\left[\log\{1+\delta(T-t)\} - \log\left\{1+\delta\left(T-T_1\right)\right\}\right] \tag{4}$$

Inventory model before and after the deflation is illustrated in Figures 1 and 2.

3.1 Case I (M < T_1): Payment before depletion

The holding cost during [0, *T*] is:

$$HC = h\sum_{n=0}^{m-1} C(\eta T)\int_0^{T_1} q(t)\,dt = h\,c_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left[\int_0^{T_1} a\left(\alpha\beta T_1^{\beta} + b\right)\left\{e^{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - t^{\beta+1}) + b(T_1 - t)} - 1\right\}dt\right] \tag{5}$$

$$= h\,c_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left[a\left(\alpha\beta T_1^{\beta} + b\right)\left\{e - \alpha\beta T_1^{\beta} - \frac{e^{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1} + bT_1 + 1}}{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1} + bT_1 + 1} - T_1\right\}\right] \tag{6}$$

(Ignoring the higher order of α)

The number deteriorating items during [0, T_1] is:

$$= q(0) - \int_0^{T_1}(a+bt)dt = S - \left(aT_1 + \frac{bT_1}{2}\right) = \frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2} \tag{7}$$

The deteriorated cost *DC* is:

$$DC = C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left\{\frac{a\alpha T_1^{\beta+1}}{\beta+1} + \frac{b\alpha T_1^{\beta+2}}{\beta+2}\right\} \tag{8}$$

Figure 1. Inventory model before the deflation.

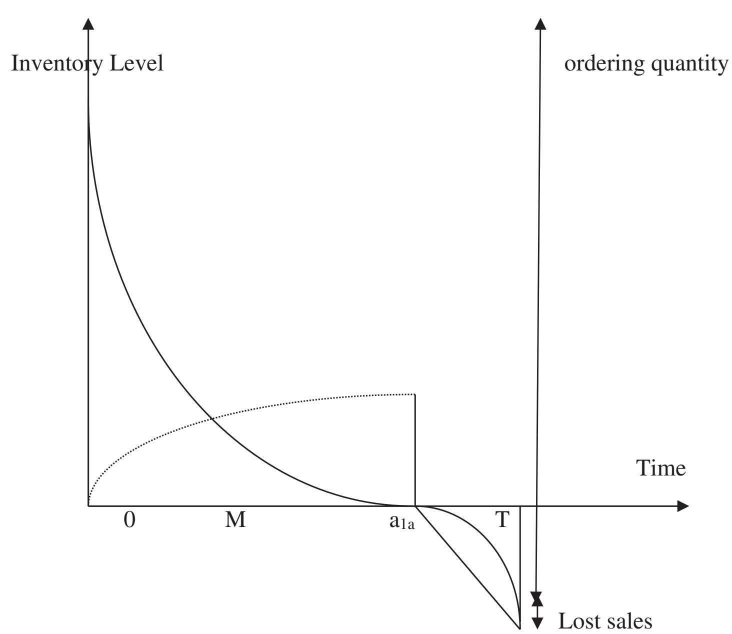
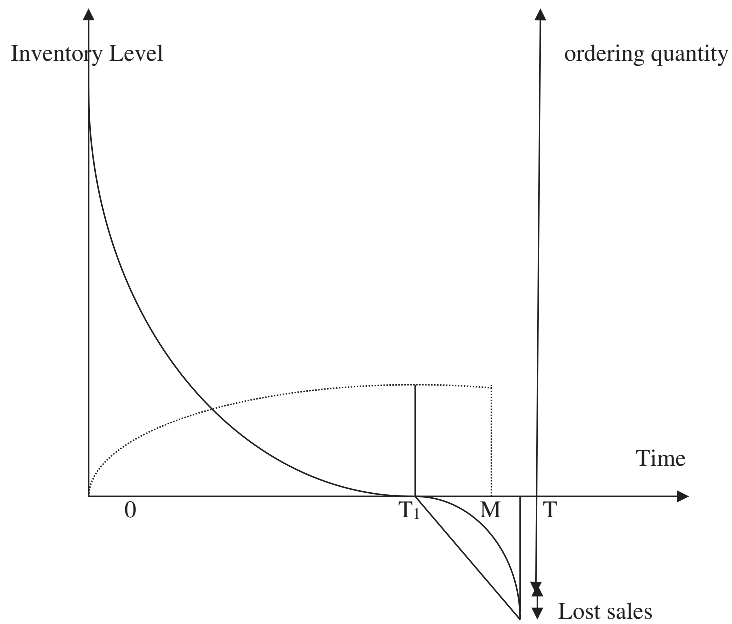


Figure 2. Inventory model after the deflation.



The shortage cost $[0, T_1]$ is:

$$SHC = C_2.C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\int_{T_1}^{T} q(t)\,dt \tag{9}$$

$$= C_2.C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\int_{T_1}^{T} \frac{a}{\delta}\left[\log\{1+\delta(T-t)\} - \log\left\{1+\delta\left(T-T_1\right)\right\}\right] dt \tag{10}$$

$$= C_2.C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta}+\delta\left(T-T_1\right)-\left(T-T_1\right)\right\}\right] \tag{11}$$

The total variable cost is comprised of the sum of the ordering cost, holding cost, backorder cost, deterioration cost, and interest payable minus the interest earned. They are grouped together after evaluating the above cost individually.

The interest earned IE_1 during the time $[0, T]$ is:

$$IE_1 = i_e.C_0\frac{e^{KH}-1}{e^{KT}-1}\int_0^T \left(T_1-t\right)\left(a+bq(t)\right)dt \tag{12}$$

$$= i_e.C_0\frac{e^{KH}-1}{e^{KT}-1}\left[\frac{aT_1^2}{2}+\frac{ab\alpha^2\beta^2}{\beta+1}\left(T_1^{2(\beta+1)}-T\,T_1^{2(\beta+1)}-\frac{T_1^{(\beta+3)}}{2}+\frac{T_1^{(2\beta+3)}}{\beta+1}\right)\right.$$
$$-\frac{ab\alpha\beta}{\beta+1}\left(\frac{T_1^{(\beta+2)}}{\beta+2}-b\,T_1^{(\beta+2)}+\frac{T_1^{(\beta+2)}}{\beta+2}\right)-ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2}+\frac{T_1^{(\beta+3)}}{3}\right)$$
$$\left.+abT\,T_1^2-\frac{ab^3\,T_1^3}{3}+ab^3T\,T_1\left(T-\frac{T_1}{2}\right)\right]$$

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The interest payable *IP* per cycle for the inventory not being sold after due date *M*:

$$IP_1 = i_p.C_0 \frac{e^{KH} - 1}{e^{KT} - 1} \int_M^T q(t)\, dt \tag{13}$$

$$= i_p.C_0 \frac{e^{KH} - 1}{e^{KT} - 1} \int_M^T a(\alpha\beta T_1^{\beta} + b)\left[e^{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - t^{\beta+1}) + b(T_1 - t)} - 1\right] dt \tag{14}$$

$$= i_p.C_0 \left(\frac{e^{KH} - 1}{e^{KT} - 1}\right)\left[a(\alpha\beta T_1^{\beta} + b)\left\{(\alpha\beta M^{\beta} + b)\left(\frac{e^{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - M^{\beta+1}) + b(T_1 - M) + 1}}{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - M^{\beta+1}) + b(T_1 - M) + 1}\right) - e(\alpha\beta T_1^{\beta} + b)\right\}\right] \tag{15}$$

So, the total variable cost, TVC_1 is defined as:

TVC_1=Ordering cost + holding cost + deterioration cost + shortage cost + interest payable – interest earned.

$$TVC_1 = C_3 + HC + DC + SHC + IP_1 - IE_1 \tag{16}$$

$$\begin{aligned}
TVC_1 = {} & C_0\left(\frac{e^{KH} - 1}{e^{KT} - 1}\right)\left[a(\alpha\beta T_1^{\beta} + b)\left\{h\left(e - \alpha\beta T_1^{\beta} - \frac{e^{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1} + bT_1 + 1}}{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1} + bT_1 + 1} - T_1\right)\right\}\right. \\
& C_3 + i_p\left\{(\alpha\beta T_1^{\beta} + b)\left(\frac{e^{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - M^{\beta+1}) + b(T_1 - M) + 1}}{\frac{\alpha\beta}{\beta+1}(T_1^{\beta+1} - M^{\beta+1}) + b(T_1 - M) + 1}\right) - e(\alpha\beta T_1^{\beta} + b)\right\} \\
& + C_2\left\{\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta} + \delta(T - T_1) - (T - T_1)\right\}\right]\right\} \\
& + \left\{\frac{a\alpha T_1^{\beta+1}}{\beta + 1} + \frac{b\alpha T_1^{\beta+2}}{\beta + 2}\right\} - i_c\left\{\frac{aT_1^2}{2} + \frac{ab\alpha^2\beta^2}{\beta + 1}\left(T_1^{2(\beta+1)} - T\,T_1^{2(\beta+1)} - \frac{T_1^{(\beta+3)}}{2} + \frac{T_1^{(2\beta+3)}}{\beta + 1}\right)\right. \\
& - \frac{ab\alpha\beta}{\beta + 1}\left(\frac{T_1^{(\beta+2)}}{\beta + 2} - b\,T_1^{(\beta+2)} + \frac{T_1^{(\beta+2)}}{\beta + 2}\right) - ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2} + \frac{T_1^{(\beta+3)}}{3}\right) \\
& \left.\left. + abT\,T_1^2 - \frac{ab^3\,T_1^3}{3} + ab^3T\,T_1\left(T - \frac{T_1}{2}\right)\right\}\right].
\end{aligned} \tag{17}$$

The total variable cost per unit time, *TCU*, during the cycle period [0, *T*] is given by:

$$TCU_1 = \frac{TVC_1}{T} = TVC_1 = C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left[a\left(\alpha\beta T_1^{\beta}+b\right)\left\{h\left(e-\alpha\beta T_1^{\beta}-\frac{e^{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}}{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}-T_1\right)\right\}\right.$$
$$+C_3+i_p\left\{\left(\alpha\beta T_1^{\beta}+b\right)\left(\frac{e^{\frac{\alpha\beta}{\beta+1}\left(T_1^{\beta+1}-M^{\beta+1}\right)+b\left(T_1-M\right)+1}}{\frac{\alpha\beta}{\beta+1}\left(T_1^{\beta+1}-M^{\beta+1}\right)+b\left(T_1-M\right)+1}\right)-e\left(\alpha\beta T_1^{\beta}+b\right)\right\}$$
$$+C_2\left\{\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta}+\delta\left(T-T_1\right)-\left(T-T_1\right)\right\}\right]\right\}$$
$$+\left\{\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{\beta+2}\right\}-i_c\left\{\frac{aT_1^2}{2}+\frac{ab\alpha^2\beta^2}{\beta+1}\left(T_1^{2(\beta+1)}-T\,T_1^{2(\beta+1)}-\frac{T_1^{(\beta+3)}}{2}+\frac{T_1^{(2\beta+3)}}{\beta+1}\right)\right.$$
$$-\frac{ab\alpha\beta}{\beta+1}\left(\frac{T_1^{(\beta+2)}}{\beta+2}-b\,T_1^{(\beta+2)}+\frac{T_1^{(\beta+2)}}{\beta+2}\right)-ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2}+\frac{T_1^{(\beta+3)}}{3}\right)$$
$$\left.\left.+abT\,T_1^2-\frac{ab^3\,T_1^3}{3}+ab^3T\,T_1\left(T-\frac{T_1}{2}\right)\right\}\right]/T \tag{18}$$

3.2. Case II (T < M): Payment after depletion

The ordering cost C_3, the holding cost HC, the shortage cost SHC, and the deterioration cost DC during the cycle period (0, *T*) is the same as in case I. The payable per cycle is $P_T = 0$ when $T_1 < M < T$ because the supplier can be paid in full (at time *M*) the permissible delay. The interest earned per cycle is:

$$I\,E_2 = i_e C_0.\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\left\{\int_0^{T_1}\left(T_1-t\right)\left(a+bq(t)\right)dt+\left(M-T_1\right)\int_0^{T_1}\left(a+bq(t)\right)dt\right\} \tag{19}$$

$$=i_e.C_0\frac{e^{KH}-1}{e^{KT}-1}\left[\left\{\frac{aT_1^2}{2}+\frac{ab\alpha^2\beta^2}{\beta+1}\left(T_1^{2(\beta+1)}-T\,T_1^{2(\beta+1)}-\frac{T_1^{(\beta+3)}}{2}+\frac{T_1^{(2\beta+3)}}{\beta+1}\right)\right.\right.$$
$$-\frac{ab\alpha\beta}{\beta+1}\left(\frac{T_1^{(\beta+2)}}{\beta+2}-b\,T_1^{(\beta+2)}+\frac{T_1^{(\beta+2)}}{\beta+2}\right)-ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2}+\frac{T_1^{(\beta+3)}}{3}\right)$$
$$\left.+abT\,T_1^2-\frac{ab^3\,T_1^3}{3}+ab^3T\,T_1\left(T-\frac{T_1}{2}\right)\right\}$$
$$\left.+a\left(M-T_1\right)\left\{T_1\left(1-bT\right)-\frac{bT_1^2}{2}+\frac{\alpha\beta}{\beta+2}\left(b\alpha\beta T_1^{2(\beta+1)}-T_1^{\beta+2}\right)\right\}\right] \tag{20}$$

The total variable cost, TVC_2 is defined as:

$$TVC_2 = C_3 + HC + DC + SHC - IE_2 \tag{21}$$

$$
\begin{aligned}
TVC_2 = {} & C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\Bigg[h\Bigg\{a\left(\alpha\beta T_1^{\beta}+b\right)\left(e-\alpha\beta T_1^{\beta}-\frac{e^{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}}{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}-T_1\right)\Bigg\} \\
& + C_3 + C_2\left\{\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta}+\delta(T-T_1)-(T-T_1)\right\}\right]\right\} \\
& + \left\{\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{\beta+2}\right\} - i_c\Bigg\{\frac{aT_1^2}{2}+\frac{ab\alpha^2\beta^2}{\beta+1}\left(T_1^{2(\beta+1)}-T\,T_1^{2(\beta+1)}-\frac{T_1^{(\beta+3)}}{2}+\frac{T_1^{(2\beta+3)}}{\beta+1}\right) \\
& - \frac{ab\alpha\beta}{\beta+1}\left(\frac{T_1^{(\beta+2)}}{\beta+2}-b\,T_1^{(\beta+2)}+\frac{T_1^{(\beta+2)}}{\beta+2}\right)-ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2}+\frac{T_1^{(\beta+3)}}{3}\right)+abT\,T_1^2 \\
& - \frac{ab^3\,T_1^3}{3}+ab^3T\,T_1\left(T-\frac{T_1}{2}\right)\Bigg\} \\
& + a(M-T_1)\left\{T_1(1-bT)-\frac{bT_1^2}{2}+\frac{\alpha\beta}{\beta+2}\left(b\alpha\beta T_1^{2(\beta+1)}-T_1^{\beta+2}\right)\right\}\Bigg]
\end{aligned}
\tag{22}
$$

The total variable cost per unit time $TCU\ (T_1, T)$ is:

$$
\begin{aligned}
TCU_2 = \frac{TVC_2}{T} = {} & C_0\left(\frac{e^{KH}-1}{e^{KT}-1}\right)\Bigg[\Bigg[h\Bigg\{a\left(\alpha\beta T_1^{\beta}+b\right)\left(e-\alpha\beta T_1^{\beta}-\frac{e^{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}}{\frac{\alpha\beta}{\beta+1}T_1^{\beta+1}+bT_1+1}-T_1\right)\Bigg\} \\
& + C_3 + C_2\left\{\frac{a}{\delta}\left[\log\left\{\frac{1}{\delta}+\delta(T-T_1)-(T-T_1)\right\}\right]\right\} \\
& + \left\{\frac{a\alpha T_1^{\beta+1}}{\beta+1}+\frac{b\alpha T_1^{\beta+2}}{\beta+2}\right\} - i_c\Bigg\{\frac{aT_1^2}{2}+\frac{ab\alpha^2\beta^2}{\beta+1}\left(T_1^{2(\beta+1)}-T\,T_1^{2(\beta+1)}-\frac{T_1^{(\beta+3)}}{2}+\frac{T_1^{(2\beta+3)}}{\beta+1}\right) \\
& - \frac{ab\alpha\beta}{\beta+1}\left(\frac{T_1^{(\beta+2)}}{\beta+2}-b\,T_1^{(\beta+2)}+\frac{T_1^{(\beta+2)}}{\beta+2}\right)-ab^2\alpha\beta\left(\frac{T\,T_1^{(\beta+2)}}{2}+\frac{T_1^{(\beta+3)}}{3}\right) \\
& + abT\,T_1^2-\frac{ab^3\,T_1^3}{3}+ab^3T\,T_1\left(T-\frac{T_1}{2}\right)\Bigg\} \\
& + a(M-T_1)\left\{T_1(1-bT)-\frac{bT_1^2}{2}+\frac{\alpha\beta}{\beta+2}\left(b\alpha\beta T_1^{2(\beta+1)}-T_1^{\beta+2}\right)\right\}\Bigg]\Bigg/T
\end{aligned}
\tag{23}
$$

4. Genetic algorithm

A genetic algorithm (GA) is based on natural selection process to optimized tools that minimizes the total costs in supply chain management. It is an evolutionary computation method to solve inventory problems. This is the more effective methods to find the optimized solution. The genetic algorithm uses three main types of rules at each step to create the next generation from the current population.

4.1. The basic steps to find the optimized solution

Step 1: First one is ***Selection rules***, In this we select the individuals, called *parents* that contribute to the population at the next generation.

Step 2: Next one is ***Crossover rules***, *In this,* we perform crossover operation between two parents to form children for the next generation.

Step 3: Last one is ***Mutation rules***, In mutation, we apply some random changes to individual parents.

4.2. Parameters

Firstly, we set the different parameters on which the specific GA depends. These are the number of generations (MAXGEN), population size (POPSIZE), the probability of crossover (PCROS), probability of mutation (PMUTE).

4.3. Chromosome representation

An important issue in applying a GA is to design an appropriate chromosome representation of solutions of the problem together with genetic operators. Traditional binary vectors used to represent the chromosomes are not effective in many non-linear problems. Since the proposed problem is highly non-linear, hence to overcome the difficulty, a real-number representation is used. In this representation, each chromosome V_i is a string of n numbers of genes G_{ij}, ($j = 1, 2, \ldots n$) where these n numbers of genes, respectively, denote n number of decision variables (X_i, $i = 1, 2, \ldots n$).

4.4. Initial population production

The population generation technique proposed in the present GA is illustrated by the following procedure: For each chromosome V_i, every gene G_{ij} is randomly generated between its boundary (LB_j, UB_j) where LB_j and UB_j are the lower and upper bounds of the variables X_i, $i = 1, 2, \ldots n$, POPSIZE.

4.5. Evaluation

Evaluation function plays the same role in GA as that, which the environment plays in natural evolution. Now, evaluation functions (EVAL) for the chromosome V_i is equivalent to the objective function PF(X). These are steps of evaluation:

Step 1: Find EVAL(V_i) by EVAL(V_i) = $f(X_1, X_2, X_3, \ldots X_n)$ where the genes G_{ij} represent the decision variable X_j, $j = 1, 2, \ldots n$, POPSIZE and f is the objective function.

Step 2: Find total fitness of the population: $\mathrm{F} = \sum_{i=1}^{POPSIZE} EVAL(V_i)$..

Step 3: Calculate the probability pi of selection for each chromosome V_i as $Y_i = \sum_{j=1}^{i} pj$

4.6. Selection

The selection scheme in GA determines which solutions in the current population are to be selected for recombination. Many selection schemes, such as stochastic random sampling, roulette wheel selection have been proposed for various problems. In this paper, we adopt the roulette wheel selection process.

This roulette selection process is based on spinning the roulette wheel POPSIZE times, each time we select a single chromosome from the new population in the following way:

(a) Generate a random (float) number r between 0 and 1.

(b) If $r < Y_i$ then the first chromosome is V_i otherwise select the ith chromosome V_i ($2 \le i \le$ POPSIZE) such that $T_i - 1 \le r \le Y_i$.

4.7. Crossover

Crossover operator is mainly responsible for the search of the new string. The exploration and exploitation of the solution space are made possible by exchanging genetic information of the current chromosomes. Crossover operates on two parent solutions at a time and generates off spring solutions by recombining both parent solution features. After selection of chromosomes for the new population, the crossover operator is applied. Here, the whole arithmetic crossover operation is used. It is defined as a linear combination of two consecutive selected chromosomes V_m and V_n and the resulting offspring's V'_m and are V'_n calculated as:

$$V'_m = c \cdot V_m + (1 - c) \cdot V_n$$

$$V'_n = c \cdot V_n + (1 - c) \cdot V_m$$

where c is a random number between 0 and 1.

4.8. Mutation

Mutation operator is used to prevent the search process from converging to local optima rapidly. It is applied to a single chromosome V_i. The selection of a chromosome for mutation is performed in the following way:

Step 1. Set i $\leftarrow$ 1.

Step 2. Generate a random number u from the range [0, 1].

Step 3. If u < PMUTE, then go to Step 2.

Step 4. Set i $\leftarrow i + 1$.

Step 5. If $i \leq$ POPSIZE, then go to Step 2.

Then the particular gene G_{ij} of the chromosome V_i, selected by the above-mentioned steps is randomly selected. In this problem, the mutation is defined as G_{ij}^{mut} = random number from the range [0, 1].

4.9. Termination

If the number of iterations is less than or equal to MAXGEN then the process is going on, otherwise it terminates. The GA's procedure is given below:

```
begin
do {
  t ← 0
  while (all constraints are not satisfied)
  {
    initialize Population (t)
  }
    evaluate Population(t)
    while (not terminate)
    {
    t ← t + 1
      select Population(t) from Population(t-1)
      crossover and mutate Population(t)
      evaluate Population(t)
    }
  print Optimum Result
}
end.
```

5. Numerical example and sensitivity analysis

In this paper, the ordering policies have been discussed in two scenarios: payment before total depletion (Case I) and payment after total depletion (Case II). An example is considered to illustrate the effect of the developed model in this paper.

The following inventory parametric values are using $a = 600$, $b = 70$, $\alpha = 0.00010$, $\beta = 1.0$, $M = 0.1$, $\delta = 5.0$, $C_0 = 0.5$, $i_e = 0.18$, $i_p = 0.20$, $K = 0.1$, $H = 1$ year, h = \$ 2.00/unit, $C_3 = 100.0$, $C_2 = 0.8$/unit.

To solve this problem we used genetic algorithm. In this problem, GA consists of the parameters, POPSIZE = 50, MAXGEN = 50, Cross over probability = 0.75, Mutation probability = 0.005. The solutions of two cases for different parametric values of α, β, M and δ, are given in Tables 1 and 2.

If we plot the total cost function $TCU1$ and $TCU2$ with some values of T_1 and T vs. various parameters then we get strictly convex graph of total cost function given in Figures 3 and 4.

From the above tables and figures, the results can be discussed as follows:

1. Table 1 indicates as the values of T_1 and T reduce when the parameter α increases, but the total cost function (TCU) increases in both cases.
2. The values of T_1, T decrease when the parameter β increases, and total cost function (TCU) increases in both cases.
3. The values of T_1, T increase when the parameter M increases, and the total average inventory cost (TCU) decreases both in cases.
4. In Table 2 cycle time T decreases as the parameter δ increases, but total cost function (TCU) increases in both the cases.

Table 1. Case I: Payment before depletion

Changing parameters	Change in parameter	T_1	T	TCU_1
α	0.00010	0.0822	0.0883	4585.08
	0.00015	0.0716	0.0853	6375.53
	0.00020	0.0593	0.0676	7845.87
β	1.0	0.0822	0.0883	4585.08
	1.5	0.0721	0.0813	6405.02
	2.0	0.0543	0.0759	6968.14
M	0.1	0.0822	0.0883	4585.08
	0.3	0.0919	0.0953	3695.52
	0.5	0.0958	0.1053	2655.38
δ	5.0	0.0822	0.0853	4585.08
	6.0	0.0873	0.0927	4915.57
	7.0	0.0892	0.1073	5289.64

Table 2. Case II: Payment after depletion

Changing parameters	Change in parameter	T_1	T	TCU_2
α	0.00010	0.0926	0.1033	3245.50
	0.00015	0.0816	0.0953	4175.53
	0.00020	0.0798	0.0886	4855.84
β	1.0	0.0926	0.1033	3245.50
	1.5	0.0791	0.0913	3405.02
	2.0	0.0643	0.0759	4968.14
M	0.10	0.0926	0.1033	3245.50
	0.12	0.1004	0.1243	2295.56
	0.13	0.1058	0.1355	2055.30
δ	5.0	0.0926	0.1033	3245.50
	6.0	0.973	0.0927	3985.67
	7.0	0.0976	0.0873	4287.24

Figure 3. Effect of parameters α, β, M, and δ on total cost before the deflation.

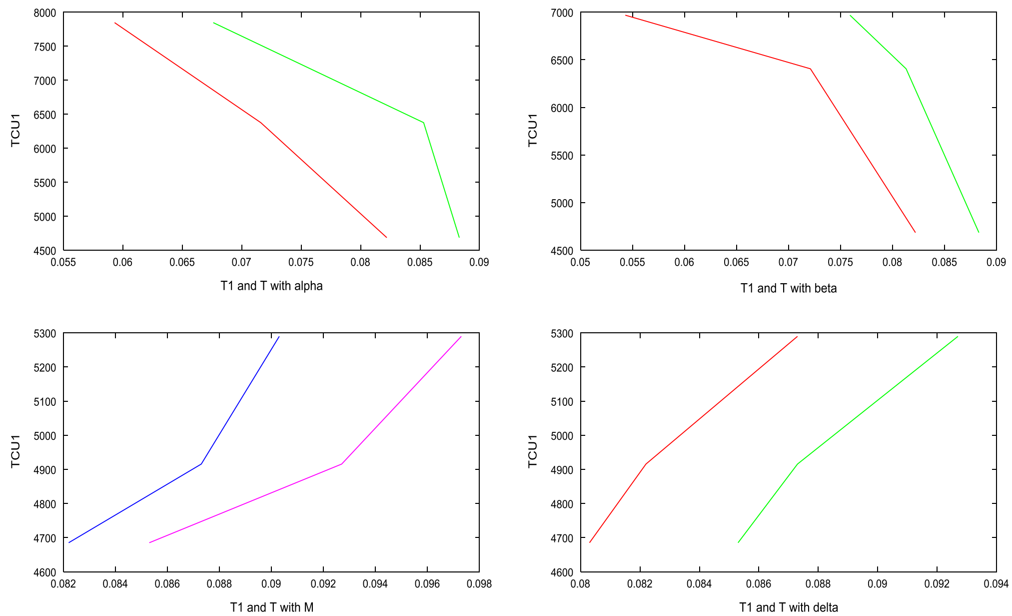
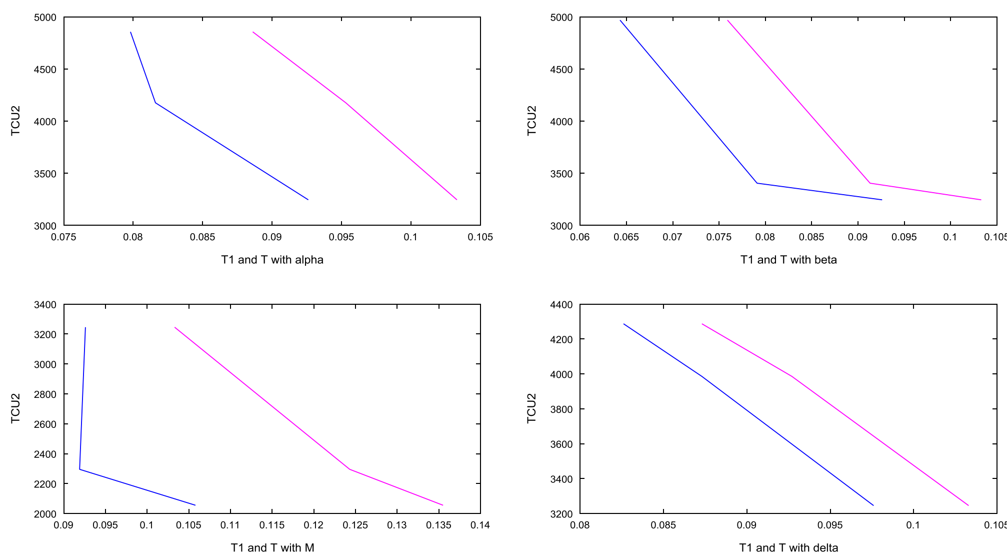


Figure 4. Effect of parameters α, β, M, and δ on total cost after the deflation.



In Tables 1 and 2 the parameters α, β have a bigger influence than the parameters M and δ. Therefore, the parameters α, β are necessitate privileged compassion toward the cycle time as well as total average inventory cost.

6. Conclusion

In the present article, we have designed a model has been illustrated for determination of optimal ordering time and total cost with stock dependent demand for deteriorating items following the Weibull distribution. Two cases specifically (i) payment before depletion and (ii) payment after depletion have been taken into account for the consideration of the model which can assist the decision-maker to find the optimal cycle time to minimize the total average inventory cost. From the sensitivity analysis, it is observed that as the rate of deterioration (α and β) and backlogging rate δ increases, the total average inventory cost increases, which is obvious. Moreover it is also observed that as permissible delay increases, the total average inventory cost decreases i.e. there is an opposite relation between permissible delay period and the total average inventory cost. The sensitive motive behind this is that the conservatory in permissible delay period offers an opportunity to the consumer to earn more by investing the resource otherwise from the sale proceeds of the inventory which result in lower cost.

A further study would be to extend the purposed model for finite replenishment rate, price dependent demand, fuzzy demand, variable lead time, and many more.

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