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MATERIALS ENGINEERING | RESEARCH ARTICLE

Centrifugal separation experimentation and optimum predictive model development for copper recovery from waste copper smelter dust

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Abstract: This research has presented a three level-two factors full factorial experimental design that investigated the process parameterization of a centrifugal concentrator for the separation of a waste copper smelter dust (CSD). This was followed by a theoretical contribution involving the development of a scheme of predictive models premised on the concept of constrained interpolant models. These were used for the experimental trend, pattern investigation and furthermore to provide expressions that depicts optimal experimental conditions in this research. Based on the experimental outputs, it was observed that a maximum grade of about 35.02 wt% Cu was achieved at a Rotational Bowl Speed of 120G, Water Flow Rate of 3.0l/min and constant experimental flow rate of 1.48l/min with a Liquid to Solid Ratio of 0.5. Similarly, a minimum output of 14.58% SiO₂ and 10.29% Al₂O₃ was achieved at same experimental conditions. This clearly depicts a trend geared towards optimum experimental conditions aimed at maximizing Cu output and



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Mr. Daniel Okanigbe is a trained geoscientist with a bachelor's degree from the department of applied geology and geophysics, Obafemi Awolowo University, Ile-ife, Osun State, Nigeria. He obtained a Master's degree in metallurgy from University of Lagos, Akoka, Nigeria. He is about completing his doctoral study in the field of Engineering from the department of Chemical, Metallurgical and Materials Engineering, Faculty of Engineering and the Built Environment, Tshwane University of Technology, Pretoria, Republic of South Africa. His area of research interest stretches from igneous and metamorphic petrology, applied exploration and economic geology, structural geology, geological mapping, geochemistry, geochronology, hydrometallurgy, pyrometallurgy, mineral processing, nanotechnology, flowsheet design, with particular emphasis on waste management and treatment. At the moment, he is working on a project aimed at producing a novel process flowsheet for the recovery of copper from a waste CSD, a metallurgical waste material from primary copper ore processing.

PUBLIC INTEREST STATEMENT

The research was divided into two phases viz: the experimental and modeling phases. The experimentation which preceded the modeling phase was carried out via a three level-two factors experimental design. This investigated the process parameterization of a centrifugal concentrator for the separation of copper smelter dust (CSD). Following, was the development of a predictive model that aimed at recovering the experimental data pattern. The modeling is premised on developed constrained interpolation models. This was able to generate the optimal experimental conditions for this research and a trend capable of predicting future experimental output based on the set conditions. An additional experimental trend recovery was carried out using a tenth-order polynomial curve fitting in Matlab software. Both the constrained interpolation modeling and polynomial curve fitting produced a good output. However, the level of recovery accuracy was higher with the constrained interpolant modeling.

minimizing SiO₂ and Al₂O₃ impurities. The predicted outputs premised on the use of Matlab software are in good conformance with the experimental outputs with a high degree of accuracy and confidence level over 97% as shown in Table 1 and corresponding Figures.

Subjects: Applied & Economic Geology; Resource Management - Environmental Studies; Environment & Resources; Metallurgical Engineering

Keywords: Waste copper smelter dust; mathematical modeling; optimization; density separation; classification

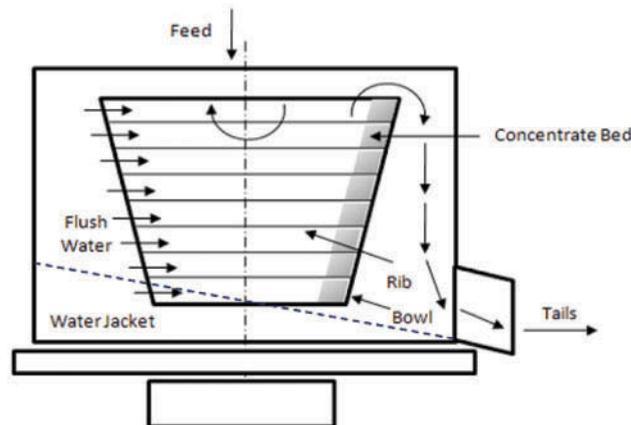
1. Introduction

In the last few decades, the literature of extractive metallurgy has extended its research needs to proffering alternative and sustainable solutions for the readily availability of metallic resources. This measure has become very necessary for sustainability of the earth amongst other deteriorating environmental factors. One promising alternative measure for metallic extraction other than exploring the earth surface is the approach of scrap materials smelting. The smelting of abandoned scrap materials for the purpose of harnessing specific metals does not only aid in sustainability of the earth but controls and minimizes environmental pollution posed through non-biodegradable solid wastes in form of metallic solid wastes and powdery materials amongst others. This research is focused at providing both experimental and mathematical modeling solutions capable of exploring the effectiveness of a proposed experimental technique. A predictive modelling scheme premised on the use of constraint interpolants was used to model the percentage output proportion from the extraction and refinement of copper concentrates via smelting of scrap metallic copper concentrate also referred to as waste CSD. The process is not without its own challenges characterized usually with waste generation. The waste CSD contains a substantial amount of copper in close association with environmentally toxic compounds such as Arsenic, Bismuth, Lead, Antimony and Cadmium capable of affecting the purity of the refined copper. The generation of waste CSD is almost becoming a global menace requiring that stringent environmental regulations be put in place to inhibit its generation (Montenegro, Sano, & Fujisawa, 2008). However, the waste CSD if well harnessed and managed can be a sustainable secondary source of copper. Hence, the need for an appropriate technology becomes inevitable, principally from the stand point of mineral conservation, utilization of scanty copper resource and its sustainability. Mineral processing techniques can be employed as a pretreatment method to reduce the amount of contaminants in the waste CSD before subjecting the produced concentrate to hydrometallurgical treatments (Geldenhuis, 2002). Additionally, most of the heavy minerals are treated in gravity concentration at different stages of upgradation (Demi, Koci, & Boci, 2006; Meloy, Williams, Bevilacqua, & Ferrara, 1994).

Furthermore, gravity concentration is considered advantageous because of its simplicity, low operating cost and ease of operation. However, the average size of this waste CSD falls within the size range of 5–50 µm; (Ha, Kwon, Park, & Mohapatra, 2015; Morales, Cruells, Roca, & Bergó, 2010; Okanigbe, Popoola, & Adeleke, 2017a, 2017b), a size range only the modern gravity techniques e.g. Knelson (Figure 1) and Falcon concentrators have proven useful in separation of materials into different fractions (Wills & Finch, 2015). As expected in mineral processing a rigorous and costly experimental evaluation of laboratory and pilot-scale equipment trials are carried out to ascertain the performance of any unit operation. Consequently, designing a low cost, time saving tool (model), with the capacity to correctly predict the separation results of the unit operation would be advantageous.

The success of separation with centrifugal concentrator is a function of the selection of appropriate process variables at which the response attains its optimum. One of the approaches for achieving the optimum results is the Design of Experiments (DOE) using the full Factorial Methodology (FFM). Several experimental designs can be used for different objectives, just as the

Figure 1. Cut away schematic of a Knelson Centrifugal Concentrator (Rahil Khan, 2016).



randomized block designs can be used for screening relevant factors (Aslan, 2008; Xiao & Vien, 2004). Some reports exist in literature of modeling methods been used in mineral and coal processing operations (Akar Sen, 2016). Optimization and modeling studies of hydrocyclone for treating Indian iron ore slime is an example (Mohanty & Das, 2010) as well as low-grade bentonite (Özgen, Yıldız, Çalışkan, & Sabah, 2009). Response surface methodology has also been used to assess the performance of the froth floatation on coal fines (Kalyani, Pallavika, Gouri Charan, & Chaudhuri, 2005). A Second order quadratic model was created by using the response surface methodology for separation of titanium bearing minerals from Indian beach sand minerals (Gunaraj & Murugan, 1999). In this investigation, the experiments were carried out as per the full factorial design, with an objective to develop the empirical models for predicting the maximum percentage grade in concentrate fraction of the centrifugal concentrator while treating waste CSD. The optimum conditions were obtained for achieving maximum grade and recovery of the concentrate fraction. The effects of different operating parameters Fluidized water flow rate (FWFR), Rotational bowl speed (RBS) on percentage grade of copper and its closely associated contaminants in concentrate fraction while treating waste CSD are presented.

2. Experimental

2.1. Materials

2.1.1. Ore handling and sampling

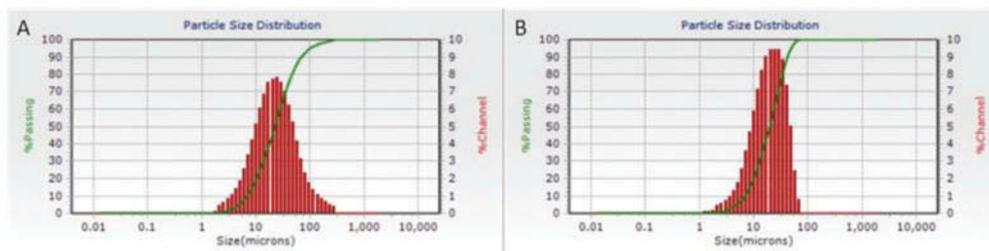
Precisely 120 kg of waste CSD was received from Palabora Copper (PTY) Ltd, Limpopo, Republic of South Africa for the purpose of this study. The homogenization of the as-received waste CSD sample was carried out first by re-weighing it before subjecting it to coning and quartering method. The homogenized waste CSD was separate into two categories—undersized and oversized particles with a 300 micron sieve. The oversized particles were afterwards milled and sieved with the same 300 micron sieve and the produced undersized particles were mixed with the previously produced undersized waste CSD (segment diameter $d_{97} = 300$ micron). After the milling and sieving exercise, the amount of particles with diameter less than 300 micron increased from 90.82% of total number of particles in the sample to 95.59%. This change is shown by the distribution curve before (Figure 2(a)) and after sample preparation (Figure 2(b)). After milling, the distribution curve (Figure 2(b)) became skewed to the left making the distribution asymmetric.

2.2. Methods

2.2.1. Design of experiment (DOE) for centrifugal separation experiment

The DOE is based on principles of experimental design, mathematical equations or models and outcomes of the factors. In the present study, the FFM was used to determine the interaction

Figure 2. Particle size distribution: A) As-received waste CSD B) milled waste CSD (Okanigbe, Popoola, Adeleke, & Popoola, 2018).



between the response functions of maximum copper content, minimum contaminant content (quartz and mullite) and the two operating variables (FWFR and RBS). The FFM contains all likely combinations of the operating variables (factors). The effect of all factors and interaction effects on the responses are investigated methodically. The centrifugal separation experiments were conducted using a laboratory scale Knelson gravity concentrator (KGC) with model number KC-MD3, at the Gravity Concentrators Africa (GCA), Republic of South Africa. The variables together with the levels considered and the DOE for the test program are shown in Tables 2 and 3.

2.2.2. Modeling

2.2.2.1. *Modeling procedure for output prediction.* Software MATLAB 7.1 was used to perform data processing according to full factorial design procedure.

Step #1: Study trend of experimental samples

Step #2: Set-up constraint models to categorize and group samples into sub-classes based on #1.

Step #3: Compute absolute difference between input and output samples in same class as grouped in #2.

Step #4: identify different experimental levels for selected classes.

Step #5: Apply interpolant model

Step #6: End

Table 4 is a generalized representation of the interpolant model variables. Herein, the assumed unknown output is for row one hence ϖ_1 is first computed. The Predicted output model O_{o_1} is trend driven and constraint based as presented in the following sub-section. Table 4 can however be adapted to compute ϖ_2 and ϖ_3 as the case may be.

Given: Output =

$$\text{Output} = f(\text{speed, flow rate, input, feed rate, liquid solid ratio})$$

Let: serial number for inputs: $s_i = \{1, \dots, n - 1, n\}$ and serial number for outputs $s_o = \{1, \dots, n - 1, n\}$ for $\forall n \in R$

Where:

$$ts_{(i)ij} = \text{Total available input samples}$$

$$ts_{(o)ij} = \text{Total available output samples}$$

$$\text{exp}_{(i)ij} = \text{Experimental inputs}$$

$\text{exp}_{(o)ij} = \text{Experimental outputs}$

$\text{Pre}_{(o)ij} = \text{Predictive outputs}$

$I_{ij} = \% \text{ input proportion of selected samples}$

$O_j = \% \text{ output proportion of selected samples}$

$\Delta IO_{ij} = |I_{ij} - O_j|$ absolute difference between I_{ij} and O_j

Where:

$j = \{1, \dots, k - 1, k\}$

representing experimental levels

Given the following absolute differences between I_{ij} and O_j

$$|I_{i_1} - O_{o_1}| = \varpi_1$$

$$|I_{i_2} - O_{o_2}| = \varpi_2$$

$$|I_{i_3} - O_{o_3}| = \varpi_3$$

Where, p_{o_j} (output) is constraint specific.

“Absolute difference” model with respect to Table 4 is given as:

$$\varpi_1 = \left[\frac{\varpi_3(\phi_2 - \phi_1) - \varpi_2(\phi_2 - \phi_1) - \varpi_2(\phi_3 - \phi_2)}{(\phi_2 - \phi_3)} \right] \tag{1}$$

$$\varpi_2 = \left[\frac{\varpi_3(\phi_2 - \phi_1) + \varpi_1(\phi_3 - \phi_2)}{(\phi_3 - \phi_2) + (\phi_2 - \phi_1)} \right] \tag{2}$$

$$\varpi_3 = \left[\frac{\varpi_1(\phi_2 - \phi_3) + \varpi_2(\phi_3 - \phi_2) + \varpi_2(\phi_2 - \phi_1)}{(\phi_2 - \phi_1)} \right] \tag{3}$$

2.2.2.2. Different experimental conditions and constraints.

i. Modeling constraints for [CuO] prediction

Given: $LSR = 0.5$; $FR = 1.48$ constants for all experiments

$$\text{Predictive Output : } O_{1,1} = I_{1,1} + \Delta IO_{1,1}$$

$$O_{1,2} = I_{1,2} + \Delta IO_{1,2}$$

$$O_{1,3} = I_{1,3} + \Delta IO_{1,3}$$

$$\text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) = \begin{cases} s : 1 \rightarrow s \rightarrow 3 \\ i : 17.22 \rightarrow \text{CuO} \rightarrow 16.61 \\ o : 25.17 \rightarrow \text{CuO} \rightarrow 18.43 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 60 \end{cases} \tag{4}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{2,1} &= I_{2,1} + \Delta IO_{2,1} \\
 O_{2,2} &= I_{2,2} + \Delta IO_{2,2} \\
 O_{2,3} &= I_{2,3} + \Delta IO_{2,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 4 \leq s \leq 6 \\ i : 16.42 \rightarrow \text{CuO} \rightarrow 16.05 \\ o : 30.08 \rightarrow \text{CuO} \rightarrow 23.74 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 90 \end{cases} \quad (5)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{3,1} &= I_{3,1} + \Delta IO_{3,1} \\
 O_{3,2} &= I_{3,2} + \Delta IO_{3,2} \\
 O_{3,3} &= I_{3,3} + \Delta IO_{3,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 7 \leq s \leq 9 \\ i : 16.56 \rightarrow \text{CuO} \rightarrow 15.99 \\ o : 35.02 \rightarrow \text{CuO} \rightarrow 24.03 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 120 \end{cases} \quad (6)
 \end{aligned}$$

ii. Modeling constraints for [Fe₂O₃] prediction

$$\begin{aligned}
 \text{Predictive Output : } O_{1,1} &= I_{1,1} + \Delta IO_{1,1} \\
 O_{1,2} &= I_{1,2} + \Delta IO_{1,2} \\
 O_{1,3} &= I_{1,3} + \Delta IO_{1,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 1 \leq s \leq 3 \\ i : 11.26 \rightarrow \text{Fe}_2\text{O}_3 \rightarrow 10.63 \\ o : 21.90 \rightarrow \text{Fe}_2\text{O}_3 \rightarrow 15.75 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 60 \end{cases} \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{3,1} &= I_{3,1} + \Delta IO_{3,1} \\
 O_{3,2} &= I_{3,2} + \Delta IO_{3,2} \\
 O_{3,3} &= I_{3,3} + \Delta IO_{3,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 7 \leq s \leq 9 \\ i : 11.36 \rightarrow \text{Fe}_2\text{O}_3 \rightarrow 11.03 \\ o : 26.18 \rightarrow \text{Fe}_2\text{O}_3 \rightarrow 21.49 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 120 \end{cases} \quad (8)
 \end{aligned}$$

iii. Modelling constraints for [SiO₂] prediction

$$\begin{aligned}
 \text{Predictive Output : } O_{1,1} &= I_{1,1} - \Delta IO_{1,1} \\
 O_{1,2} &= I_{1,2} - \Delta IO_{1,2} \\
 O_{1,3} &= I_{1,3} - \Delta IO_{1,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 1 \leq s \leq 3 \\ i : 35.34 \rightarrow \text{SiO}_2 \rightarrow 36.29 \\ o : 22.97 \rightarrow \text{SiO}_2 \rightarrow 31.36 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 60 \end{cases} \quad (9)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{2,1} &= I_{2,1} - \Delta IO_{2,1} \\
 O_{2,2} &= I_{2,2} - \Delta IO_{2,2} \\
 O_{2,3} &= I_{2,3} - \Delta IO_{2,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 4 \leq s \leq 6 \\ i : 35.3 \rightarrow S_i O_2 \rightarrow 35.69 \\ o : 18.10 \rightarrow S_i O_2 \rightarrow 24.93 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 90 \end{cases} \quad (10)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{3,1} &= I_{3,1} - \Delta IO_{3,1} \\
 O_{3,2} &= I_{3,2} - \Delta IO_{3,2} \\
 O_{3,3} &= I_{3,3} - \Delta IO_{3,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 7 \leq s \leq 9 \\ i : 35.4 \rightarrow S_i O_2 \rightarrow 35.85 \\ o : 14.58 \rightarrow S_i O_2 \rightarrow 24.14 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 120 \end{cases} \quad (11)
 \end{aligned}$$

iv. Modelling constraints for $[Al_2O_3]$ prediction

$$\begin{aligned}
 \text{Predictive Output : } O_{1,1} &= I_{1,1} - \Delta IO_{1,1} \\
 O_{1,2} &= I_{1,2} - \Delta IO_{1,2} \\
 O_{1,3} &= I_{1,3} - \Delta IO_{1,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 1 \leq s \leq 3 \\ i : 27.45 \rightarrow Al_2O_3 \rightarrow 28.38 \\ o : 18.08 \rightarrow Al_2O_3 \rightarrow 24.67 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 60 \end{cases} \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{2,1} &= I_{2,1} - \Delta IO_{2,1} \\
 O_{2,2} &= I_{2,2} - \Delta IO_{2,2} \\
 O_{2,3} &= I_{2,3} - \Delta IO_{2,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 4 \leq s \leq 6 \\ i : 27.64 \rightarrow Al_2O_3 \rightarrow 27.96 \\ o : 13.98 \rightarrow Al_2O_3 \rightarrow 19.32 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 90 \end{cases} \quad (13)
 \end{aligned}$$

$$\begin{aligned}
 \text{Predictive Output : } O_{3,1} &= I_{3,1} - \Delta IO_{3,1} \\
 O_{3,2} &= I_{3,2} - \Delta IO_{3,2} \\
 O_{3,3} &= I_{3,3} - \Delta IO_{3,3} \\
 \text{Output} = f(\text{FWFR}, \text{RBS}, \text{LSR}, \text{FR}) &= \begin{cases} s : 7 \leq s \leq 9 \\ i : 27.59 \rightarrow Al_2O_3 \rightarrow 28.04 \\ o : 10.29 \rightarrow Al_2O_3 \rightarrow 18.93 \\ 3 \leq \text{FWFR} \leq 6 \\ \text{RBS} : 120 \end{cases} \quad (14)
 \end{aligned}$$

Expressions (4) to (14) represent the different constraint interpolant models proposed for this research. The constraint models are experiment specific with a corresponding positive or negative incremental difference between the input and output experimental proportion.

A complimentary curve fitting model of 10th order polynomial was also presented as shown in Equations (15)–(19). The 10th order polynomial curve fitting was adjudged the best fitment for this experiment amongst the different fitment option. While (15) represents a generalized form of the polynomial functions, (16)–(19) are specific functions with-respect-to the different concentrates in the mix.

$$y = P_1x^{10} + P_2x^9 + P_3x^8 + P_4x^7 + P_5x^6 + P_6x^5 + P_7x^4 + P_8x^3 + P_9x^2 + P_{10}x + P_{11} \pm e \quad (15)$$

CuO (w%)

$$y = -1.1 * 10^{-7}x^{10} + 10^{-5}x^9 - 0.00039x^8 + 0.0081x^7 - 0.1x^6 + 0.75x^5 - 3.3x^4 + 8.1x^3 - 9.8x^2 + 1.9x + 28 \pm e \quad (16)$$

Fe₂O₃ (w%)

$$y = -3.1 * 10^{-8}x^{10} + 2.5 * 10^{-6}x^9 - 7.4 * 10^{-5}x^8 + 0.00092x^7 + 0.00058x^6 - 0.15x^5 + 1.8x^4 - 9.5x^3 + 26x^2 - 35x + 39 \pm e \quad (17)$$

SiO₂ (w%)

$$y = 10^{-7}x^{10} - 8.9 * 10^{-6}x^9 + 0.00033x^8 - 0.0065x^7 + 0.073x^6 - 0.45x^5 + 1.3x^4 + 0.12x^3 - 9.4x^2 + 21x + 10 \pm e \quad (18)$$

Al₂O₃ (w%)

$$y = 9.610^{-8}x^{10} - 8.7 * 10^{-6}x^9 + 0.00033x^8 - 0.0068x^7 + 0.081x^6 - 0.58x^5 + 2.3x^4 - 4.5x^3 + 2.3x^2 + 5.9x + 13 \pm e \quad (19)$$

where $e = \text{error_factor}$

2.2.2.3. Optimum experimental trend and condition. This section presents both the optimum experimental condition and optimum experimental trend represented in a generalized mathematical expression. The optimum and most viable experimental condition is directly linked to the maximization of the reduction process of dominant SiO₂ and Al₂O₃ (Silica and Alumina) impurity in the resultant concentrates while also maximizing the recovery of CuO and Fe₂O₃ (Copper oxide and Iron oxide) in the same concentrates. Having a prior knowledge of optimum conditions and optimum experimental trend can be of tremendous benefit in setting up a future design of experiment for a similar behaved waste CSD. The optimum experimental condition for this research takes place when the RBS increases with extremely low values for the FWFR as shown in Table 1 and Figures 4–6. The optimum experimental condition was recorded for a RBS of 120G and FWFR of 3.0l/min. This experimental condition resulted in a maximum output of CuO and Fe₂O₃ at respective values of 35.02 wt% and 26.18 wt%. The corresponding global minimum outputs for SiO₂ and Al₂O₃ are respectively 14.58 wt% and 10.29 wt%. The first expression in (20) is a trend indicator which shows that increasing the value of RBS along the positive Cartesian plane combined with a decreased value of FWFR along the positive Cartesian plane would result in a maximum recovery of Fe₂O₃ and CuO and minimum recovery of SiO₂ and Al₂O₃. The second expression indicates that a decrement of RBS along the negative Cartesian plane coupled with an increment of FWFR along the negative Cartesian plane would also result in a maximum recovery of CuO and Fe₂O₃ and a minimum recovery of both SiO₂ and Al₂O₃ impurities. Similarly, expression number three of (20) indicates that an increment in RBS combined with a corresponding increase of FWFR along a negative Cartesian plane would result in a maximum recovery of the desired CuO and Fe₂O₃ and a minimum recovery of the contaminants SiO₂ and Al₂O₃. Finally, the fourth expression of (20) indicates that a decrease of RBS along the negative Cartesian plane coupled with a decrease of FWFR along the positive Cartesian plane would result in a maximum recovery of Fe₂O₃ and CuO concentrates and a minimum recovery of the undesired SiO₂ and Al₂O₃ concentrates.

Table 1. Experimental and predictive outputs for feed compositions

S/N	Chemical Composition (Feed)					FWFR (l/min)	RBS (G)	Experimental Output					Chemical Composition (Concentrate)				
	Experimental Input							Experimental Output					Predicted Output				
	CuO (wt %)	Fe ₂ O ₃ (wt%)	SiO ₂ (wt%)	Al ₂ O ₃ (wt%)	Al ₂ O ₃ (wt%)			CuO (wt%)	Fe ₂ O ₃ (wt%)	SiO ₂ (wt%)	Al ₂ O ₃ (wt%)	Al ₂ O ₃ (wt%)	CuO (wt %)	Fe ₂ O ₃ (wt%)	SiO ₂ (wt%)	Al ₂ O ₃ (wt%)	
1	17.22	11.26	35.34	27.45	27.45	3.0	60	25.17	21.90	22.97	18.08	25.17	21.9	22.97	18.08		
A						3.75	60	No exp.	No exp.	No exp.	No exp.	23.13	20.28	25.47	20.04		
2	15.86	11.00	35.94	28.15	28.15	4.5	60	21.09	18.66	27.97	22.00	21.09	18.66	27.97	22.00		
B						5.25	60	No exp.	No exp.	No exp.	No exp.	19.76	17.21	29.67	23.34		
3	16.61	10.63	36.29	28.38	28.38	6.0	60	18.43	15.75	31.36	24.67	18.43	15.76	31.37	24.68		
C						5.25	60	No exp.	No exp.	No exp.	No exp.	19.76	17.21	29.67	23.34		
D						4.5	60	No exp.	No exp.	No exp.	No exp.	21.09	18.66	27.98	22.01		
4	16.42	11.23	35.30	27.64	27.64	3.0	90	30.08	24.41	18.10	13.98	30.08	24.42	18.10	13.98		
E						3.75	90	No exp.	No exp.	No exp.	No exp.	28.81	23.78	19.48	15.04		
5	16.68	11.79	34.95	27.40	27.40	4.5	90	27.54	23.14	20.86	16.09	27.54	23.14	20.86	16.10		
F						5.25	90	No exp.	No exp.	No exp.	No exp.	26.27	21.99	22.90	17.71		
6	16.05	11.21	35.69	27.96	27.96	6.0	90	23.74	20.83	24.93	19.32	25	20.84	24.94	19.32		
G						5.25	90	No exp.	No exp.	No exp.	No exp.	26.27	21.99	22.9	17.71		
H						4.5	90	No exp.	No exp.	No exp.	No exp.	27.54	23.14	20.87	16.1		
7	16.56	11.36	35.34	27.59	27.59	3.0	120	35.02	26.18	14.58	10.29	35.02	26.18	14.58	10.29		
I						3.75	120	No exp.	No exp.	No exp.	No exp.	32.07	25.13	16.92	12.55		
8	16.50	11.49	34.74	27.63	27.63	4.5	120	29.12	24.08	19.25	14.81	29.12	24.08	19.26	14.81		
J						5.25	120	No exp.	No exp.	No exp.	No exp.	26.58	22.79	21.70	16.87		
9	15.99	11.03	35.85	28.04	28.04	6.0	120	24.03	21.49	24.14	18.93	24.04	21.5	24.14	18.93		

Key: RBS = Rotational Bowl Speed; WFR = Water Flow Rate; FR = Flow rate = 1.48(l/min)(constant); LSR = Liquid to Solid Ratio = 0.5 (constant); No exp. = No experiment was conducted

Table 2. Parameters considered at three levels for the KGC

S/N	Variable (factor)	Low (0)	Medium (1)	High (2)
X1	Rotational bowl speed (G)	60	90	120
X2	water flow rate (l/min)	3.0	4.5	6.0

Table 3. DOE for centrifugal separation of the waste CSD using the 3₂ full factorial design

Tests	Rotational bowl speed (G)	Water Flow rate (l/min)	Treatment Combination
1	0	0	00
2	0	1	01
3	0	2	02
4	1	0	10
5	1	1	11
6	1	2	12
7	2	0	20
8	2	1	21
9	2	2	22

Key: 0 = low Water Flow rate (3.0l/min) or Rotational bowl speed (60G); 1 = Water Flow rate (4.5l/min) or Rotational bowl speed (90G); 2 = Water Flow rate (6.0l/min) or Rotational bowl speed (120G)

Table 4. Generalized representation of model variables

Level	Data Acquisition Procedure	Input value for variant factor I_{ij}	Output values for variant factor O_{oj}	Expt. Levels ϕ_1	Absolute difference between I_{ij} and O_{oj}
1-First	Prediction	I_{i1}	O_{o1}	ϕ_1	$ I_{i1} - O_{o1} = \varpi_1$
2-second	Experiment	I_{i2}	O_{o2}	ϕ_2	$ I_{i2} - O_{o2} = \varpi_2$
3-Third	Experiment	I_{i3}	O_{o3}	ϕ_3	$ I_{i3} - O_{o3} = \varpi_3$

$$\begin{aligned}
 &[(\uparrow +RBS) \cap (\downarrow +FWFR)] \\
 &\quad \text{or} \\
 &[(\downarrow -RBS) \cap (\uparrow -FWFR)] \\
 &\quad \text{or} \\
 &[(\uparrow +RBS) \cap (\uparrow -FWFR)] \\
 &\quad \text{or} \\
 &[(\downarrow -RBS) \cap (\downarrow +FWFR)]
 \end{aligned} \tag{20}$$

Where \uparrow = increment in the positive(+ve) or negative(-ve) direction
 \downarrow = decrement in the positive(+ve) or negative(-ve) direction

3. Results and discussion

This section presents the simulation results of this research in two folds viz: experimentation and predictive outputs Firstly, considering the experimental procedure, it should be noted that the experiments were carried out using the KGC to separate the copper minerals from the closely associated impurities in the waste CSD. The experimental outputs of the beneficiation process alongside the respective operating conditions are presented in Table 4. The predictive outputs presented in Table 1 were obtained using an interpolant interval of 0.75units within the FWFR levels. At this interval, the entire experimental outputs as presented in Table 1 were adequately recovered for the different input feed concentrates viz: CuO, Fe₂O₃, SiO₂, and Al₂O₃. The maximum error obtained between the predicted and experimental outputs is 1.86units. This is contained in

serial number 6 of Table 1 and under the CuO column. The experimental output herein is 23.74 wt% while the predicted is 25wt%. The predicted intervals without experimentations for varying levels of RBS and FWFR are with a high degree of accuracy ranging between 95 and 99% based on the extremely minimal discrepancy between the experimented and predicted outputs. In addition, Figures 3 through 6 present a graphical representation of specific predicted outputs and corresponding FWFR levels.

The graphical trend for CuO as contained in Figure 3 presents the lowest predictive value of 18.43 wt% when FWFR is 6.0 l/min with a RBS value of 60G. The highest predictive value of 35.02 wt% was obtained as RBS increased to 120G and for a corresponding lowest value of 3.0l/min for FWFR. In Figure 4, results for Fe₂O₃ are presented with an observed output trend similar to that of Figure 3. The least predictive value of 15.76 wt% was obtained for RBS of 60G and FWFR of 6.0 l/min. A higher value was obtained for CuO over Fe₂O₃ as the RBS increased to a peak value of 120G perhaps due to a much higher concentration criterion (CC) between CuO

Figure 3. Experimental and Predictive Output for CuO.

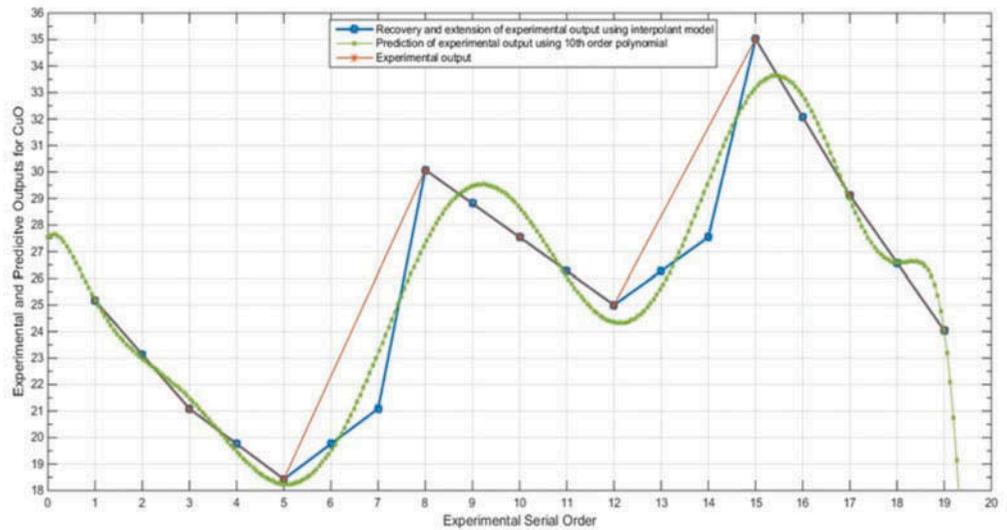
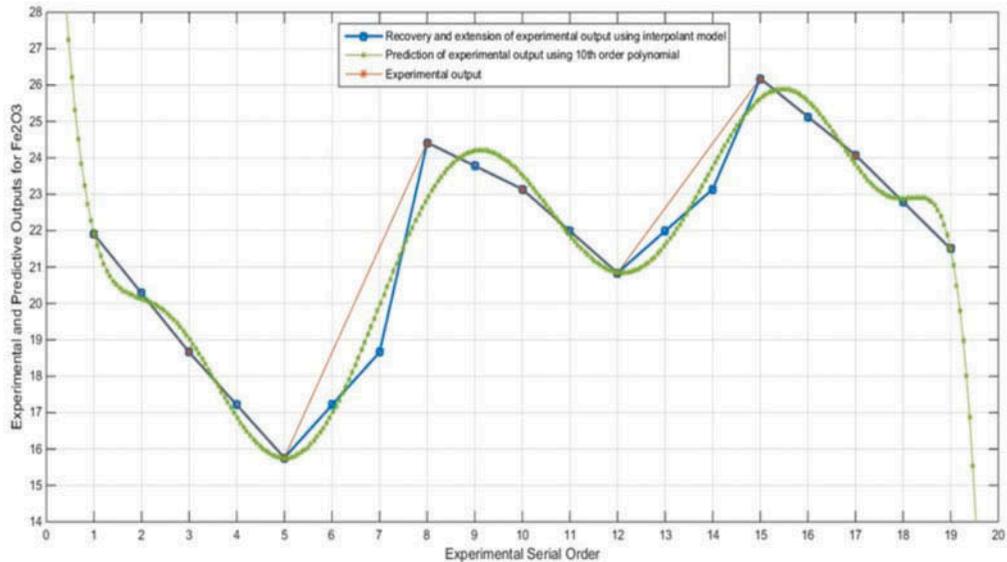


Figure 4. Experimental and Predictive Output for Fe₂O₃.



(specific gravity (S.G.) of 6.50) and the impurities-SiO₂- (specific gravity of 2.13) and Al₂O₃ -(specific gravity of 3.4) than for the CC between Fe₂O₃ (specific gravity of 4.2) and the same impurities in the waste CSD amongst other factors. As mentioned earlier, the two major impurities present in the experimental waste CSD includes: SiO₂ and Al₂O₃. Unlike the behavioral pattern presented by outputs of CuO and Fe₂O₃ compounds, SiO₂ and Al₂O₃ compounds as respectively shown in Figures 5 and 6 both have peak predictive outputs of 31.37 wt% and 24.68 wt% at the least RBS of 60G and a corresponding FWFR value of 6.0 l/min. Similarly, the least predictive output values for SiO₂ and Al₂O₃ were obtained while RBS was high at 120G at a corresponding low FWFR value of 3l/min. The accuracy of the experimentation process is reflected in the output data. Also, the impact of the predicted data in this research firstly, lies in the area of experimental cost savings by avoiding set-up costs, resource acquisition cost, cost of safety adherence and accidents amongst others. Furthermore is the facilitation of an efficient and effective long and short-term planning process prior to the conduct of an experiment. In addition, on comparing the predicted data (premised on the constrained interpolant

Figure 5. Experimental and Predictive Output for SiO₂.

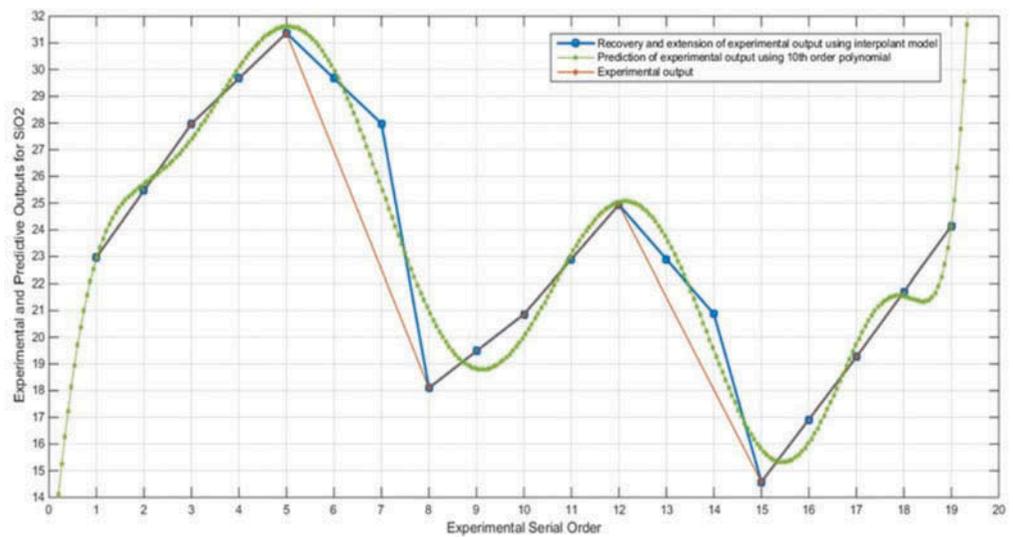
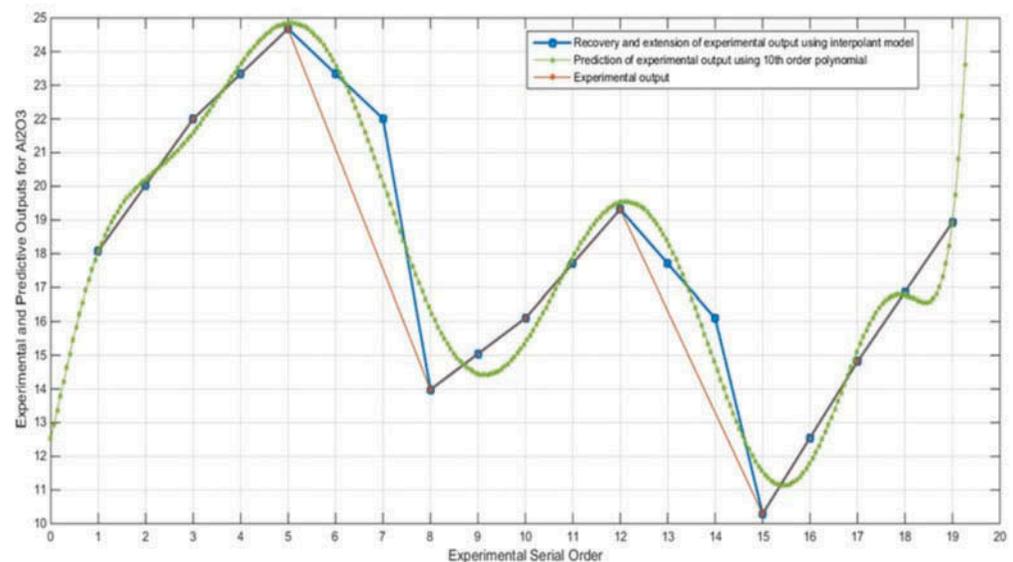


Figure 6. Experimental and Predictive Output for Al₂O₃.



model) with the experimental data, a high level of statistical correlation was recorded. While CuO had a least square correlation coefficient of 0.992, Fe₂O₃, SiO₂ and Al₂O₃, respectively, had correlation coefficients of 1.000.

4. Conclusion

On a generalized note, this research has effectively demonstrated some competence by showing how the interactive effects of statistical attributes such as factorial design, optimum conditions etc., related to an experiment can be applied on experimental data for the accurate prediction of computational data. This can be linked to the work presented by Fagade, Tande, Cho, Seames, Sakodinskaya, Muggli, & Kozliak (2013) which stated that statistical study-specific terms such as “effect,” “factorial design,” “interaction effect,” “factors,” “optimum condition” etc. are not common terminologies for general readers. Furthermore, this research has presented the experimental classification and mathematical prediction of Copper mineral in close association with contaminants in the waste CSD using the Knelson centrifugal gravity concentrator. The classification and predictive efforts in this research were achieved by setting up optimum combinations of the process parameters for the centrifugal concentrator. The four process parameters considered in this study include: FWFR, RBS, Flow rate of 1.48l/min and a Liquid to Solid Ratio of 0.5. The model proposed in this research was developed to maximize Cu output and minimize the output of SiO₂ and Al₂O₃ impurities. The following deductions were reached in this research:

- KGC separated the waste CSD's particles into light and heavy minerals with maximum CuO recovery and minimum SiO₂ and Al₂O₃ recoveries in the resultant concentrates.
- Optimum separation conditions (giving highest CuO content and reduced amount of contaminants in concentrate) were: FWFR 3.0 l/min. and RBS 120G.
- Maximum grade of copper was achieved under Test 7 (As-received waste CSD –16.56 wt%, concentrate –35.02 wt%) compared to that obtained under Test 4 (As- received waste CSD –16.42wt%, concentrate –30.08 wt %), the closest to it.
- The same Test 7 produced a concentrate with the least quartz content (As- received waste CSD –35.34 wt%, concentrate –14.58wt %) and mullite content (As- received waste CSD –27.59 wt%, concentrate –10.29wt %) compared to that produced under Test 4 with content of quartz (As- received waste CSD –35.30wt%, concentrate –18.10wt %) and mullite (As-received waste CSD –27.64wt%, concentrate—13.98 wt%).
- The predicted values obtained using the models were in good agreement with the observed values with an error margin between 0.00 and 0.06%.
- Both variables considered i.e. FWFR and RBS had major influence on the experimentation process of the concentrates from the centrifugal concentrator.
- From the model, further outputs can be generated without any laboratory experimental effort.
- Future predictive work would present an automated algorithm capable of considering two or more experimental varying factors.

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References

- Akar Sen, G. (2016). Application of full factorial experimental design and response surface methodology for chromite beneficiation by Knelson concentrator. *Minerals*, 6(1), 5. doi:10.3390/min6010005
- Aslan, N. (2008). Application of response surface methodology and central composite rotatable design for modeling and optimization of a multi-gravity separator for chromite concentration. *Powder Technology*, 185(1), 80–86. doi:10.1016/j.powtec.2007.10.002
- Demi, G., Koci, B., & Boci, S. (2006). *Gravity concentration of poor chromium ores in Albania*. In XXIII International Mineral Processing Congress, Istanbul, Turkey (pp. 310–312).
- Fegade, S. L., Tande, B. M., Cho, H., Seames, W. S., Sakodynskaya, I., Muggli, D. S., & Kozliak, E. I. (2013). Aromatization of propylene over Hzsm-5: A design of experiments (DOE) approach. *Chemical Engineering Communications*, 200(8), 1039–1056. doi:10.1080/00986445.2012.737385
- Geldenhuis, J. M. A. (2002). Recovery of valuables from flue dust fines. *Minerals Engineering*, 15(1–2), 95–98. doi:10.1016/S0892-6875(01)00197-2
- Gunaraj, V., & Murugan, N. (1999). Application of response surface methodology for predicting weld bead quality in submerged arc welding of pipes. *Journal of Materials Processing Technology*, 88(1–3), 266–275. doi:10.1016/S0924-0136(98)00405-1
- Ha, T. K., Kwon, B. H., Park, K. S., & Mohapatra, D. (2015). Selective leaching and recovery of bismuth as Bi₂O₃ from copper smelter converter dust. *Separation and Purification Technology*, 142, 116–122. doi:10.1016/j.seppur.2015.01.004
- Kalyani, V. K., Pallavika, Gouri Charan, T., & Chaudhuri, S. (2005). Optimization of a laboratory-scale froth flotation process using response surface methodology. *Coal Preparation*, 25(3), 141–153. doi:10.1080/07349340590962793
- Khan, R. (2016). Retrieved from <https://www.911metallurgist.com/blog/gold-extraction-recovery-processes/cut-away-schematic-of-a-knelson-centrifugal-concentrator>
- Meloy, T. P., Williams, M. C., Bevilacqua, P., & Ferrara, G. (1994). Shaking tables-Effects of riffles. *Transactions-Society For Mining Metallurgy And Exploration Incorporated*, 296, 1870–1877.
- Mohanty, S., & Das, B. (2010). Optimization studies of hydrocyclone for beneficiation of iron ore slimes. *Mineral Processing & Extractive Metallurgy Review*, 31(2), 86–96. doi:10.1080/08827500903397142
- Montenegro, V., Sano, H., & Fujisawa, T. (2008). Recirculation of Chilean copper smelting dust with high arsenic content to the smelting process. *Materials Transactions*, 49(9), 2112–2118. doi:10.2320/matertrans.M-MRA2008817
- Morales, A., Cruells, M., Roca, A., & Bergó, R. (2010). Treatment of copper flash smelter flue dusts for copper and zinc extraction and arsenic stabilization. *Hydrometallurgy*, 105(1–2), 148–154. doi:10.1016/j.hydromet.2010.09.001
- Okanigbe, D. O., Popoola, A. P. I., & Adeleke, A. A. (2017a). Characterization of copper smelter dust for copper recovery. *Procedia Manufacturing*, 7, 121–126. doi:10.1016/j.promfg.2016.12.032
- Okanigbe, D. O., Popoola, A. P. I., & Adeleke, A. A. (2017b). *Hydrometallurgical processing of copper smelter dust for copper recovery as nano-particles: A review*. In *Energy Technology 2017* (pp. 205–226). Cham: Springer.
- Okanigbe, D. O., Popoola, A. P. I., Adeleke, A. A., & Popoola, O. M. (2018, March). *Upgrading the copper value in a waste copper smelter dust with the falcon gravity concentrator*. In *TMS Annual Meeting & Exhibition* (pp. 283–295). Cham: Springer.
- Özgen, S., Yıldız, A., Çalışkan, A., & Sabah, E. (2009). Modeling and optimization of hydrocyclone processing of low grade bentonites. *Applied Clay Science*, 46(3), 305–313. doi:10.1016/j.clay.2009.08.029
- Wills, B. A., & Finch, J. (2015). *Wills' mineral processing technology: An introduction to the practical aspects of ore treatment and mineral recovery*. Oxford: Butterworth-Heinemann Elsevier Ltd.
- Xiao, Z., & Vien, A. (2004). Experimental designs for precise parameter estimation for non-linear models. *Minerals Engineering*, 17(3), 431–436. doi:10.1016/j.mineng.2003.11.018



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