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Ultimate strength and axial strain of FRP strengthened circular concrete columns

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Abstract: In this paper, a comprehensive study of CFRP and GFRP strengthened circular concrete columns is presented. Previous experimetal work is reviewed including the effect of different parameters affecting the strenght and behaviour of circular concrete columns confined with FRP and new equations are propsed which has more accurate prediction of the enhanced circular concrete column strength and behvaviour than previously proposed ones.

Subjects: Civil, Environmental and Geotechnical Engineering; Concrete & Cement; Structural Engineering

Keywords: strengthening; Fiber Reinforced Polymer (FRP); compressive strength; axial strain

1. Introduction

Strengthening of concrete columns is one of the most important tasks in civil engineering, as well as strengthening other elements and structures such as beams, bridges, municipal buildings, transportation systems, and parking. This might be required for a change in the structural system of the building such as removing of some load-bearing members or the damage caused by external factors or when the function of the building is changed from previously planned which could lead to an increase in the applied load and consequently the need to increase the column

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Shamil K. Ahmed has been working as an assistant lecturer at the Headquarter of University of Anbar, Iraq. His research interest includes producing of eco-friendly lightweight concrete using waste, strengthening reinforced concrete columns, the fire behaviour of the buildings which contains (PVC and Aluminum Composite) panels.

PUBLIC INTEREST STATEMENT

In this paper, a comprehensive study of CFRP and GFRP strengthened circular concrete columns is presented. Previous experimental work is reviewed including the effect of different parameters affecting the strength and behavior of circular concrete columns confined with FRP and new equations are proposed which has more accurate prediction of the enhanced circular concrete column strength and behavior than previously proposed ones.

For the ultimate compression strength, a new factor is taking into consideration, namely, the number of FRP layers (n). For the ultimate strain, the k_2 factor was modified to vary with the value of unconfined ultimate axial strain while it is usually take constant in other studies.

The results of the proposed equations were compared with the experimental results and it has been shown that the coefficient of variation for the proposed equations to be within "9–11.2" in comparison to previous equations were "12–64".

capacity. Conventional methods like concrete or steel jacketing were used for repair, strengthening, or providing a lateral confinement for the RC columns (Chai, Priestly, & Seible, 1991; Priestly, Seible, Xiao, & Verma, 1994a, 1994b). Although both methods are effective in increasing the structural capacity, they are labor consuming, mostly results in a substantial increase in the cross-section of the strengthened column, high density, long time of installation, difficulty in application, and the need for continuous maintenance. For these reasons and to replace outdated techniques, the innovative rehabilitation and strengthening methods for reinforced concrete structures, especially with composite materials, have taken a large portion of the research and application work in the field of repair and restoration of structural elements.

lately emerged fiber reinforced polymer FRP as a new material which has been used in aerospace, marine and automobile industries and to be used in different structural elements for its mechanical properties such as high tensile strength, low density, high resistance for corrosion, low thermal coefficient, short time of installation, easy application, and low cost for maintenance (Ferguson, Breen, & Jirsa, 1981). Moreover it is considered as an alternative to steel in reinforced concrete structures because of the continuing decline in the cost of FRP. FRP sheets for strengthening and rehabilitation of structural concrete elements has attracted great attention in the recent years (Hollaway, 2003; Mufti, 2003; Mufti, Labossiere, & Neale, 2002; Nanni, 1993). First applications for FRP were in the form of rebar. Then, FRP laminates were used for strengthening of concrete bridge girders by binding them to the tension face (Meier, 1992) as well as for rehabilitation of concrete columns (Saadatmanesh, 1994). FRP are available in the form of rods, grids, sheets, and winding strands. Review of literature up to 1996 can be found in ACI Committee 440 (ACI Committee 440, 1996). Another general review on class of materials including FRP used in civil construction was presented by Bakis et al. (Bakis et al., 2002). Some research by Parvin and Wang on FRP confined columns, the effect of FRP jacket thickness were investigated (Parvin & Wang, 2001a, 2001b). Shear strength of RC columns strengthened with FRP was studied by Ye et al. (Ye, Yue, Zhao, & Li, 2002) who finally concluded that shear strength of RC column can be effectively increased through external strengthening by using FRP sheets.

In this paper, several methods for predicting strength and behavior of CFRP strengthened concrete columns were reviewed and a new equation is proposed.

2. Types and properties fiber reinforced polymer

There is more than one type of fiber reinforced polymer (FRP), including composites with aramid (AFRP), basalt (BFRP), carbon (CFRP), and glass (GFRP) fibers, versus steel reinforcing should be understood prior to undertaking the design of structures using these reinforcements. Typical mechanical properties of fiber reinforced polymers are illustrated in Table 1 (Karian, 2003; American Concrete Institute (ACI) Committee 440, 440.6-08, 2008; Prince Engineering, PLC, 2011).

3. Repair and rehabilitation of columns

The structural strengthening might be required for inadequate design, damage due to seismic activities, poor quality construction, to meet current design requirement, repair of old structures, degradation problems which may arise from environmental exposure and other natural hazards. Therefore, the structural strengthening has received much attention over the last two decades all over the world. The experimental and analytical research have demonstrated that the use of FRP for strengthening and structural repair is less costly, more effective and requires less effort and time than the traditional methods. FRP composites were first used as strengthening materials for reinforced concrete columns and walls against earthquake forces, in addition strengthening of beams, bridge girders, beam-column joints, used in bridge decks and in cable stayed bridges (ACI Committee 440, 1996; Fardis & Khalili, 1982).

4. Parameters affecting the performance of FRP strengthened columns

The parameters affecting the performance of confined columns systems includes concrete strength, number of FRP layers, (depth-width) ratio, longitudinal steel, stirrups, corrosion of steel,

Table 1. Typical mechanical properties fiber reinforced polymers (Molded Fiber Glass Companies, 2003; American Concrete Institute (ACI) Committee 440, 440.6–08, 2008; Prince Engineering, PLC, 2011)

Reinforcing Material	Yield Strength ksi (MPa)	Tensile Strength ksi (MPa)	Elastic Modulus ksi (GPa)	Strain at Break percent (%)
Glass FRP	N/A	70–230 (480–1600)	5100–7400 (35–51)	1.2–3.1
Basalt FRP	N/A	150–240 (1035–1650)	6500–8500 (45–59)	1.6–3.0
Aramid FRP	N/A	250–368 (1720–2540)	6000–18,000 (41–125)	1.9–4.4
Carbon FRP	N/A	250–585 (1720–3690)	15,900–84,000 (120–580)	0.5–1.9

type of fiber, direction of fiber, thickness of FRP sheets, slenderness ratio, deformability of the concrete, stiffness of the jacket in the lateral direction, concrete dilation ratio, geometric, and loading imperfection (Demers, Hebert, Labossiere, & Neale, 1996; El-Hacha, Green, & Wight, 2010; Homan & Sheikh, 0000; Karbhari & Eckel, 1994; Karbhari, Rivera, & Dutta, 2000; Li & Hadi, 2003; Mirmiran et al., 1998; Nisticò, Pallini, Rousakis, Wu, & Karabinis, 2014; Rochette & Labossiere, 2000; Sadeghian, Rahai, & Ehsani, 2010; Scholefield, 2003; Teng & Lam, 2002; Touhari & Mitiche-Kettab, 2016). In this paper, concrete strength, number of FRP layers, and type of FRP material are studied.

4.1. Concrete strength

According to a study to Touhari, and Mitiche-Kettab (Touhari & Mitiche-Kettab, 2016) it could be noted that, the carbon fiber reinforced polymer (CFRP) and glass fiber reinforced polymer (GFRP) confinement on low strength concrete specimens produced higher results in terms of strength and strain than that those of normal and high strength concrete. For example, when CFRP is used to confine low-strength concrete cylinders with one layer of reinforcement, specimens revealed an increase of 69% and 416% in terms of compressive strength and axial strain over the reference specimen. While for normal strength concrete cylinders similarly confined, the specimen exhibited an increase of only 40% in compressive strength and 280% in axial strain. However, in high strength concrete cylinders, the specimen exhibited an increase of only 31% in compressive strength and 237% in axial strain for one layer of GFRP. The gain in terms of strength of low-strength concrete (LSC), normal strength concrete (NSC), and high strength concrete (HSC) was 69%, 40%, and 31%, respectively. The gain in terms of ductility of LSC, NSC, and HSC is about 416%, 390%, and 237%, respectively.

It can be seen that the axial strength and strain enhancement ratios of FRP confined concrete cylinders decrease as the strength of unconfined concrete increases. In other words, higher concrete compressive strength reduces the effect of confinement for the same number of FRP layers. This might be due to the fact that concrete with higher compressive strength exhibits lower lateral expansion under compression compared to concrete with lower compressive strength (Scholefield, 2003).

4.2. Number of FRP layers

The number of layers of FRP materials is one of the major parameters, having a significant effect on the behavior of specimens. According to Hadi (Li & Hadi, 2003) the test results proved that the benefit of confinement could be enhanced by increasing the stiffness of external confinement by applying multiple layers. However, the influence of the number of layers of FRP on the specimens under eccentric loading is not so pronounced as that of the specimens under concentric loading. In addition, the difference in strength between CFRP confined cylindrical specimens and GFRP ones increases more and more with the increase in the number of layers of FRP (Touhari & Mitiche-

Kettab, 2016). It should also be noted that the enhancement in strength and strain of FRP confined concrete cylinders is not proportional to the number of FRP layers, especially when high number of FRP layers strengthening is used.

4.3. Type of FRP material

The results of the tests showed carbon fiber reinforced polymer (CFRP) jacketing attains higher strength and strain than that of glass fiber reinforced polymer (GFRP) confined specimens, leading to a significant effect on the mechanical properties of FRP materials and the strength and strains enhancement ratios (Li & Hadi, 2003; Touhari & Mitiche-Kettab, 2016).

5. Predicting the ultimate compressive strength of FRP circular confined concrete columns

The first well-known study on the stress-strain curve of concrete with and without steel confinement was conducted by Richart et al (Richart, Brandtzaeg, & Brown, 1929). The following well known relation was based on a linear relationship for expressing the enhancement of compressive strength based on their test results (Eq. 1).

$$f'_{cc} = f'_{co} + k_1 \times f'_L \tag{1}$$

where f'_{cc} and f'_{co} are the compressive strength of confined and unconfined concrete, respectively, $\boldsymbol{k}_1$ is the confinement effectiveness factor, and f'_L is the effective lateral confining stress.

Since then, the majority of models devoted to predict the compressive strength of FRP confined concrete columns are based on the general equation proposed by Richart et al. (Richart et al., 1929) (Eq. 1) which has been developed to estimate the confined concrete with steel which have been based either on tests of plain concrete specimens or reinforced concrete columns. Most of these models used a constant value for $\boldsymbol{k_1}$ and it was limited between 2 and 5 (Fardis & Khalili, 1982; Lam & Teng, 2002; Micelli & Modarelli, 2013; Valdmanis, DeLorenzis, Rousakis, & Tepfers, 2007). Moreover, other researchers expressed $\boldsymbol{k_1}$ in a non-linear form (Afifi, Mohamed, & Benmokrane, 2015; Mohamed & Masmoudi, 2010; Realfonzo & Napoli, 2011; Rousakis, Nistico, & Karabinis, 2012).

Fardis and Khalil (Fardis & Khalili, 1982) developed a linear relationship between the ultimate strength and the effective lateral confining stress.

$$f'_{cc} = f'_{co} + 4.1 \times f'_L \tag{2}$$

Mander et al. (Mander, Priestley, & Park, 1988) also derived a non-linear relationship between the ultimate strength and the effective lateral confining pressure of confined concrete cylinders based on the tri-axial test data.

$$f'_{cc} = f'_{co}\left[-1.254 + 2.254\sqrt{1 + \frac{7.94 \times f'_L}{f'_{co}}} - 2\frac{f'_L}{f'_{co}}\right] \tag{3}$$

Li et al. (Li, Chih, & S A, 2003) proposed a constitutive model for confined concrete columns reinforced with CFRP materials. They studied the behaviour of cylinders with various strengths of concrete:

$$f'_{cc} = f'_{co} + f'_L tan^2(45^\circ + \theta/2) \tag{4}$$

$$\theta = 36^\circ + 1^\circ\left(f'_{co}/35\right) \leq 45^\circ \tag{5}$$

where $\boldsymbol{\theta}$ is the angle of internal friction of concrete.

Ozbakkaloglu and Jian (Ozbakkaloglu & Jian, 2013) developed a new model based on over 500 experimental results for CFRP and GFRP confined concrete cylinders:

For CFRP confined concrete cylinders:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 3.64 \times \frac{f'_L}{f'_{co}} \tag{6}$$

For GFRP confined concrete cylinders:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 2.64 \times \frac{f'_L}{f'_{co}} \tag{7}$$

Pham and Hadi (T X & Hadi, 2014) proposed new confinement model for FRP confined normal- and high-strength concrete circular columns:

$$f'_{cc} = 0.7f'_{co} + 1.8f'_L + 5.7(t/D) + 13 \tag{8}$$

where t is the thickness of the composite jacket, and D is the diameter of the concrete core.

Touhari and Mitiche-Kettab (Touhari & Mitiche-Kettab, 2016) proposed new confinement model for FRP confined normal- and high-strength concrete circular columns:

For CFRP confined concrete cylinders:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 2.8\frac{f'_L}{f'_{co}} \tag{9}$$

For GFRP confined concrete cylinders:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 1.85\frac{f'_L}{f'_{co}} \tag{10}$$

Youssef et al. (Youssef, Feng, & Mosallam, 2007) proposed the following strength model:

$$\frac{f'_{cc}}{f'_{co}} = 1 + 2.25 \times \left(\frac{f'_L}{f'_{co}}\right)^{\frac{5}{4}} \tag{11}$$

All of the above models assumed that the compressive strength of confined concrete is a function of the unconfined concrete strength and the effective lateral confining pressure.

5.1. Mechanism of confinement

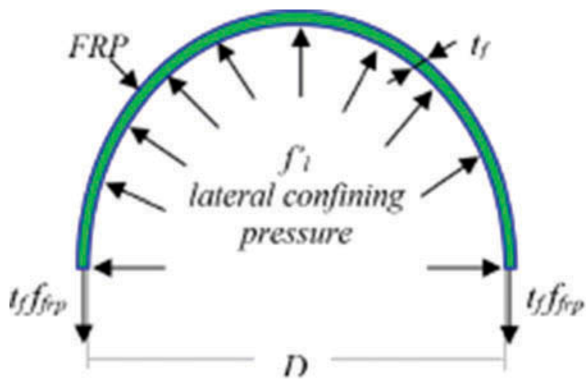
The lateral confinement pressure provided by a FRP jacket to concrete is naturally passive. In FRP confined concrete cylinders, the concrete core extends laterally and this expansion is restrained by the FRP material. The action of expansion and the reaction of the confinement are represented by a uniform lateral pressure f'_L in the interface and the response of FRP material (Figure 1). This expansion of the concrete core is confined by the FRP jackets, and thus transforms the concrete core to a 3-D compressive stress condition. The mechanism of confinement goes from uniaxial loading to tri-axial loading. The maximum confinement pressure is reached when the circumferential strain in the FRP reaches its ultimate strain ε_{frp} corresponding to the failure of the cylinder. Based on static analysis, equilibrium of forces, deformation compatibility, and by considering one unit length section along the column span, the forces acting on the section shown in (Figure 1) can be written as (Touhari & Mitiche-Kettab, 2016):

$$Df'_L = 2t_f \times f_{frp} \tag{12}$$

The lateral confining pressure reaches its maximum value f'_L at the rupture of FRP, with:

$$f'_L = \frac{2t_f \times f_{frp}}{D} = \frac{2E_{frp} \times \varepsilon_{frp} \times t_f}{D} \tag{13}$$

Figure 1. Mechanism of confinement (Touhari & Mitiche-Kettab, 2016).



where $f_L^{'}$ presents the lateral confining pressure, $\boldsymbol{E_{frp}}$ is the tensile modulus of FRP material, $\boldsymbol{t_f}$ is the thickness of the composite jacket, ε_{frp} is the ultimate circumferential strain in the composite jacket, D is the diameter of the concrete core.

The FRP volumetric ratio $\boldsymbol{\rho_{frp}}$ is given by the following equation for entirely wrapped circular cross section:

$$\rho_{frp} = \frac{4t_f}{D} \tag{14}$$

where $\boldsymbol{\rho_{frp}}$ is the FRP volumetric ratio.

6. Predicting the ultimate axial strain of FRP circular confined concrete columns

For FRP confined concrete cylinders, numerous studies suggested that the ultimate axial strain can be correlated to the lateral confining pressure (Lam & Teng, 2003; Micelli & Modarelli, 2013; Scholefield, 2003). Existing models can be classified into two categories, empirical or analytical models and numerical models or plasticity analysis. Ozbakkaloglu and Jian (Ozbakkaloglu & Jian, 2013) proposed a two-part equation depending on the type of the used FRP where the strain enhancement ratio FRP confined concrete can be represented as follows:

For CFRP:

$$\frac{\varepsilon_{cc}^{'}}{\varepsilon_{co}^{'}} = 2 + 17.41\frac{f_L^{'}}{f_{co}^{'}} \tag{15}$$

For GFRP:

$$\frac{\varepsilon_{cc}^{'}}{\varepsilon_{co}^{'}} = 2 + 24.47\frac{f_L^{'}}{f_{co}^{'}} \tag{16}$$

where $\varepsilon_{co}^{'}$ is the strain of unconfined concrete and $\varepsilon_{cc}^{'}$ is the ultimate strain of FRP confined concrete.

Kwan and Dong (Kwan & Dong, 2015) proposed a new axial strength model for FRP confined concrete which can be given in the following equation:

$$\frac{\varepsilon_{cc}^{'}}{\varepsilon_{co}^{'}} = 1 + 17.4\left(\frac{\sigma_r}{f_{co}^{'}}\right)^{1.06} \tag{17}$$

where $\boldsymbol{\sigma_r}$ is the confining stress.

Last but not the least Youssef et al.(Youssef et al., 2007) propsed the following model to represent the ultimate concrete compressive strain:

$$\varepsilon_{cc} = 0.003368 + 0.2590\left(\frac{f'_L}{f'_{co}}\right)\left(\frac{f_{ju}}{E_j}\right)^{\frac{1}{2}} \tag{18}$$

where f_{ju} is tensile strength of FRP jacket, E_j is tensile modulus of FRP jacket in the hoop direction.

All previous equations database covers unconfined concrete strength between (20 - 80) MPa, specimens with a diameter ranging from (150 - 160) mm. Furthermore, this database includes specimens with a variety of FRP types: carbon FRP (CFRP), glass FRP (GFRP) and number of layer ranging from (1–3), and all the previous models take the value of modification factor k_1 constant without taking into account the increase in the number of FRP layers and compressive strength of concrete which leads to differences in the value of the effective lateral confining pressure.

7. Proposed equations

7.1. The ultimate compressive strength of FRP circular confined concrete columns

Testing the proposed equations in previous studies (Cui & Sheikh, 2010; Jiang & Teng, 2007; Lam & Teng, 2004; Teng, Yu, Wong, & Dong, 2007; Touhari & Mitiche-Kettab, 2016; Valdmanis et al., 2007; Vincent & Ozbakkaloglu, 2013) against the available experimental results, large differences has been noticed, as can be shown in Table 2. In this table the ratio between the experimental results and the theoretical one is shown to be between (0.37 - 2) and the coefficient of variation between (12 - 64)%. This high variation does not imply that the equations are safe and rigorous for modeling FRP confined concrete columns, especially when it comes for a wide range of test results involving many different parameters. This might partly be attributed to the fact of neglecting the change in the tensile modulus of FRP material E_{frp} in the previous equations because of the composite action of the laminate where the number of FRP layers, the type of resin and the orientations of the fibers should be taken into considerations, leading to a decrease in the tensile modulus of FRP material and consequently, E_{frp} the lateral confining pressure f'_L also decreases.

Where μ is sample mean, σ is standard deviation, C.% is coefficient of variation. Because it is difficult to calculate the composite FRP tensile modulus due to lack of experimental data for most of the reviewed tests, a new model has been proposed as shown in Eqs. (19 and 20) for CFRP and GFRP confined concrete columns. A new term have been added to the original model by Richart et al. (Eq. 1) (Richart et al., 1929) to compensate for the decrease in the tensile modulus of FRP material E_{frp}. These terms are $\left[\left(f'_L - n \times 5\right)\right]$ and $\left[\left(f'_L - n \times 8\right)\right]$ for GFRP and CFRP, respectively will lead to a decrease in the lateral confining pressure as a consequence to the reduced composite tesnsile modulus of the composite jacketing with increasing the number of FRP lamina.

For CFRP confined concrete cylinders:

$$f'_{cc} = f'_{co} + k \times f'_L + \left[\left(f'_L - n \times 5\right)\right] \tag{19}$$

For GFRP confined concrete cylinders:

$$f'_{cc} = f'_{co} + k \times f'_L + \left[\left(f'_L - n \times 8\right)\right] \tag{20}$$

where k is the effective lateral confining factor according to the Table 3, n is the number layer.

In this work, the factor k in the new proposed equations (19 and 20), which originally accounts for the effectiveness of the strengthening technique due to the column sectional geometrical properties, has been modified using multivariate regression analysis to account for the differences in the number of FRP layers and the value of the effective lateral confining pressure. The values of k can be found in Table 3 both for GFRP and CFRP strengthening systems.

Table 2. Comparison between the experimental and predicted results from previous equations of FRP confined concrete columns

(CFRP)										(GFRP)		
f'_{cc} theo. $/f'_{cc}$ exp. Eq. (2)	f'_{cc} theo. $/f'_{cc}$ exp. Eq. (3)	f'_{cc} theo. $/f'_{cc}$ exp. Eq. (4)	f'_{cc} theo. $/f'_{cc}$ exp. Eq. (6)	f'_{cc} theo. $/f'_{cc}$exp. Eq. (8)	f'_{cc} theo. $/f'_{cc}$exp. Eq. (9)	f'_{cc} theo. $/f'_{cc}$exp. Eq. (11)	ε'_{cc} theo. $/\varepsilon'_{cc}$ exp. Eq.(15)	ε'_{cc} theo. $/\varepsilon'_{cc}$ exp. Eq.(16)	ε'_{cc} theo. $/\varepsilon'_{cc}$ exp. Eq.(17)	f'_{cc} theo. $/f'_{cc}$ exp. Eq. (7)	f'_{cc} theo. $/f'_{cc}$exp. Eq. (10)	ε'_{cc} theo. $/\varepsilon'_{cc}$ exp. Eq.(18)
Touhari and Mitiche-Kettab (2016)										Touhari and Mitiche-Kettab (2016)		
0.98	1.12	0.55	0.93	0.84	0.83	0.51	0.61	0.70	0.95	1.02	0.92	1.16
1.01	1.16	0.57	0.96	0.87	0.86	0.53	0.64	0.75	1.02	1.12	1.01	1.36
1.12	1.25	0.84	1.08	1.14	1.02	0.81	0.50	0.81	1.04	1.03	0.93	1.26
1.17	1.27	0.49	1.09	0.86	0.94	0.43	0.81	0.74	1.05	1.03	0.98	1.47
1.21	1.31	0.49	1.12	0.88	0.96	0.44	0.92	0.85	1.20	1.03	0.88	1.20
1.16	1.26	0.47	1.08	0.84	0.93	0.42	0.83	0.74	1.04	0.99	0.84	1.16
1.08	1.11	0.37	0.99	0.72	0.84	0.32	1.13	0.74	1.06	0.99	0.82	1.17
1.07	1.09	0.36	0.98	0.71	0.82	0.32	1.11	0.72	1.03	1.06	0.88	1.24
1.11	1.15	0.39	1.02	0.75	0.86	0.34	1.12	0.78	1.12	1.09	0.91	1.27
1.07	1.18	0.89	1.04	0.95	1.00	0.84	0.48	0.78	1.00	0.97	0.91	1.23
1.01	1.15	0.75	0.97	0.83	0.90	0.68	0.52	0.76	1.01	0.98	0.92	1.28
1.00	1.13	0.74	0.96	0.82	0.89	0.66	0.53	0.75	1.00	0.95	0.88	1.19
1.05	1.20	0.67	0.99	0.79	0.89	0.57	0.65	0.79	1.10	0.99	0.88	1.31
1.08	1.23	0.66	1.02	0.78	0.91	0.54	0.64	0.74	1.05	0.97	0.86	1.26
1.08	1.23	0.66	1.02	0.78	0.91	0.54	0.60	0.69	0.97	0.95	0.85	1.22
1.04	1.16	0.57	0.97	0.70	0.85	0.43	0.76	0.68	1.00	1.04	0.90	1.35
1.04	1.16	0.57	0.97	0.71	0.85	0.43	0.81	0.74	1.07	1.03	0.88	1.21
1.06	1.18	0.58	0.99	0.72	0.87	0.45	0.79	0.74	1.08	1.02	0.88	1.29
1.03	1.15	0.96	1.00	0.81	0.95	0.77	0.47	0.74	0.95	1.05	1.00	1.29

(*Continued*)

Table2. (*Continued*)

(CFRP)										(GFRP)		
f'_{cc} theo. /f'_{cc} exp. Eq. (2)	f'_{cc} theo. /f'_{cc} exp. Eq. (3)	f'_{cc} theo. /f'_{cc} exp. Eq. (4)	f'_{cc} theo. /f'_{cc} exp. Eq. (6)	f'_{cc} theo. /f'_{cc}exp. Eq. (8)	f'_{cc} theo. /f'_{cc}exp. Eq. (9)	f'_{cc} theo. /f'_{cc}exp. Eq. (11)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(15)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(16)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(17)	f'_{cc} theo. /f'_{cc} exp. Eq. (7)	f'_{cc} theo. /f'_{cc}exp. Eq. (10)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(18)
1.02	1.13	0.95	0.99	0.81	0.94	0.78	0.49	0.79	1.01	1.01	0.96	1.28
1.01	1.13	0.94	0.98	0.80	0.93	0.76	0.52	0.82	1.06	0.97	0.92	1.14
1.04	1.18	0.93	0.99	0.76	0.91	0.64	0.65	0.89	1.20	1.08	0.99	1.23
1.01	1.15	0.90	0.97	0.74	0.89	0.62	0.57	0.76	1.03	1.12	1.03	1.30
1.05	1.20	0.93	1.00	0.76	0.91	0.63	0.54	0.72	0.98	1.12	1.02	1.26
1.05	1.20	0.92	1.00	0.74	0.90	0.59	0.59	0.75	1.03	1.07	0.95	1.31
1.05	1.20	0.91	0.99	0.71	0.89	0.53	0.67	0.76	1.06	1.07	0.95	1.24
1.09	1.24	0.95	1.03	0.75	0.93	0.58	0.62	0.77	1.07	1.06	0.94	1.24
Lam and Teng (2004)										Jiang and Teng (2007)		
1.31	1.50	0.81	1.24	1.02	1.12	0.71	0.44	1.52	0.88	1.04	0.96	1.51
1.40	1.60	0.86	1.33	1.09	1.20	0.76	0.49	1.76	1.01	1.06	0.98	1.17
1.24	1.42	0.77	1.18	0.97	1.06	0.67	0.45	1.50	0.86	1.40	1.32	1.24
1.43	1.55	0.67	1.33	0.95	1.14	0.53	0.73	2.15	1.11	1.40	1.32	0.95
1.41	1.53	0.66	1.31	0.94	1.13	0.52	0.63	1.84	0.96	1.28	1.16	1.53
1.37	1.49	0.64	1.27	0.91	1.10	0.51	0.68	1.95	1.01	1.23	1.11	1.23
1.53	1.55	0.58	1.41	0.94	1.18	0.44	0.96	2.48	1.17	1.22	1.06	1.33
1.40	1.42	0.53	1.29	0.86	1.08	0.41	0.89	2.12	1.00	1.40	1.23	2.06
1.30	1.32	0.49	1.20	0.80	1.00	0.38	0.92	2.03	0.96	Cui and Sheikh(2010)		
1.28	1.46	0.77	1.21	1.00	1.09	0.68	0.53	1.84	1.01	1.18	1.07	1.02
1.29	1.47	0.77	1.22	1.01	1.10	0.69	0.50	1.74	0.96	1.17	1.06	1.20

(*Continued*)

Table2. (*Continued*)

(CFRP)										(GFRP)		
f'_{cc} theo. /f'_{cc} exp. Eq. (2)	f'_{cc} theo. /f'_{cc} exp. Eq. (3)	f'_{cc} theo. /f'_{cc} exp. Eq. (4)	f'_{cc} theo. /f'_{cc} exp. Eq. (6)	f'_{cc} theo. /f'_{cc} exp. Eq. (8)	f'_{cc} theo. /f'_{cc} exp. Eq. (9)	f'_{cc} theo. /f'_{cc} exp. Eq. (11)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(15)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(16)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(17)	f'_{cc} theo. /f'_{cc} exp. Eq. (7)	f'_{cc} theo. /f'_{cc} exp. Eq. (10)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(18)
1.14	1.30	0.68	1.08	0.89	0.97	0.61	0.50	1.62	0.89	1.03	0.88	1.05
1.33	1.52	0.89	1.27	1.09	1.17	0.81	0.61	2.16	1.30	1.04	0.89	1.05
1.33	1.53	0.89	1.28	1.09	1.17	0.81	0.57	2.03	1.22	1.01	0.83	1.14
1.36	1.55	0.91	1.30	1.11	1.19	0.82	0.57	2.03	1.22	1.02	0.84	1.63
1.36	1.52	0.74	1.27	0.96	1.12	0.60	0.59	1.92	1.03	**Teng et al. (2007)**		
1.31	1.46	0.71	1.22	0.92	1.07	0.58	0.56	1.76	0.94	1.36	1.27	1.25
1.44	1.61	0.75	1.35	1.02	1.18	0.62	0.72	2.39	1.27	1.30	1.22	1.42
1.34	1.50	1.30	1.30	1.04	1.22	0.98	0.71	2.19	1.61	1.12	1.00	0.86
1.27	1.43	1.24	1.24	0.99	1.17	0.94	0.75	2.26	1.67	1.09	0.97	1.00
1.30	1.46	1.23	1.26	1.01	1.19	0.95	0.66	2.02	1.48	1.10	0.95	0.98
1.56	1.78	1.49	1.49	1.10	1.35	0.89	0.67	2.35	1.50	1.18	1.02	1.40
1.62	1.85	1.46	1.54	1.14	1.39	0.91	0.88	3.16	2.01			
1.65	1.89	1.56	1.57	1.16	1.42	0.94	0.73	2.67	1.71			
1.61	1.81	1.52	1.51	1.05	1.33	0.75	0.76	2.59	1.55			
1.72	1.94	1.62	1.62	1.13	1.43	0.80	0.72	2.59	1.55			
1.62	1.79	1.52	1.51	1.01	1.31	0.65	0.89	2.82	1.64			
1.57	1.73	1.36	1.47	0.98	1.27	0.62	0.93	2.84	1.64			
1.56	1.73	1.54	1.46	0.98	1.27	0.63	0.91	2.81	1.63			
Valdmanis et. al. (2007)												
0.93	1.06	0.67	0.89	0.76	0.83	0.61	0.64	1.69	1.15			

(*Continued*)

Table2. (*Continued*)

(CFRP)										(GFRP)		
f'_{cc} theo. /f'_{cc} exp. Eq. (2)	f'_{cc} theo. /f'_{cc} exp. Eq. (3)	f'_{cc} theo. /f'_{cc} exp. Eq. (4)	f'_{cc} theo. /f'_{cc} exp. Eq. (6)	f'_{cc} theo. /f'_{cc}exp. Eq. (8)	f'_{cc} theo. /f'_{cc}exp. Eq. (9)	f'_{cc} theo. /f'_{cc}exp. Eq. (11)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(15)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(16)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(17)	f'_{cc} theo. /f'_{cc} exp. Eq. (7)	f'_{cc} theo. /f'_{cc}exp. Eq. (10)	ε'_{cc} theo. /ε'_{cc} exp. Eq.(18)
0.95	1.08	0.56	0.89	0.69	0.79	0.46	0.73	1.71	1.04			
1.08	1.19	0.55	1.01	0.72	0.87	0.42	0.87	1.91	1.10			
0.90	1.02	0.68	0.86	0.73	0.80	0.60	0.65	1.71	1.19			
1.05	1.20	0.67	1.00	0.76	0.89	0.54	1.19	3.12	1.92			
0.94	1.04	0.53	0.88	0.63	0.77	0.39	1.23	2.53	1.47			
μ 1.22	1.36	0.83	1.15	0.89	1.03	0.62	0.71	1.55	1.17	1.09	0.98	1.25
σ 0.22	0.24	0.33	0.20	0.14	0.18	0.17	0.19	0.78	0.27	0.13	0.13	0.19
C.% 18	20	27	16	12	14	14	16	64	22	12	12	17

*Where μ is sample mean, σ is standard deviation, *C*.% is coefficient of variation

Table 3. The effective lateral confining factor k

(CFRP)			(GFRP)		
If $n = 1$ & $f'_L > 5.6$	$k = 1.75$	else $k = 3.7$	If $n = 1$ & $f'_L \le 4.1$	$k = 0.75$	else $k = 3$
If $n = 2$ & $f'_L > 11$	$k = 1.5$		If $n = 2$ & $f'_L \le 9$	$k = 2.1$	
If $n = 3$ & $f'_L > 16$	$k = 1.35$		If $n = 3$ & $f'_L \le 12.4$	$k = 2.6$	
If $n = 4$ & $f'_L > 20$	$k = 1.35$		If $n = 4$ & $f'_L \le 17$	$k = 3.4$	
If $n = 5$ & $f'_L > 24$	$k = 1.25$		If $n = 5$ & $f'_L \le 20$	$k = 3.45$	
If $n = 6$ & $f'_L > 27.5$	$k = 1.15$		If $n = 6$ & $f'_L \le 25$	$k = 3.6$	

7.2. The ultimate axial strain of FRP circular confined concrete columns

There are a few approaches to develop an equation for ultimate axial strain of confined concrete. All the above-mentioned models take the value of modification factor k_2 constant. In this work, the value of k_2 is taken variable depending upon the ultimate strain of the unconfined concrete as shown in Table 4. These values were based on regression analysis of the available test results in (Cui & Sheikh, 2010; Jiang & Teng, 2007; Lam & Teng, 2004; Teng et al., 2007; Touhari & Mitiche-Kettab, 2016; Valdmanis et al., 2007; Vincent & Ozbakkaloglu, 2013). Based on this, equation (21) is proposed for predicting the ultimate strain for confined concrete cylinders.

$$\varepsilon'_{cc} = \varepsilon'_{co} + k_2\left(\frac{f'_L}{f'_{co}}\right) \tag{21}$$

Where $\boldsymbol{k_2}$ is the effective strain of unconfined concrete according to the Table 4.

7.3. Validation of the proposed model

The strength and strain enhancement proposed model of FRP confined concrete cylinders is compared to the test data obtained from (Cui & Sheikh, 2010; Jiang & Teng, 2007; Lam & Teng, 2004; Teng et al., 2007; Touhari & Mitiche-Kettab, 2016; Valdmanis et al., 2007; Vincent & Ozbakkaloglu, 2013), as shown in Tables 5 and 6, for (CFRP) and (GFRP) confined concrete cylinders, respectively. These comparisons indicate that the results of the proposed equations is in agreement with the experimental data. So that the ratio between the experimental results and the theoretical results are typically close to (1) in comparison to a ratio between (0.37 - 2) for the previous equations and the coefficient of variation between (9 - 11.2) in comparison to a ratio between (12 - 64) for the previous equations. This leaves no doubts that the proposed equation are capable of predicting the ultimate strength and strain of confined concrete cylinders more accurately than previous equations.

8. Conclusions

- In this work, a review of several previous studies pertaining strengthening of concrete columns with FRP material (jacketing) is performed. It has been shown that the parameters affecting the performance of confined columns systems by using FRP sheets are concrete strength, shape of column section, (depth-width) ratio, longitudinal steel, stirrups, corrosion of steel, type of fiber, direction of fiber, thickness of FRP sheets, slenderness ratio, deformability of the

Table 4. The effective strain of unconfined concrete k_2

(CFRP)		(GFRP)	
If $\varepsilon'_{co} \le 0.19\%$	$k_2 = 3.2$	If ε'_{co} (0.2–0.29) %	$k_2 = 5.5$
If ε'_{co} (0.2–0.29) %	$k_2 = 3.45$	If ε'_{co} (0.3–0.35) %	$k_2 = 6$
If ε'_{co} (2.6–2.9) %	$k_2 = 60$	If ε'_{co} (2.6–2.9) %	$k_2 = 60$
If ε'_{co} (3–3.3) %	$k_2 = 65$	If ε'_{co} (3–3.3) %	$k_2 = 65$

Table 5. Comparison between the experimental and predicted results of circular confined concrete by (CFRP) for (proposed model)

D mm	f'_{co} Mpa	t mm	n	E Gpa	ε'_{frp} (%)	ε'_{co} (%)	k	k_2	f'_l exp Mpa	f'_{cc} theo Mpa	f'_{cc} exp Mpa	ε'_{cc} theo (%)	ε'_{cc} exp (%)	ε'_{cc} theo $/\varepsilon'_{cc}$ exp	f'_{cc} theo $/f'_{cc}$ exp
Touhari and Mitiche-Kettab (Touhari & Mitiche-Kettab, 2016)															
160	24	0.13	1	234	14	2.71	3.7	60	5.41	44.43	47	16.24	16.9	0.96	0.95
160	24	0.13	1	234	14	2.71	3.7	60	5.35	44.15	45.3	16.09	15.6	1.03	0.97
160	24	0.13	1	234	14	2.71	3.7	60	2.17	29.20	29.5	8.14	9.31	0.87	0.99
160	24	0.26	2	234	14	2.71	3.7	60	10.1	61.47	55.8	27.96	24.1	1.16	1.10
160	24	0.26	2	234	14	2.71	3.7	60	10.5	63.35	55.5	28.96	26.5	1.09	1.14
160	24	0.26	2	234	14	2.71	3.7	60	10.6	63.82	58	29.21	25.2	1.16	1.10
160	24	0.39	3	234	14	2.71	3.7	60	14.5	77.15	77.3	38.96	32.9	1.18	1.00
160	24	0.39	3	234	14	2.71	3.7	60	14.7	78.09	79	39.46	33.4	1.18	0.99
160	24	0.39	3	234	14	2.71	3.7	60	13.8	73.86	72.9	37.21	35.8	1.04	1.01
160	41.6	0.13	1	234	14	3.11	3.7	65	2.84	49.95	49.8	7.55	8.92	0.85	1.00
160	41.6	0.13	1	234	14	3.11	3.7	65	4.94	59.82	61.3	10.83	12.5	0.87	0.98
160	41.6	0.13	1	234	14	3.11	3.7	65	5.14	60.76	62.9	11.14	12.9	0.86	0.97
160	41.6	0.26	2	234	14	3.11	3.7	65	8.54	71.74	73.2	16.45	15.7	1.05	0.98
160	41.6	0.26	2	234	14	3.11	3.7	65	10	78.60	76.6	18.74	18.4	1.02	1.03
160	41.6	0.26	2	234	14	3.11	3.7	65	10.1	79.07	77	18.89	19.9	0.95	1.03
160	41.6	0.39	3	234	14	3.11	3.7	65	14.5	94.75	96.9	25.77	25.2	1.02	0.98
160	41.6	0.39	3	234	14	3.11	3.7	65	14.2	93.34	95.9	25.30	23	1.10	0.97
160	41.6	0.39	3	234	14	3.11	3.7	65	13.8	91.46	92.7	24.67	22.4	1.10	0.99
160	61.5	0.13	1	234	14	3.02	3.7	65	5.08	80.38	80	8.39	10.25	0.82	1.00
160	61.5	0.13	1	234	14	3.02	3.7	65	4.54	77.84	78.9	7.82	9.78	0.80	0.99
160	61.5	0.13	1	234	14	3.02	3.7	65	4.99	79.95	81.1	8.29	9.72	0.85	0.99
160	61.5	0.26	2	234	14	3.02	3.7	65	9.25	94.98	96	12.80	11.6	1.10	0.99

(Continued)

Table5. (*Continued*)

D mm	f'_{co} Mpa	t mm	n	E Gpa	ε'_{frp} (%)	ε'_{co} (%)	k	k_2	f'_l exp Mpa	f'_{cc} theo Mpa	f'_{cc} exp Mpa	ε'_{cc} theo (%)	ε'_{cc} exp (%)	ε'_{cc} theo /ε'_{cc} exp	f'_{cc} theo /f'_{cc} exp
160	61.5	0.26	2	234	14	3.02	3.7	65	9.46	95.96	99.4	13.02	13.7	0.95	0.97
160	61.5	0.26	2	234	14	3.02	3.7	65	10.1	98.97	98.2	13.69	14.9	0.92	1.01
160	61.5	0.39	3	234	14	3.02	3.7	65	11.8	101.96	104.9	15.49	15.6	0.99	0.97
160	61.5	0.39	3	234	14	3.02	3.7	65	15.1	117.47	117.1	18.98	17.8	1.07	1.00
160	61.5	0.39	3	234	14	3.02	3.7	65	12.9	107.13	105.4	16.65	15.9	1.05	1.02
Lam and Teng (Lam & Teng, 2004)															
150	35.9	0.16	1	230	15	0.2	1.75	3.45	7.36	51.14	50.4	0.91	1.27	0.78	1.01
150	35.9	0.16	1	230	15	0.2	1.75	3.45	7.36	51.14	47.2	0.91	1.1	0.83	1.08
150	35.9	0.16	1	230	15	0.2	1.75	3.45	7.36	51.14	53.2	0.91	1.29	0.76	0.96
150	35.9	0.33	2	230	15	0.2	1.5	3.45	15.18	63.85	68.7	1.66	1.68	0.99	0.93
150	35.9	0.33	2	230	15	0.2	1.5	3.45	15.18	63.85	69.6	1.66	1.96	0.85	0.92
150	35.9	0.33	2	230	15	0.2	1.5	3.45	15.18	63.85	71.6	1.66	1.85	0.90	0.89
150	34.3	0.49	3	230	15	0.18	1.35	3.2	22.54	72.27	82.6	2.28	2.06	1.11	0.87
150	34.3	0.49	3	230	15	0.18	1.35	3.2	22.54	72.27	90.4	2.28	2.41	0.95	0.80
150	34.3	0.49	3	230	15	0.18	1.35	3.2	22.54	72.27	97.3	2.28	2.51	0.91	0.74
150	34.3	0.16	1	230	15	0.18	1.75	3.2	7.36	49.54	50.3	0.87	1.02	0.85	0.98
150	34.3	0.16	1	230	15	0.18	1.75	3.2	7.36	49.54	50	0.87	1.08	0.81	0.99
150	34.3	0.16	1	230	15	0.18	1.75	3.2	7.36	49.54	56.7	0.87	1.16	0.79	0.87
Vincent and Ozbakkaloglu T (Vincent & Ozbakkaloglu, 2013)															
150	**35.5**	**0.11**	**1**	**240**	**16**	**0.21**	**1.75**	**3.45**	**5.63**	**45.99**	**44**	**0.76**	**0.77**	**0.98**	**1.05**
150	**35.5**	**0.11**	**1**	**240**	**16**	**0.21**	**1.75**	**3.45**	**5.63**	**45.99**	**43.9**	**0.76**	**0.82**	**0.92**	**1.05**
150	**35.5**	**0.11**	**1**	**240**	**16**	**0.21**	**1.75**	**3.45**	**5.63**	**45.99**	**43.1**	**0.76**	**0.82**	**0.92**	**1.07**
150	**38**	**0.23**	**2**	**240**	**16**	**0.21**	**1.5**	**3.45**	**11.78**	**56.26**	**63.5**	**1.28**	**1.51**	**0.85**	**0.89**

(*Continued*)

Table5. (*Continued*)

D mm	f'_{co} Mpa	*t* mm	*n*	*E* Gpa	ε'_{frp} (%)	ε'_{co} (%)	*k*	k_2	f'_l exp Mpa	f'_{cc} theo Mpa	f'_{cc} exp Mpa	ε'_{cc} theo (%)	ε'_{cc} exp (%)	ε'_{cc} theo $/\varepsilon'_{cc}$ exp	f'_{cc} theo $/f'_{cc}$ exp
150	**38**	**0.23**	**2**	**240**	**16**	**0.21**	**1.5**	**3.45**	**11.78**	**56.26**	**66.1**	**1.28**	**1.65**	**0.82**	**0.85**
150	**36.1**	**0.23**	**2**	**240**	**16**	**0.21**	**1.5**	**3.45**	**11.78**	**54.36**	**58.6**	**1.34**	**1.27**	**1.06**	**0.93**
150	**64.5**	**0.11**	**1**	**240**	**16**	**0.27**	**1.75**	**3.45**	**5.63**	**74.99**	**65.6**	**0.57**	**0.59**	**0.97**	**1.14**
150	**64.5**	**0.11**	**1**	**240**	**16**	**0.27**	**1.75**	**3.45**	**5.63**	**74.99**	**68.7**	**0.57**	**0.57**	**1.00**	**1.09**
150	**62.9**	**0.11**	**1**	**240**	**16**	**0.27**	**1.75**	**3.45**	**5.63**	**73.39**	**66.3**	**0.58**	**0.65**	**0.89**	**1.11**
150	**64.5**	**0.23**	**2**	**240**	**16**	**0.27**	**1.5**	**3.45**	**11.78**	**82.76**	**72.3**	**0.90**	**0.93**	**0.97**	**1.14**
150	**62.4**	**0.23**	**2**	**240**	**16**	**0.27**	**1.5**	**3.45**	**11.78**	**80.66**	**68.4**	**0.92**	**0.71**	**1.14**	**1.18**
150	**64.2**	**0.23**	**2**	**240**	**16**	**0.27**	**1.5**	**3.45**	**11.78**	**82.46**	**68.2**	**0.90**	**0.82**	**1.10**	**1.21**
150	**64.5**	**0.35**	**3**	**240**	**16**	**0.27**	**1.35**	**3.45**	**17.92**	**91.61**	**85.9**	**1.23**	**1.19**	**1.03**	**1.07**
150	**64.5**	**0.35**	**3**	**240**	**16**	**0.27**	**1.35**	**3.45**	**17.92**	**91.61**	**80.3**	**1.23**	**1.19**	**1.03**	**1.14**
150	**64.5**	**0.46**	**4**	**240**	**16**	**0.27**	**1.35**	**3.45**	**23.55**	**99.85**	**99.4**	**1.53**	**1.38**	**1.11**	**1.00**
150	**62.4**	**0.46**	**4**	**240**	**16**	**0.27**	**1.35**	**3.45**	**23.55**	**97.75**	**101**	**1.57**	**1.41**	**1.12**	**0.97**
150	**65.8**	**0.46**	**4**	**240**	**16**	**0.27**	**1.35**	**3.45**	**23.55**	**101.15**	**104**	**1.50**	**1.36**	**1.09**	**0.97**
Valdmanis et. all (Valdmanis et al., 2007)															
150	**40**	**0.17**	**1**	**230**	**10**	**0.17**	**3.7**	**3.2**	**5.21**	**59.50**	**66**	**0.59**	**0.63**	**0.93**	**0.90**
150	**40**	**0.34**	**2**	**230**	**10**	**0.17**	**3.7**	**3.2**	**10.43**	**79.01**	**87.2**	**1.00**	**1.07**	**0.94**	**0.91**
150	**40**	**0.51**	**3**	**230**	**10**	**0.17**	**3.7**	**3.2**	**15.64**	**98.51**	**96**	**1.42**	**1.36**	**1.05**	**1.03**
150	**44.3**	**0.17**	**1**	**230**	**10**	**0.17**	**3.7**	**3.2**	**5.21**	**63.80**	**73.3**	**0.55**	**0.58**	**0.94**	**0.87**
150	**44.3**	**0.34**	**2**	**230**	**10**	**0.17**	**3.7**	**3.2**	**10.43**	**83.31**	**82.6**	**0.92**	**0.54**	**1.08**	**1.01**
150	**44.3**	**0.51**	**3**	**230**	**10**	**0.17**	**3.7**	**3.2**	**15.64**	**102.81**	**115**	**1.30**	**0.94**	**1.11**	**0.89**
μ														0.98	0.99
σ														0.11	0.09
C.%														11.2	9.0

Table 6. Comparison between the experimental and predicted results of circular confined concrete by (GFRP) for (proposed model)

D mm	f'_{co} Mpa	t mm	n	E Gpa	f'_{frp} (%)	f'_{co} (%)	k	k_2	f'_l exp Mpa	f'_{cc} theo Mpa	f'_{cc} exp Mpa	ε'_{cc} theo (%)	ε'_{cc} exp (%)	ε'_{cc} theo $/\varepsilon'_{cc}$ exp	f'_{cc} theo $/f'_{cc}$ exp
Touhari and Mitiche-Kettab (Touhari & Mitiche-Kettab, 2016)															
160	26.2	0.17	1	76	19	2.67	3	60	4.83	37.52	38.3	13.73	15	0.92	0.98
160	26.2	0.17	1	76	19	2.67	3	60	4.71	37.04	34.6	13.46	12.6	1.07	1.07
160	26.2	0.17	1	76	19	2.67	3	60	4.87	37.68	38	13.82	13.9	0.99	0.99
160	26.2	0.34	2	76	19	2.67	2.1	60	1.86	15.97	30.2	6.93	6.81	1.02	0.53
160	26.2	0.34	2	76	19	2.67	3	60	9.42	47.88	49.4	24.24	24.1	1.01	0.97
160	26.2	0.34	2	76	19	2.67	3	60	9.75	49.20	52.5	25.00	25.5	0.98	0.94
160	26.2	0.51	3	76	19	2.67	3	60	13.71	57.04	62.8	34.06	33.9	1.00	0.91
160	26.2	0.51	3	76	19	2.67	3	60	12.7	53.00	56.4	31.75	29.8	1.07	0.94
160	26.2	0.51	3	76	19	2.67	3	60	12.6	52.60	54.7	31.52	28.9	1.09	0.96
160	42.6	0.17	1	76	19	2.89	3	60	4.66	53.24	56.5	9.45	11	0.86	0.94
160	42.6	0.17	1	76	19	2.89	3	60	4.55	52.80	55.5	9.30	10.4	0.89	0.95
160	42.6	0.17	1	76	19	2.89	3	60	5.31	55.84	59.8	10.37	12.3	0.84	0.93
160	42.6	0.34	2	76	19	2.89	3	60	9.52	64.68	68.5	16.30	16.5	0.99	0.94
160	42.6	0.34	2	76	19	2.89	3	60	9.6	65.00	70	16.41	17.2	0.95	0.93
160	42.6	0.34	2	76	19	2.89	3	60	9.77	65.68	71.7	16.65	18.1	0.92	0.92
160	42.6	0.51	3	76	19	2.89	3	60	13.6	73.00	75.5	22.04	21	1.05	0.97
160	42.6	0.51	3	76	19	2.89	3	60	14.6	77.00	78.8	23.45	24.9	0.94	0.98
160	42.6	0.51	3	76	19	2.89	3	60	13.9	74.20	77.5	22.47	22.4	1.00	0.96
160	61.7	0.17	1	76	19	3.11	3	65	4.2	70.50	69.4	7.53	8.85	0.85	1.02
160	61.7	0.17	1	76	19	3.11	3	65	4.71	72.54	73.1	8.07	9.37	0.86	0.99
160	61.7	0.17	1	76	19	3.11	3	65	5.2	74.50	77.5	8.59	11.1	0.77	0.96
160	61.7	0.34	2	76	19	3.11	3	65	9.78	84.82	80.8	13.41	14.9	0.90	1.05

(Continued)

Table6. (*Continued*)

D mm	f'_{co} *Mpa*	*t mm*	*n*	*E Gpa*	f'_{frp} *(%)*	f'_{co} *(%)*	*k*	k_2	f'_l *exp Mpa*	f'_{cc} *theo Mpa*	f'_{cc} *exp Mpa*	ε'_{cc} *theo (%)*	ε'_{cc} *exp (%)*	ε'_{cc} *theo* $/\varepsilon'_{cc}$ *exp*	f'_{cc} *theo* $/f'_{cc}$ *exp*
160	61.7	0.34	2	76	19	3.11	3	65	9.23	82.62	76.7	12.83	13.5	0.95	1.08
160	61.7	0.34	2	76	19	3.11	3	65	9.67	84.38	78	13.30	14.4	0.92	1.08
160	61.7	0.51	3	76	19	3.11	3	65	13.1	90.10	90.1	16.91	17.1	0.99	1.00
160	61.7	0.51	3	76	19	3.11	3	65	13.8	92.90	92.1	17.65	18.8	0.94	1.01
160	61.7	0.51	3	76	19	3.11	3	65	14.6	96.10	94.4	18.49	19.5	0.95	1.02
Jiang and Teng J (Jiang & Teng, 2007)															
150	33.1	0.17	1	80.1	23	0.30	3	6	4.12	41.58	42.4	1.05	1	1.05	1.06
150	33.1	0.17	1	80.1	23	0.30	3	6	4.12	41.58	41.6	1.05	1.29	0.81	1.06
150	45.9	0.17	1	80.1	23	0.24	3	5.5	4.12	54.38	40.5	0.73	0.81	0.90	1.08
150	45.9	0.17	1	80.1	23	0.24	3	5.5	4.12	54.38	40.5	0.73	1.06	0.69	0.91
150	45.9	0.34	2	80.1	23	0.24	2.1	5.5	8.24	55.44	52.8	1.23	1.12	1.10	0.87
150	45.9	0.34	2	80.1	23	0.24	2.1	5.5	8.24	55.44	55.2	1.23	1.25	0.98	0.95
150	45.9	0.51	3	80.1	23	0.24	2.6	5.5	12.36	66.41	64.6	1.72	1.55	1.11	1.15
150	45.9	0.51	3	80.1	23	0.24	2.6	5.5	12.36	66.41	55.9	1.72	1.68	1.02	1.10
Cui and Sheikh (Cui & Sheikh, 2010)															
150	47.8	1.25	2	22	23	0.22	2.1	5.5	8.32	57.59	59.1	1.18	1.35	0.87	0.97
150	47.8	1.25	2	22	23	0.22	2.1	5.5	8.32	57.59	59.8	1.18	1.15	1.03	0.96
150	47.8	2.5	4	22	23	0.22	3.4	5.5	16.65	89.04	88.9	2.14	2.21	0.97	1.00
150	47.8	2.5	4	22	23	0.22	3.4	5.5	16.65	89.04	88	2.14	2.21	0.97	1.01
150	47.8	3.75	6	22	23	0.22	3.6	5.5	24.97	114.65	113	3.09	2.85	1.08	1.01
150	47.8	3.75	6	22	23	0.22	3.6	5.5	24.97	114.65	112	3.09	2.94	1.05	1.02
Teng et. all (Teng et al., 2007)															
152	39.6	0.17	1	80.1	23	0.26	0.75	5.5	4.1	38.78	37.2	0.83	0.94	0.88	1.04

(*Continued*)

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Table6. (*Continued*)

D mm	f'_{co} Mpa	*t* mm	*n*	*E* Gpa	f'_{frp} (%)	f'_{co} (%)	*k*	k_2	f'_L exp Mpa	f'_{cc} theo Mpa	f'_{cc} exp Mpa	ε'_{cc} theo (%)	ε'_{cc} exp (%)	ε'_{cc} theo $/\varepsilon'_{cc}$ exp	f'_{cc} theo $/f'_{cc}$ exp
152	39.6	0.17	1	80.1	23	0.26	0.75	5.5	4.1	38.78	38.8	0.83	0.83	1.00	1.00
152	39.6	0.34	2	80.1	23	0.26	2.1	5.5	8.21	49.05	54.6	1.40	1.77	0.79	0.90
152	39.6	0.34	2	80.1	23	0.26	2.1	5.5	8.21	49.05	56.3	1.40	1.83	0.77	0.87
152	39.6	0.51	3	80.1	23	0.26	2.6	5.5	12.32	59.96	65.7	1.97	2.56	0.77	0.91
152	39.6	0.51	3	80.1	23	0.26	2.6	5.5	12.32	59.96	60.9	1.97	1.79	1.10	0.98
μ														0.95	0.97
σ														0.10	0.09
C.%														10.5	9.3

concrete, stiffness of the jacket in the lateral direction, concrete dilation ratio, and heating and cooling cycles etc. However, in this study new equations are proposed to predict the ultimate compressive strength and the ultimate strain based on modifications to previous equations taking into consideration some of the most important parameters.

- For the ultimate compression strength, a new factor is taking into consideration, namely, the number of FRP layers (n). This factor has been incorporated in Equations (19 and 20) to account for the reduction in the tensile modulus of the FRP due to the composite action of the lamina. This will consequently reduce the effective lateral confining pressure and obviously affect the prediction of the ultimate compressive strength.
- For the ultimate strain (Eq. 21), the k_2 factor was modified to vary with the value of unconfined ultimate axial strain while it is usually take constant in other studies.
- The results of the proposed equations was compared with the experimental results both for CFRP and GFRP cases and it has been shown that the proposed equations should very good agreement with test results in comparison to previous equation. The coefficient of variation are shown to be within (9–11.2) in comparison to previous equations were (12–64).

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