Finite element analysis and optimization of bonded post-tensioned concrete slabs

Abbas H. Mohammed1*, Nildem Tayşi1, Dia Eddin Nassani2 and Ali K. Hussein1

Abstract: Optimization techniques may be effective in finding alternative design of post-tensioned slabs to improve their mechanical behaviour, particularly reducing the bending moments. Post-Tensioned (PT) concrete one-way slabs are one of the widely used slabs because of their good performance and cost effectiveness compared with other slab types. The objective of this study is to develop three-dimensional Finite Element (FE) model for the optimization of bonded PT concrete one-way slab. The FE software package ANSYS was used to find the optimum strain energy and area of PT tendons. The interface between PT tendon and the surrounding concretes was also modeled, allowing the tendon to keep its shape during slab deformation. Contact algorithm and material nonlinearity were considered in the FE model. The total strain energy of bonded PT concrete slab was considered as the objective function. The design variables are the area of PT tendons, the initial stress in tendons and the eccentricity of the tendon profile while the constraints are the normal stress in concrete, the stress in steel tendon, the shear stress in concrete and the displacement at mid-span of the slab. The optimization results indicate that the area of PT tendons may be reduced by approximately 38% using an appropriate optimization algorithm. We conclude that the optimization of area of PT tendons of the post tensioned slab is very important because, it enable us to find the optimal area of PT tendon with low cost and suitable service conditions.
1. Introduction
PT concrete slab is broadly used for residential buildings, hotels, factories and hospitals because of its good performance and cost effectiveness compared with other slab types. The use of post-tensioning system allows for material savings due to reduced slab thickness. Moreover, cracking and deflection of the concrete slabs can be reduced by using post-tensioning. Several design variables and many practical constraints are contributory in the design of PT slab; so that the optimum design of PT slabs is challenging to the designers.

The behavior of PT concrete structural members have previously been investigated experimentally by Williams and Waldron (1989), Yang, Mun, and Kim (2013), Ranzi, Al-Deen, Ambrogi, and Uy (2013, Ranzi, Al-Deen, Hollingum et al., 2013), Bailey and Ellobody (2009), Aimin, Yuli, and Litang (2013), Hussien, Elafandy, Abdelrahman, Abdel Baky, and Nasr (2012), Kim and Lee (2016) and others.

Numerical and theoretical models have been previously developed by other researchers to study the behavior of bonded and unbonded PT concrete members. Kim and Lee (2012) proposed a flexural behavior model for continuous unbonded PT members, which was a nonlinear analysis model that reflects the moment redistribution.

Using the commercial Finite Element (FE) package ANSYS, Fanning (2001) recommended some numerical strategies to model the PT beams without using interface element. Prestressed concrete beams using FE analysis were studied by Kasat and Varghese (2012) to understand the load deflection response and the stress distributions of prestressed concrete beams due to transverse loading. Ellobody and Bailey (2008) investigated the structural behaviour of bonded PT one-way spanning concrete slabs in fire conditions. They developed a nonlinear FE model for the analysis of post-tensioned bonded concrete slabs at elevated temperatures.

Many articles and researches have been published addressing the optimization of concrete structures in recent years (Amir, 2013; Atabay & Gulay, 2009; Bennegadi, Sereir, & Amziane, 2013; Chaitanya Kumar & Venkat, 2013; Peng & Liu, 2013; Sahab, Ashour, & Toropov, 2005a). The techniques of optimization play a significant role in the design of structures, one of the most important objectives of optimization is finding the optimum solutions from which a designer can gain an utmost advantages from the available resources. There are a huge number of available designs, but the best design should be selected. The optimum design can be in terms of maximum performance, minimum weight, minimum cost or a combination of these (Kumar, 2014).

Krauser (2009) carried out a parametric study on two-way slab designed using post-tensioning methods. The design was carried out by hand calculations through a series of Excel spreadsheets and compared with the results found by the computer program, ADAPT-PT. The designs from the hand analysis and the ADAPT-PT model provided similar results for the PT slab, and both methods provided an appropriate design. The use of ADAPT-PT was recommended because of its simply of use and quick calculation capabilities.

Lounis and Cohn (1993) studied the optimum design of PT slab and a PT highway bridge. The study purpose was to minimize two objective functions, the first was cost and the second was the initial camber. ε-constraint approach was used in the analysis considering one function as the objective function and the other as the constraint function. Optimization performed using projected Lagrangian algorithm while analysis performed using force-in-tendon method and sectional stress analysis. Although several design variables were studied in the design optimization, the method application was limited to simple structures only.
El Semelawy, Nassef, and El Damatty (2012) developed numerical tools that were able to find the optimum design of a pre-stressed concrete slab using modern heuristic search algorithms. Structural analysis of the system conducted by using FE method. Consistent triangular shell elements were used to model the concrete slab which was originally developed by Koziey and Mirza (1995). They investigated the optimum design of a square pre-stressed flat slab carried by four columns at the corners. The design code was the Canadian code CSA A23.3.

Sahab, Ashour, and Toropov (2005b) studied the cost optimization of reinforced concretes flat slab structures. The British Code (BS8110) was chosen as the design code. The objective function was the structure cost (foundations, columns and floors). The structure was modeled and analyzed using the equivalent frame method. It was concluded that with the increase in the number of structural elements, the increase in the cost savings achieved using design optimization. Moreover, floors cost is the maximum part of the total structural cost.

In this paper, three-dimensional FE model was developed using ANSYS to study the optimum design of bonded PT concrete slab. Experimental bonded PT concrete one-way slab from literature (Bailey & Ellobody, 2009) was chosen for numerical analyses verification, and good agreement was obtained between numerical and test results. The optimum Strain Energy (SE) and the area of PT tendons were calculated. ANSYS optimization routines utilize three types of variables that characterize the design optimization process: design variables, constraints, and the objective function. ANSYS Parametric Design Language (APDL) represents these variables by scalar parameters. The use of APDL is a fundamental step in the optimization process.

2. Finite element analysis and optimization

Nowadays FE method is often used to analyze structures and to find the stresses, strains and deformations of a structure subjected to different boundary conditions and loads. In all engineering fields, structural design techniques and especially the methods of numerical optimization have been developed during the last decades. Schmit (1960) explained the design of elastic structures by using the nonlinear programming techniques. Today, many FE codes that have optimization capabilities are usable.

Optimization questions based on FE could be revealed as:

Minimize:

\[ f(x, U) \]

Subject to:

\[ g_i(x, U) \leq 0 \quad i = 1, \ldots, m \]

\[ h_j(x, U) = 0 \quad j = 1, \ldots, l \]

where \( f \) is the objective or cost function, \( h \)'s are equality constraints, \( g \)'s are inequality constraints (the inequality constraints include explicit upper and lower bounds on the design variables). \( x = (x_1, x_2, \ldots, x_n)^T \) is a column vector of \( n \) design variables. \( U \) is a \( \text{ndof} \times 1 \) nodal displacement vector from which the displacement field \( u(x, y, z) \) is easily determined. Where \( \text{ndof} \) indicates the number of freedom degrees in the structure. \( U \) is an implicit function of \( x \) and a partial differential equation expresses the relation between \( U \) and \( x \) as shown:

\[ K(x) U = F(x) \]

where \( F \) is a \( \text{ndof} \times 1 \) load vector and \( K \) is a \( \text{ndof} \times \text{ndof} \) square stiffness matrix (Simpson et al., 2004).
2.1. Finite element modeling

The computer program ANSYS (2012) is used to model and analyze PT concrete one-way slab. SOLID65 (or 3D reinforced concrete solid) is utilized for 3D modeling of concrete, which is capable of crushing in compression and cracking in tension. The element is modeled by 8-nodes having 3-degrees of freedom at each node.

Tendons and reinforcement were represented using 2-node discrete link elements (LINK8), which are included within the properties of 8-node brick elements. The link element is assumed to be capable of transmitting only axial forces. For tendon cable, since it is placed inside the slab section (throw the hole) and PT force is transferred to concrete through end anchorages and profile of tendon, the cable is linked to slab only at the end of anchorages. To prevent stress concentration, steel plates were added at location of loading. SOLID45 is used for 3D modeling of steel plates having 8-nodes with 3-degrees of freedom at each node.

The tendons must be anchored at both ends of the beam members to maintain the internal forces. These forces create high stresses at both ends. To avoid the ends from crushing the shell plates must be applied at ends. SHELL181 (ANSYS, 2012) is suitable for analyzing shell structures. The element is modeled by 4-nodes having 6-degrees of freedom at each node.

The contact between the concrete and the tendon is modeled by contact elements (using the CONTACT PAIR MANAGER). The method requires the definition of two surfaces that are the target and the contact surfaces. The target surface within this model (TARGE170) represents rigid surface which is defined as the concrete surface surrounding the tendon. The contact surface (CONTA175) represents a contact, sliding and deformable surface, which is the tendon surface in this case. This element is located on surface of a 3D rigid element such as the 8-node brick element and has the same geometric characteristics as the rigid element face with which it is connected. The contact elements are formed using these two elements and monitors the displacement of the contact surface in relation to the target surface.

TARGE170 is used to represent various 3-D “target” surfaces for the associated contact elements CONTA175, the contact elements themselves overlay the solid, shell, or line elements describing the boundary of a deformable body and are potentially in contact with the target surface, defined by TARGE170. This target surface discretized by a set of target segment elements TARGE170 and paired with its associated contact surface via a shared real constant set. It can impose any translational or rotational displacement, temperature, and magnetic potential on the target segment element. For rigid target surfaces, these elements can easily model complex target shapes. For flexible targets, these elements will overlay the solid, shell, or line elements describing the boundary of the deformable target body.

The contact between concrete and the tendon was assumed perfect bonded, therefore no need to define the frictional properties for the contact elements.

Material plays a significant role in ANSYS modelling. Real values of material properties should be given as an input in ANSYS. The stress-strain relationship for concrete in tension is almost linearly elastic up to the maximum tensile strength. Then, the concretes start cracking and the strength decreases continuously to zero.

The multi-linear stress-strain relationship is considered for concrete in compression in this study. The adopted stress-strain relation is based on work done by Desayi and Krishnan (1964); as shown in Figure 1(a).

The bilinear stress-strain relationship indicated in Figure 1(b) is considered for reinforcing steel bars in this study. Since the steel bars are slender, it could be assumed that bars transmit only axial force. On the other hand, the strands are considered as multilinear isotropic material in this study.
2.1.1. Validation and application of the numerical model
Bonded PT concrete slab (TB1) tested by Bailey and Ellobody (2009) was chosen for numerical analyses verification. The designated compressive strength of the used concrete was 41.2 MPa. The slab was simply supported at both ends and subjected to four concentrated point loads. PT slab was designed according to BS8110-1. The general layout of the bonded PT one-way concrete is shown in Figure 2(a). The tendons were plain mono-strands with each strand made of seven high-strength steel wires. The tendons had a measured tensile strength of 1,846 MPa, diameter of 15.7 mm and area of 150 mm². The load was applied gradually in equal increments of 5 kN. Three longitudinal ducts have been used in the slab (one in the middle and the other two were on both sides, at a spacing of 530 mm) as illustrated in Figure 2(b). At the middle of the slab height, the duct ends were fixed. The strength of grout was 47 MPa after 28 days. The full applied design PT force to the slabs was 195 kN, with the average measured force in the three tendons being 169 kN, equating to 13% losses (Bailey & Ellobody, 2009).

The behavior of bonded PT concrete slabs is more complicated by reason of the presence of ducts, the bond between concretes and ducts, the bond between ducts and grout, and the bond between grout and tendons. Two stages of analysis are needed for bonded slabs. At the first stage, the slab is analyzed as unbonded PT slab under prestress loading and gravity loading only. Next, the slab is analyzed as bonded PT slab. The two stages are needed because the grout is added after the prestressing of the strands.
In this paper, the bonded PT concrete slab was modeled using 3D solid elements available within ANSYS (2012). Because of symmetry, only one-quarter of the slab was modeled with 11,100 elements, including the interface elements. Load applications and boundary conditions were the same as that utilized in the test. The measured post-tensioning force in the tendons (169 kN) was initially applied in a separate step. The dead load representing the weight of the slab was applied as a static body load.

Bailey and Ellobody (2009) observed that there was no slip at the interface between the duct and grout as well as at the interface between the duct and concretes. Hence, only the contact between the tendon and grout was modeled utilizing interface elements (using the contact pair option) available within the ANSYS element library. The interface elements composed of two matching contact faces from the tendon elements and surrounding grout elements. The FE mesh of the concrete and PT strand for a quarter of the slab TB1 are shown in Figure 3. The material properties adopted in the analysis are given in Table 1.

The current analysis of a one way slab is a static nonlinear analysis under vertical live, dead and prestress loads. In nonlinear FE analysis, the total load applied to a FE model is divided into a series of load increments called load steps. The analysis is carried out up to failure, thus it enables determination of failure load. When each load step increments are completed, the stiffness matrix is adjusted to reflect the nonlinear changes in the structural stiffness before the next load increment step is proceeding. For updating the model stiffness ANSYS uses the Newton-Raphson equilibrium iterations. Convergence criteria were based on displacement are used for the reinforced concrete solid elements, and the convergence tolerance limit is 5% in order to obtain convergence of the solutions. The load increment increase up to a maximum load step size if the convergence behaviour is smooth and the load increment bisect until it is equal to a minimum load step size if the convergence behaviour is abrupt.

**Table 1. Material properties of concrete, reinforcement and strand for bonded slab**

<table>
<thead>
<tr>
<th>Property</th>
<th>Concrete</th>
<th>Steel plate</th>
<th>Strand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate compressive strength, ( f_{cu} ) (MPa)</td>
<td>41.2</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Ultimate tensile strength, ( f_t ) (MPa)</td>
<td>3.0</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Modulus of elasticity, ( E ) (MPa)</td>
<td>30,000</td>
<td>200,000</td>
<td>202,000</td>
</tr>
<tr>
<td>Poisson’s ratio, ( \nu )</td>
<td>0.2</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>–</td>
<td>400</td>
<td>1,846</td>
</tr>
</tbody>
</table>
The results observed from the FE analysis in terms of the ultimate loads and load-central deflection curves were compared against the test results (Bailey & Ellobody, 2009). Figure 4 represents the load–deflection curve obtained from FE analysis and experimental results for the slab TB1, and good agreement can be noticed from the figure. The failure load and central deflection obtained from the experiment were 216 kN and 120 mm respectively, while the failure load and central deflection obtained from FE analysis were 211 kN and 161 mm respectively. The failure load predicted from FE analysis was 2.3% less than that obtained from the experiment.

2.2. Optimization strategy

Many studies concerning the design optimization of concrete structural elements were published (Holko & Dicky, 2015; Zhang & Bai, 2011). Few studies were concerned with the design of PT concrete slabs. In this study, FE analysis is applied to optimize the area of tendons used to reinforce PT slab subjected to static loading. Optimization flow chart is given in Figure 5, it was noticed that in ANSYS package (ANSYS, 2012) that two optimization methods are available: the first order method and
sub-problem approximation method. The first-order method utilizes gradients of the dependent variables with respect to design variables. The sub-problem approximation method could be explained as an advanced zero-order method that it needs only the dependent variable values, and not their derivatives. Three types of variables characterize the design process in both optimization methods: objective function, constraints and design variables.

The aim of the optimization procedures is to minimize the SE of the slab while satisfying all relevant strength and serviceability limit states mandatory according to the design code. The optimization problem will be defined as:

Minimize:

\[ SE = \frac{1}{2} \int \sigma \varepsilon \, dV \]  

Subjected to:

\[ g_i(x) \leq 0 \quad i = 1, \ldots, m \]  
\[ h_j(x) = 0 \quad j = 1, \ldots, l \]

where SE is the objective function, \( \sigma \) is the stresses, \( \varepsilon \) is the strains, \( g_i \) is the inequality constraints, \( h_j \) is the equality constraints and \( x \) is the design variable \( (x_1, x_2, \ldots, x_n) \). The design variables are as \( x_1 \), which is the area of PT tendons, \( x_2 \), which is the initial stress in tendons and \( x_3 \), which is the eccentricity of the tendon profile at mid-span of the slab. The constraints are defined as follows:

- Maximum normal stress in concrete.
- Maximum normal stress in steel tendons.
- Maximum shear stress in concrete.
- Maximum displacement at mid-span of the slab.

2.2.1. Strain energy minimization of bonded post-tensioned slab

In this part, the Bonded Post-Tensioned (BPT) one-way concrete slab is optimized. The dimension of BPT slab is shown in Figure 6, and the loading considered for the proposed study is shown in Figure 7.

The slab is designed to behave as a fully PT concrete member, reinforced with fully bonded PT tendons with no regular rebar. The applied loads are the gravitational dead and static point loads. There are no lateral loads considered in the design. Pre-stressing was considered as a variable in the optimization process.

Design optimization was entirely conducted using the APDL. By adding penalty functions to the objective function, the optimization problem with constraints will be converted to unconstrained problem. For each iteration, various steepest descent and conjugate direction searches are performed until convergence is reached to determine a search direction. A line search strategy is adopted to minimize the unconstrained optimization problem. SE minimization and effect of area of tendons are investigated.

The objective of the design optimization procedures developed herein is to minimize the SE of the slab. The design variables are the area of PT tendons, the initial stress in tendons and the eccentricity of the tendon profile at mid-span of the slab. Behavioral constraints dictated by the ACI 318 are explained in Table 2.
To minimize the SE, it is necessary to respect the imposed constraints conditions: normal stress in concrete, stress in steel tendon, shear stress in concrete and displacement at mid-span of the slab. These parameters are evaluated according to the lower and upper limits of the design variables given in Table 3.

Table 2. Constraints specified by the ACI 318

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Compression stresses in concrete, $f_c$</td>
</tr>
<tr>
<td>2</td>
<td>Tension stresses in concrete, $f_t$</td>
</tr>
<tr>
<td>3</td>
<td>Stresses in steel tendons, $f_{tendon}$</td>
</tr>
<tr>
<td>4</td>
<td>Shear stress, $V_r$</td>
</tr>
<tr>
<td>5</td>
<td>Displacement at mid-span of the slab, $U_y$</td>
</tr>
</tbody>
</table>

Notes: Where $f'_c$ is the specified compressive strength of concrete; $f_c$ is the compression stresses in concrete (obtained from FE analysis); $f_t$ is the tensile stresses in concrete (obtained from FE analysis); $f_{pu}$ is the specified tensile strength of PT tendons; $f_{tendon}$ is the stresses in PT tendon (obtained from FE analysis); $V_r$ is the factored shear stress resistance; $V_f$ is the factored shear stress; $U_y$ is the displacement at mid-span of the slab (obtained from FE analysis) and $l$ is the span length.

To minimize the SE, it is necessary to respect the imposed constraints conditions: normal stress in concrete, stress in steel tendon, shear stress in concrete and displacement at mid-span of the slab. These parameters are evaluated according to the lower and upper limits of the design variables given in Table 3.

After the calibration of our numerical model and clarification of the optimization strategy, minimization of the SE of the bonded PT one-way slab can be performed. To establish SE optimization, a random design iterations can be carried out against the material parameters limits. The initial data from the random design calculations can serve as starting points to feed the optimization methods described. Figure 8 shows the evolution of optimal SE of slab vs. the number of iterations for subproblem approximation method. From this figure, three distinctive zones can be recognized. The first zone is located between the first and the 6th iterations, which is characterized as a relatively high SE, close to initial SE (6,262 Nm). The second zone is the incertitude zone located between the 6th and 12th iterations, and finally the feasible zone in which we have a stable value, which is 3,865 Nm. Using this optimization, the SE is reduced by almost 38%.
In Figure 9(a) the optimized area of PT tendons is plotted against the number of iterations. Figure 9(a) has the same form as the SE of the slab. A significant reduction in the area of PT tendons from 0.00023 m² for the initial state to 0.00018 m² for the converged solution is noticed. This reduction is significant, which (approximately equals 22% of the initial area). Figure 10 shows the evolution of mid-span deflection. The mid-span deflection at optimum design ($U_y = 3.2$ mm) is smaller than at initial design ($U_y = 4.75$ mm), which is attributed to the increase the eccentricity.

The evolution of maximum compression stresses in concrete are plotted in Figure 11(a). It is noted the maximum compression stress in concrete at anchorage is decreased during the optimization iteration that is due to reduction in the prestressed force. A significant reduction in maximum compression stress at anchorage from 23.5 MPa for the initial state to 15.5 MPa for the converged solution is noticed. The concentration of PT force leads to increase the compression stress in concrete at anchorage. These high values of stresses directly affect the values of SE. Figure 11(b) shows the evolution of maximum tension stress in concrete. It is evident that the maximum tension stress is decreasing during the optimization iteration. The increase in eccentricity leads to this decrease in tension stress.

The initial and optimum design values for the SE optimization are shown in Table 3. Optimization values for design variables and constraints are within the limits. From Table 3, it can be observed that the maximum tensile stress is decreased from 3.5 MPa for the initial state to 2.69 MPa for the optimum design state. From this table, it is obvious that the maximum tensile stress controls the

<table>
<thead>
<tr>
<th>Table 3. Initial, optimum and limits of design variables and constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Objective SE (Nm)</td>
</tr>
<tr>
<td>Design variables</td>
</tr>
<tr>
<td>$a$ (m²)</td>
</tr>
<tr>
<td>$i$ (MPa)</td>
</tr>
<tr>
<td>$e$ (m)</td>
</tr>
<tr>
<td>Constraints</td>
</tr>
<tr>
<td>Max. $f_c$ (MPa)</td>
</tr>
<tr>
<td>Max. $f_t$ (MPa)</td>
</tr>
<tr>
<td>Max. $f_{tendon}$ (MPa)</td>
</tr>
<tr>
<td>Max. $V_r$ (MPa)</td>
</tr>
<tr>
<td>Max. $U_y$ (mm)</td>
</tr>
</tbody>
</table>

Notes: Where $a$ is the area of PT tendons, $i$ is the initial stress in tendons and $e$ is the eccentricity of the tendon profile.
It is found that the optimum design value for the maximum tensile stress (2.69 MPa) is closed with the upper limit (2.7 MPa).

In addition, it can be noted that the smallest reduction occurs in initial stress in tendons, which is about 2.5% while the highest reduction occurs in the SE.

After optimization a new design of the retrofitting BPT slab is obtained because the area of PT tendons is reduced. Figure 12 give a comparison of the numerical load vs. deflection curves of the post tensioned slab before and after optimization. It is obvious that the controlled load of the
optimized post tensioned slab is relatively augmented compared to the slab before the optimization. At the maximum mid span displacement, the ultimate load for the controlled slab after optimization was found to be 111 kN against 122 kN for the not optimized slab. Thus a reduction of the controlled load which is 9% compared to the area of PT tendons optimization (almost 22%). From figure, we conclude that the optimization of area of PT tendons of the post tensioned slab is very important because, it enable us to find the optimal area of PT tendon with low cost and suitable service conditions.

2.2.2. Effect of the applied load on the optimization
To study the effect of the magnitude of applied load on the optimum values for BPT slab, the SE optimization for the BPT slab is redesigned by specifying the applied total static loading to $p = 55$, $p = 65$ and $p = 75$ kN. The loading of the BPT slab is shown in Figure 13. The objective, design variables and constraints developed herein are the same as those in Section 2.2.1. Behavioral constraints dictated by the ACI-318 code are explained in Table 2 and are evaluated according to the lower and upper limits of the design variables and constraints given in Table 3 for the maximal loading.

To establish SE optimization, a random design iterations can be performed against the material parameters limits.

The initial and optimum design values for the SE optimization for the BPT slab are shown in Table 4. From Table 4, it can be observed that the value of the optimum area of tendon depends on the magnitude of applied load. From this table, it can be found that the optimum value of the area of tendons

<table>
<thead>
<tr>
<th>Total applied load (kN)</th>
<th>Initial values</th>
<th>Optimum value</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$SE$ (Nm)</td>
<td>$a$ (mm$^2$)</td>
<td>$i$ (MPa)</td>
</tr>
<tr>
<td>55</td>
<td>6,106</td>
<td>230</td>
<td>1,200</td>
</tr>
<tr>
<td>65</td>
<td>6,262</td>
<td>230</td>
<td>1,200</td>
</tr>
<tr>
<td>75</td>
<td>6,312</td>
<td>230</td>
<td>1,200</td>
</tr>
</tbody>
</table>
increases from 141 mm² for the applied loading $p = 55$ kN to 215 mm² for the applied loading $p = 75$ kN. The optimum area is increased when the applied load is increased.

The increases of the applied load on the one-way slab leads to increase in the tensile stress in concrete due to the increase in the positive bending moment and to keep the level of tensile stress in allowable limits it requires to increase the negative bending moment. For the increases of the negative bending moment it requires to increase in the area of tendons or the initial stress in tendons.

2.2.3. Effect of number of applied point loads on the optimization

To study the effect of a number of applied point loads on the optimum values for BPT slab, the SE optimization for the BPT slab is redesigned by applying four types of point loading. The four types of loading are one, two, three and four point loads. The loading of the BPT slab is shown in Figure 14. The objective, design variables and constraints developed herein are the same that in Section 2.2.1. Behavioral constraints dictated by the ACI-318 code are explained in Table 2 and are evaluated according to the lower and upper limits of the design variables given in Table 3 for the maximal loading.

Table 5 shows the initial and the design optimum values for BPT slab for different types of loading. From Table 5, it can be noted that the optimum values of SE and area of tendons are affected by the number of applied point loads. From Table 5, it can be found that the optimum value of the area of tendons decreases from 180 mm² for 1 point loading to 60 mm² for 4 point loading when the total applied load is kept constant. From this table, it is evident that the critical type of loading is when the load is applied near the mid-span of the bonded PT slab.

Figure 14. Loading of BPT slab for the number of applied point load optimization.
When the load is applied near the mid-span of the one-way slab leads to increase in the tensile stress in concrete due to the increase in the positive bending moment and because the maximum tensile stress controls the optimization convergence must keep the level of tensile stress in allowable limits. For the increases of the negative bending moment it requires to increase in the area of tendons or the initial stress in tendons.

3. Conclusions
In this paper, optimization of the bonded PT one-way concrete slab is studied. Experimental bonded PT concrete from literature was chosen for numerical analyses verification, and good agreement was achieved between numerical and test results.

This research focused on the optimization of the PT concrete slab. The objective of the optimization procedures developed herein is to minimize the SE of the slab which leads to a reduction in the area of the PT tendons and consequently decreases the construction cost. As the result, the SE was reduced approximately 38%. A significant reduction in the area of PT tendons was also reached which was approximately 22%.

The mid-span deflection and maximum tension stress were decreased during the optimization iterations within the allowable limits due to the increase the eccentricity. It was also noticed that the optimum area has increased when the magnitude of applied load is increased.

It is obvious the ultimate load for the controlled BPT slab after optimization was found to be 111 kN against 122 kN for the not optimized BPT slab. Thus a reduction of the controlled load which is 9% compared to the area of PT tendons optimization (almost 22%). We conclude that the optimization of area of PT tendons of the post tensioned slab is very important because, it enable us to find the optimal area of PT tendon with low cost and suitable service conditions.

The optimum values of SE and the area of tendons are affected by the number and location of the applied point loads. It is obvious that the critical type of loading is when the load is applied near the mid-span of the bonded PT slab.

### Table 5. Initial and design optimization values for BPT slab for different type of loading

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Initial values</th>
<th></th>
<th>Optimum value</th>
<th></th>
<th>Reduction</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SE (Nm)</td>
<td>a (mm²)</td>
<td>i (MPa)</td>
<td>e (m)</td>
<td>SE (Nm)</td>
<td>a (mm²)</td>
</tr>
<tr>
<td>1 point loading</td>
<td>6,262</td>
<td>230</td>
<td>1,200</td>
<td>0.005</td>
<td>3,865</td>
<td>180</td>
</tr>
<tr>
<td>2 points loading</td>
<td>6,148</td>
<td>230</td>
<td>1,200</td>
<td>0.005</td>
<td>2,180</td>
<td>101</td>
</tr>
<tr>
<td>3 points loading</td>
<td>6,135</td>
<td>230</td>
<td>1,200</td>
<td>0.005</td>
<td>2,115</td>
<td>99</td>
</tr>
<tr>
<td>4 points loading</td>
<td>6,120</td>
<td>230</td>
<td>1,200</td>
<td>0.005</td>
<td>1,232</td>
<td>60</td>
</tr>
</tbody>
</table>

When the load is applied near the mid-span of the one-way slab leads to increase in the tensile stress in concrete due to the increase in the positive bending moment and because the maximum tensile stress controls the optimization convergence must keep the level of tensile stress in allowable limits. For the increases of the negative bending moment it requires to increase in the area of tendons or the initial stress in tendons.

### Funding
The authors received no direct funding for this research.

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