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Interior search algorithm for optimal power flow with non-smooth cost functions

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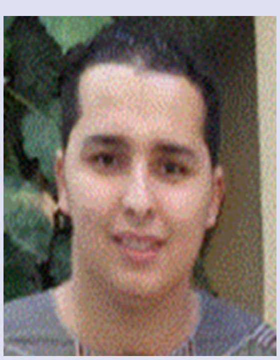
Abstract: The optimal power flow (OPF) problem considers as an optimization problem, in which the utility strives to reduce its global costs while pleasing all of its constraints. Therefore, Interior Search Algorithm (ISA) is applied to treat this problem. Where, ISA is a specific type of artificial intelligence and a mathematical programming, not a meta- heuristic algorithm. The effectiveness of this method in solving the OPF problem is evaluated on two test power systems, the IEEE 30-bus and the IEEE 57-bus test systems. For the first example, the ISA-OPF algorithm finds an answer that agrees with published results. For the 57-bus system, the ISA-OPF demonstrates its ability to transact with larger systems. Thus, the ISA-OPF algorithm is shown to be a robust tool to treat this optimization compared with other methods. Moreover, the advantage of ISA is that it has only one parameter of control which makes the simplicity in the main algorithm.

Subjects: Artificial Intelligence; Power & Energy; Systems & Control Engineering; Electrical & Electronic Engineering

Keywords: interior search algorithm; optimal power flow; valve point effect; prohibited zones

1. Introduction

Optimization techniques are becoming the inquiry of many researchers especially that the efficiency of a particular system relies on gaining an optimal solution, which is achieved by a good optimization. The optimization is a process that discovers the best solution after evaluating the cost function which indicates the relationship between the system parameters and constraints.



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ABOUT THE AUTHOR

Bachir Bentouati was born in Laghouat, Algeria, on August 22, 1990. He received license and master degrees in Electrotechnic and Electrical Power System in 2010 and 2012 respectively from Laghouat University. He is currently a PhD student at the Department of Electrical Engineering and Member in LACoSERE Laboratory, University of Laghouat, Algeria. His areas of research include optimal power flow, Artificial intelligence and optimization techniques.

PUBLIC INTEREST STATEMENT

Recently, the power system has been forced to employ on its filled ability due to the urban and economic expansion that may compel strict confines on the power transfer capability. This fear recalls building novel transmission lines with their redundancy and productions units. Consequently, the planning and operation of transport networks become more complex due to the rapid growth of electricity demand, integrations of networks and movement toward the electricity markets open. Traditional optimal power flow (OPF) has provided a tool to achieve such a task and has the first treaty the cost of fuel only. Later other objectives have been incorporated in the OPF in the form of a single objective.

In the electrical power systems, the Optimal Power Flow (OPF) problem is considered as a static non-linear, which is one of the most important operational functions of the modern energy management system. It has been used as a tool to define the level of the inter-utility power exchanges. The OPF approach has gained more importance due to the increased availability of control devices and energy prices. Since its starting point, it has proven its efficiency in dealing with various issues. The OPF can represent the same objective as a scheduling problem such as minimization of the operation cost of power systems and the thermal unit cost as well. It has been considered as an optimization process to minimize or maximize a certain objective functions of the power system while satisfying system constraint as well as minimizing the fuel cost which is the main goal of the current study.

The OPF problem has been studied for more than 60 years and many algorithms have been designed to solve the base of OPF problem after Carpentier (1962) that has defined the OPF and its derivative problems in 1962. Current algorithms for solving the OPF can be classified into the following categories: sequential algorithms (Berry & Curien, 1982), nonlinear programming algorithms (Tanaka, Fukushima, & Ibaraki, 1988) and intelligent search methods. The first two categories still have some shortcomings, which led to the appearance of several OPF algorithms based on tabu search (TS) (Abido, 2002a), black-hole-based optimization (BHBO) (Bouchekara, 2014), moth-flam optimizer (Bentouati, Chaib, & Chettih, 2016), ant colony optimization (ACO) (Soares, Sousa, Vale, Morais, & Faria, 2011), krill herd algorithm (KHA) (Mukherjee & Mukherjee, 2015), differential evolution algorithm (DE) (El & Abido, 2009), backtracking search algorithm (BSA) (Ula¸s Kılıç, 2014), multi-verse optimizer (Bentouati, Chettih, Jangir, & Trivedi, 2016) and ant-lion optimizer (Trivedi, Jangir, & Parmar, 2016) and teaching–learning-based optimization (TLBO) (Bouchekara, Abido, & Boucherma, 2014). however, such traditional methods could not achieve the desired objectives due to several factors. Algorithm is used. many optimization strategies have been incorporated into the basic algorithm, such as chaotic theory (Wang, Guo, Gandomi, Hao, & Wang, 2014; Wang, Hossein Gandomi, & Hossein Alavi, 2013), Stud (Wang, Gandomi, & Alavi, 2014), quantum theory (Wang, Gandomi, Alavi & Deb, 2016a), Lévy flights (Guo, Wang, Gandomi, Alavi, & Duan, 2014; Wang et al., 2013), multi-stage optimization (Wang, Gandomi, Alavi & Deb, 2016b), and opposition based learning (Wang, Deb, Gandomi, & Alavi, 2016). many other excellent metaheuristic algorithms have been proposed, such as monarch butterfly optimization (MBO) (Feng, Wang, Deb, Lu, & Zhao, 2015; Feng, Yang, Wu, Lu, & Zhao, 2016), earthworm optimization algorithm (EWA) (Wang, Deb, & Coelho, 2015), elephant herding optimization (EHO) (Wang, Deb, Gao, & Coelho, 2016), moth search (MS) algorithm (Wang, 2016).

All artificial intelligent techniques that mentioned above are meta-heuristic from nature. Apart from common control parameters (like population size and maximum iteration number), several specific parameters of algorithm (like inertia weight, social and cognitive parameters in the case of PSO, crossover rate in the case of GA and DE, limit in the case of ABC, etc.) are required for proper execution of all meta-heuristic based techniques. For effective application of any metaheuristic, it is crucial to tune the algorithm-specific parameters for a specific problem. This tuning procedure requires the trial and error based simulation which need rigorous computational effort. If the number of algorithm-specific parameters increases, the tuning problem of these parameters further becomes complex and time consuming. Many times, this tuning becomes more tedious than the problem itself.

Up to 2014, a new method known as the Interior Search Algorithm (ISA) has been proposed by Gandomi (2014) is inspired by interior design and decoration. ISA has just one tuned parameter which is α, this property is mainly one of the ISA advantages. So, in this paper, an optimization approach based on ISA followed by its mathematical model is proposed to solve the OPF problem for the aim of minimizing the cost of fuel, taking into account multi-fuels options, valve-point effect and other complexities as well as to other different objectives such as voltage profile improvement.

To fulfill the task achievement, the ISA method is simulated and tested on the IEEE 30-bus and the IEEE 57-bus test systems. The obtained results are compared with other methods that have been already done and others recent works.

The organization of this paper is offered as follows: Section 2 discusses the formulation of OPF problem followed by a brief description of ISA that illustrated in Section 3. Section 4 presents the simulation results and discussion. Finally, Section 5 states the conclusion of this paper.

2. The formulation of OPF problem

The mathematical formulation of the OPF problem can be stated as a nonlinearly constrained optimization problem:

$$F(x, u) \tag{1}$$

$$G(x, u) = 0 \tag{2}$$

$$H(x, u) \leq 0 \tag{3}$$

Equations (4) and (5) give respectively the vectors of control variables "u" and state variables "x" of the problem of OPF:

$$u = [P_g, V_g, T_c, Q_c] \tag{4}$$

where P_g active power generator output at PV buses except at the slack bus, V_g voltages Generation bus T_c transformer taps settings and Q_c shunt VAR compensation.

$$x = [V_L, \theta, P_s, Q_s] \tag{5}$$

where V_L voltage profile to load buses θ argument voltages of all the buses, except the beam node (slack bus) P_s active power generated to the balance bus (slack bus), Q_s reactive powers generated of generators buses.

2.1. The constraints

The OPF constraints are divided into equality and inequality constraints. The equality constraints are power/reactive power equalities, the inequality constraints include bus voltage constraints, generator reactive power constraints, reproduced with reactive source reactive power capacity constraints and the transformer tap position constraints, etc. Therefore, all the above objectives functions are subjected to the below constraints:

2.1.1. Equality constraints

Ties constraints of the OPF reflect the physical system of electrical energy. They represent the flow equations of active and reactive power in an electric network, which is represented respectively by Equations (6) and (7):

$$P_{gi} - P_{di} - V_i \sum_{j=1}^{N} V_j (g_{ij} \cos \delta_{ij} + b_{ij} \sin \delta_{ij}) = 0 \tag{6}$$

$$Q_{gi} - Q_{di} - V_i \sum_{j=1}^{N} V_j (g_{ij} \sin \delta_{ij} - b_{ij} \cos \delta_{ij}) = 0 \tag{7}$$

g_{kj}, b_{kj} elements of the admittance matrix (conductance and susceptance respectively).

2.1.2. Inequality constraints

$$P^g_{i,\min} \le P^g_i \le P^g_{i,\max} \quad i = 1 \ldots n_g \tag{8}$$

$$Q^g_{i,\min} \le Q^g_i \le Q^g_{i,\max} \quad i = 1 \ldots n_g \tag{9}$$

$$V^g_{i,\min} \le V^g_i \le V^g_{i,\max} \quad i = 1 \ldots n_g \tag{10}$$

$$Q^{sh}_{i,\min} \le Q^{sh}_i \le Q^{sh}_{i,\max} \quad i = 1 \ldots n_{sh} \tag{11}$$

$$T^g_{i,\min} \le T^g_i \le T^g_{i,\max} \quad i = 1 \ldots n_T \tag{12}$$

where $P^g_{i,\min}$, $P^g_{i,\max}$, $Q^g_{i,\min}$ and $Q^g_{i,\max}$ are the maximum active power, minimum active power, minimum reactive power and maximum reactive power of the *ith* generation unit respectively. In addition, $V^g_{i,\min}$, $V^g_{i,\max}$ are the minimum and maximum limits of voltage amplitude respectively. $Q^{sh}_{i,\min}$ stands for lower and $Q^{sh}_{i,\max}$ stands for upper limits of compensator capacitor. Finally, $T^g_{i,\min}$ and $T^g_{i,\max}$ presents lower and upper bounds of tap changer in *i*th transformer.

Security constraints: involve the constraints of voltages at load buses and transmission line loading as:

$$V^L_{i,\min} \le V^L_i \le V^L_{i,\max} \quad i = 1 \ldots n_L \tag{13}$$

$$S^L_i \le S^L_{i,\max} \quad i = 1 \ldots nl \tag{14}$$

where $V^L_{i,max}$ and $V^L_{i,\min}$ are the maximum and minimum load voltage of *i*th unit. S^L_i defines apparent power flow of *i*th branch. $S^L_{i,\max}$ defines maximum apparent power flow limit of *i*th branch.

A penalty function (Lai, Ma, Yokoyama, & Zhao, 1997) is added to the objective function, if the functional operating constraints violate any of the limits. The initial values of the penalty weights are considered as in (Aisac & Stott, 1974).

3. Interior search algorithm (ISA)

ISA is a combined optimization analysis that divides to the creative work or art relevant to Interior or internal designing (Gandomi, 2014), which consists of two stages. First one, composition stage where a number of solutions are shifted towards the optimum fitness. The second stage is reflector or mirror inspection method where the mirror is placed in the middle of every solution and best solution to yield a fancy view (Gandomi & Yang, 2012). Satisfaction of all control variables to constrained design problem using ISA is our main intention.

3.1. Algorithm description

(1) However, the position of acquired solution should be in the limitation of maximum and minimum bounds, later estimate their fitness amount (Gandomi, 2014).
(2) Evaluate the best value of a solution, the fittest solution has a maximum objective function whenever aim of an optimization problem is minimization and vice versa is always true. The solution has universally best in the *j*th run (iteration).
(3) Remaining solutions are collected into two categories mirror and composition elements with respect to a control parameter. Elements are categorized based on the value of the random number. (all used in this paper) are ranging between 0 and 1. Whether is less than equal to it moves to mirror category else moves towards composition category. For avoiding problems, elements must be carefully tuned.

(4) Being composition category elements, every element or solution is however transformed as described below in the limited uncertain search space.

$$x_i^j = lb^j + (ub^j - lb^j) \times r_2 \tag{15}$$

where x_i^j represents *i*th solution in *j*th run, ub^j, lb^j upper and lower range in *j*th run, whereas its maximum, minimum values for all elements exists (*j*–1)th run and $\text{rand}_2()$ in ranging 0–1.

(5) For *i*th solution in *j*th run spot of mirror is described:

$$x_{m,i}^j = r_2 x_i^{j-1} + (1 - r_3) \times x_{gb}^j \tag{16}$$

where $\text{rand}_3()$ ranging 0–1 .Imaginary position of solutions is dependent on the spot where mirror is situated defined as:

$$x_i^j = 2x_{m,i}^j - x_i^{j-1} \tag{17}$$

(6) It is auspicious for universally best to little movement in its position using uncertain walk defined:

$$x_{gb}^j = x_{gb}^{j-1} + r_n \times \lambda \tag{18}$$

where r_n vector of distributed random numbers having same dimension of *x*, $\lambda = (0.01*(ub–lb))$ scale vector, dependable on search space size.

(7) Evaluate fitness amount of new position of elements and for its virtual images. Whether its fitness value is enhanced then position should be updated for next design. For minimization optimization problem, updating are as follows:

$$x_i^j = \begin{cases} x_i^j & f(x_i^j) < f(x_i^{j-1}) \\ x_i^{j-1} & \text{Else} \end{cases} \tag{19}$$

(8) If termination condition is not fulfilled, again evaluate from second step.

(A) *Parameter tuning of* α

ISA has just one tuned parameter that is α which is almost fixed at 0.25, but the requirement is to increase its value ranging between 0 and 1 randomly as the increment in a maximum number of runs selected for a particular problem. It requires shifting search emphasised from exploration stage to exploitation optimum solution towards termination of maximum iteration.

(B) *Constraint manipulation*

Evolutionary edge (boundary) constraint manipulation:

$$f(z_i \rightarrow x_i) = \begin{cases} r_4 \times lb_i + (1 - r_4) x_{gb,i} & \text{if } z_i < lb_i \\ r_5 \times ub_i + (1 - r_5) x_{gb,i} & \text{if } z_i < ub_i \end{cases} \tag{20}$$

where r_4 and r_5 are random numbers between 0 and 1, and $x_{gb,i}$ is the related component of the global best solution.

(C) *Nonlinear constraint manipulation* (Landa Becerra & Coello, 2006)

Non-linear constraint manipulations have following rules:

(I) Both solutions are possible, then consider one with best objective functional value.
(II) Both solutions are impossible, then consider one with less violation of constraints.

Evaluation of constraint violation:

$$V(x) = \sum_{k=1}^{nc} \frac{g_k(x)}{g_{\max k}} \tag{21}$$

where nc represents number of constraints, $g_k(x) = k$th constraint consisting problem and $g_{\max k}$ = maximum violation in kth constraint till yet.

Pseudo code of Algorithm (Gandomi & Roke, 2014)

Initialization

```
while any stop criteria are not satisfied
  find the x^j_gb
   for i = 1 to n
    if x_gb
      Apply Equation x^j_gb = x^{j-1}_gb + r_n*λ
    else if r_1 > α
      Apply Equation x^j_i = LB^j + (UB^j − LB^j)*r_2
    else
      Apply Equation x^j_{m,i} = r_2 x^{j-1}_i + (1 − r_3)*x^j_gb
      Apply Equation x^j_i = 2x^j_{m,i} − x^{j-1}_i
    end if
   Check the boundaries except for decomposition elements.
  end for
  for i = 1 to n
      Evaluate f(x^j_i)
      Apply Equation x^j_i = { x^j_i      f(x^j_i) < f(x^{j-1}_i)
                              { x^{j-1}_i  Else
  end for
end while
```

4. Application and results

The ISA algorithm is demonstrated on the two test cases the IEEE 30-bus system, and the IEEE 57-bus system that are used as a tests systems to compare results of different case studies. The detailed results are shown for all cases.

The following eleven cases are provided to demonstrate the effectiveness of the proposed approach:

Cases studies on the IEEE 30-bus test system:

- Case 1: Fuel cost.
- Case 2: Fuel cost + Voltage profile.
- Case 3: Fuel cost + Multi-fuels.
- Case 4: Fuel cost + Voltage profile + Multi-fuels.
- Case 5: Fuel cost + Valve-point effect.
- Case 6: Fuel cost + Voltage profile + Valve-point effect.
- Case 7: Fuel cost with considering the prohibited zones.
- Case 8: Fuel cost with considering the prohibited zones + Valve-point effect.

Cases studies on the IEEE 57-bus test system:

- Case 9: Fuel cost
- Case 10: Fuel cost + Voltage profile.
- Case 11: Fuel cost + Valve-point effect.

4.1. IEEE-30-bus system

This system contains 6 generators, 41 transmission lines including 4 transformers and 9 compensators installed at the load bus. The total active power requires 283.4 MW while the reactive power requires 126.2 MVAR. The values of coefficients fuel costs of the six generators are presented in (Bouchekara, Chaib, Abido, & El-Sehiemy, 2016). The IEEE 30-bus test system has mainly 25 control variables.

- Case 1: Fuel cost

In this case, we are interested in solving the problem of OPF while minimizing the corresponding fuel cost production. The mathematical form of the objective function in this case is:

$$F = \sum_{i=1}^{n} \left(a_i P_i^2 + b_i P_i + c_i \right) + \text{Penalty} \tag{22}$$

where n total number of generators. P_i real power generated by the unit i a_i, b_i and c_i are the fuel cost coefficients of the ith generator. The obtained control variables parameters is tabulated in Table 2.

The fuel cost based objective function has achieved by 799.2776 $/hour, which is considered 11.2% lower than the normal case.

- Case 2: Fuel cost + Voltage profile

In this case, two competing objectives, namely the voltage profile improvement and fuel cost are shown in the Equation (23):

$$F = \sum_{i=1}^{n} \left(a_i P_i^2 + b_i P_i + c_i \right) + \eta \sum_{i=1}^{npq} |V_i - 1| + \text{Penalty} \tag{23}$$

where η is the weight factor, it was chosen carefully. After several experiments, the weight coefficient related to the voltage profile and fuel cost is taken 1,000.

The total generation fuel cost and voltage deviations are 807.6408 $/h and 0.1273 p.u for this case compared to Case 1 which gave us 799.2776 $/h and 1.8462 p.u. we note an increase in the fuel cost by 1% but there is an improvement in the voltage profile by 93%. The voltage profile obtained for

Cases 2 and 1 is shown in Figure 1 that illustrates in this case an improvement compared with Case 1, we can also note from this Figure that the voltage profile has improved and relieved.

- Case 3: Fuel cost + Multi-fuels

From a practical point of view, thermal generating plants may have multi-fuel sources like coal, natural gas, and oil. Hence, the fuel cost curve can be expressed by a piecewise quadratic function as follows (Bouchekara et al., 2016):

$$F = \sum_{i=1}^{n} \left(a_{ik} P_i^2 + b_{ik} P_i + c_{ik} \right) + \text{Penalty} \tag{24}$$

if $P_{ik}^{\min} \leq P_i \leq P_{ik}^{\max}$

where a_{ik}, b_{ik} and c_{ik} represent the cost coefficients of the ith generator for fuel type k. The generator fuel cost coefficients are given in Bouchekara et al. (2016), The results obtained for this case are shown in Table 2. The optimal cost obtained for this case is 646.2532 $/h.

- Case 4: Fuel cost + Voltage profile + Multi-fuels

We considered that the optimal solution is achieved using the proposed algorithm and presented in Table 2. This case is similar to Case 2, we can note from this table that the voltage profile has improved and relieved compared with Case 3. Because voltage deviation has been reduced from 1.6456 p.u to 0.1286 p.u. The voltage profile obtained for Case 3 and 4 is shown in Figure 1 that offers in this case enhancement compared with Case 3.

- Case 5: Fuel cost + Valve-point effect

For more rational and precise modeling of fuel cost function, the generating units with multi-valve steam turbines exhibit a greater variation in the fuel-cost functions (Hardiansyah, 2013). The valve-point effects are taken into consideration in the problem by superimposing the basic quadratic fuel-cost characteristics with the rectified sinusoid component as follows:

$$F = \sum_{i=1}^{n} \left(a_i P_i^2 + b_i P_i + c_i \right) + \left| d_i \times \sin(e_i \times (P_i^{\min} - P_i) \right| + \text{Penalty} \tag{25}$$

where d_i and e_i are the coefficients that represent the valve-point loading effects, the coefficients are given in (Bouchekara et al., 2016). The optimal settings obtained for adjusting control variables from the ISA method are given in Table 2. The minimum cost obtained from the proposed approach is 823.1379 $/h. It is increased compared with Case 1 by reason of taking the effect valve point.

- Case 6: Fuel cost + Voltage profile + Valve-point effect

This case is similar to Case 2, the objective is to optimize both the cost with considering the valve point effect and improve the voltage profile. Table 2 is given the optimal results obtained using ISA for this case. Figure 2 exposed the voltage profile obtained for Cases 5 and 6 that shown improvement compared with Case 5.

- Case 7: Fuel cost with considering the prohibited zones

Resolve the problem OPF with the examination of the prohibited zones is proposed in this section. The generators have a set of allowed operating zones i.e. non-prohibited zones (POZ) and they must work in one of these zones. POZ can be included as constraints as follows:

Figure 1. System voltage profile improvement.

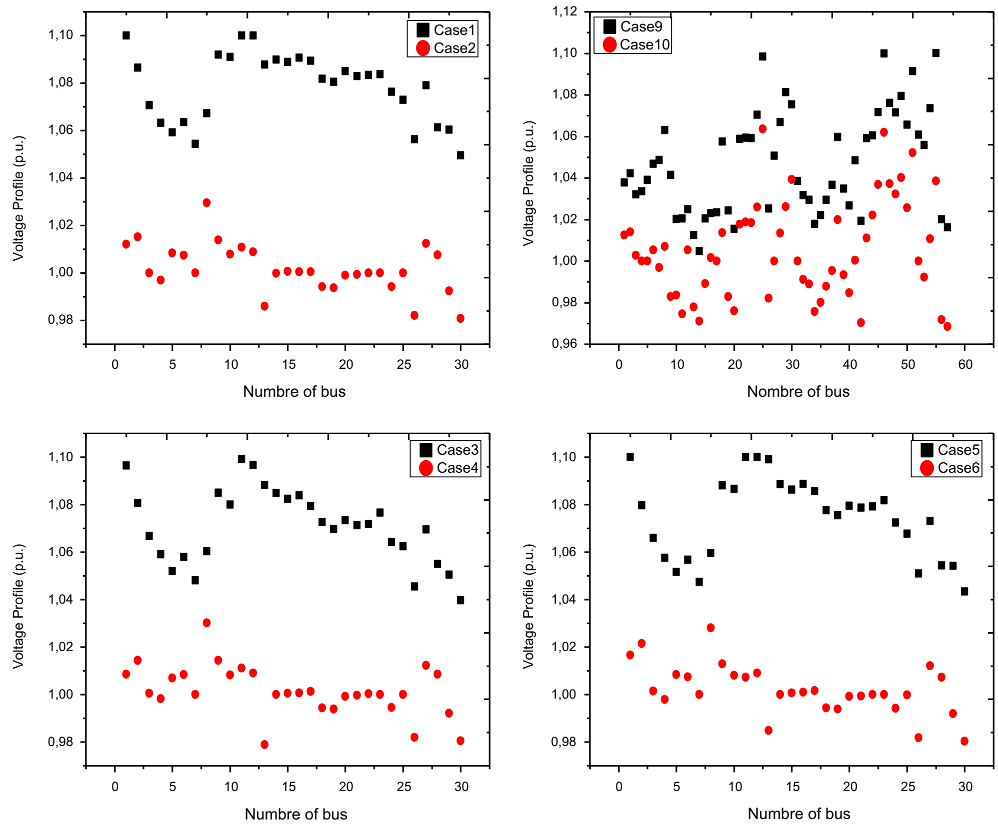
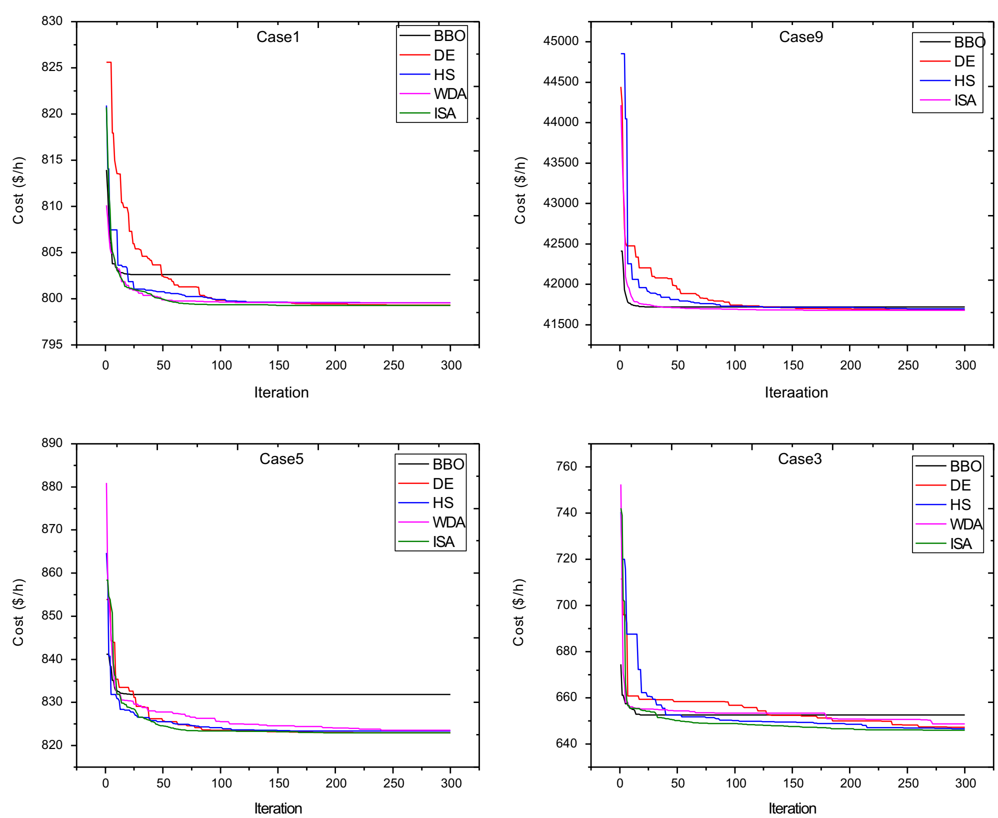


Figure 2. Minimization of fuel cost for Cases 1, 3, 5 and 9 using algorithms.



$$P_{im}^{\min} \leq P_i \leq P_{im}^{\max} \tag{26}$$

where $P_{im}^{\min}$ and $P_{im}^{\max}$ are the minimum and maximum limits prohibited zones, limits of all generators are shown in Table 1. The obtained optimal settings of control variables for this case study are shown in Table 2, it is clear that POZ slightly contributes in increasing the generation cost compared with Case 1.

- Case 8: Fuel cost with considering the prohibited zones + Valve-point effect

This Section 4.1 examines the valve point effect and prohibited zones simultaneously. The results of the present case are represented in Table 2. By comparing the results obtained here with the Case 7, it is clear that the production cost value of Case 8 is highest.

4.2. IEEE 57-bus system

In this part, we have applied ISA method to solve OPF problem in large power system and in order to prove the substance and robustness of this proposed approach, the IEEE 57-bus with 80-branch systems has been proposed, which has a 34 control variables that are given as follows: 7-generator voltage magnitudes, 17-transformer-tap settings, and 3-sbus shunt reactive compensators (Bouchekara et al., 2016). The total system demand is 12.508 p.u. for the active power while 3.364 p.u for the reactive power at 100 MVA base. The slack bus of power system has been taken in bus 1. The values of coefficients fuel costs of all generators are presented in (Sinsupan, Leeton, & Kulworawanichpong, 2010).

- Case 9: Fuel cost

In the process of ISA, the objective function of this part is given by Equation (22). The obtained results are given in Table 3. The cost obtained for this case is 41676.9466 \$/h which is less when comparing with the base case.

- Case 10: Fuel cost + Voltage profile

We have selected the voltage profiles as the objective function beside fuel cost in order to improve voltage profiles of the test system. The equation of the objective function is given by Equation (23). The weight coefficient related to the voltage profile and fuel cost is 120,000.

The obtained results are given in Table 3. The voltage deviation is 0.9931 p.u for this case compared to Case 9 which gave us 2.3863 p.u. It can be seen that there are differences between the proposed solutions and there is an improvement in the voltage profile by 59%. The voltage profile obtained for Cases 10 and 9 shown in Figure 1 that proves the improvement of the voltage profile in this case compared with Case 9.

Table 1. Prohibited zones for the IEEE 30-bus test system

Bus	Prohibited zones			
	Zone 1		Zone 2	
	Min	Max	Min	Max
1	55	66	80	120
2	21	24	45	55
5	30	36	–	–
8	25	30	–	–
11	25	28	–	–
13	24	30	–	–

Table 2. Optimal results for Case 1 through Case 8

Control variable	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7	Case 8
P_{G1} (MW)	177.124	174.21	139.852	140.114	198.66	197.97	177.34	198.967
P_{G2} (MW)	48.9332	48.909	54.9959	54.9996	26.361	29.5042	45.00	25.93146
P_{G5} (MW)	21.3175	21.808	22.525	27.3675	16.568	16.2901	21.31	16.96848
P_{G8} (MW)	21.0006	25.412	34.8961	30.8941	10	10.0161	21.036	10.00089
P_{G11} (MW)	11.8605	11.932	18.0515	15.3592	10	10.0009	11.876	10.00077
P_{G13} (MW)	11.86	11.86	19.6782	23.0195	12.14	11.8677	11.86	11.86047
V_1 (p.u)	1.1	1.0121	1.09642	1.00848	1.1	1.01667	1.1	1.1
V_2 (p.u)	1.08651	1.0151	1.08066	1.01431	1.0826	1.02147	1.0854	1.08292
V_5 (p.u)	1.05916	1.0084	1.05201	1.00702	1.0537	1.00844	1.0576	1.054499
V_8 (p.u)	1.06733	1.0295	1.06035	1.03022	1.0605	1.02803	1.0644	1.061925
V_{11} (p.u)	1.1	1.0107	1.09922	1.01115	1.1	1.00725	1.1	1.098131
V_{13} (p.u)	1.08773	0.986	1.08825	0.97892	1.0932	0.98491	1.0947	1.093675
T_{6-9}	1.06831	0.9694	0.99603	1.04552	0.9749	1.01175	1.0307	1.029261
T_{6-10}	1.04151	0.9778	1.03433	1.07346	1.0273	1.00364	1.0055	1.047304
T_{4-12}	1.00613	1.0361	1.03891	1.07783	0.9936	1.0378	0.9997	1.060403
T_{28-27}	0.97014	1.055	0.99883	1.0311	0.9665	1.00065	0.9949	0.95653
Q_{c10} (Mvar)	4.99986	4.9998	1.76102	4.99683	4.8511	4.99965	4.9996	4.998825
Q_{c12} (Mvar)	4.99982	4.9999	2.72987	4.9996	4.9989	4.99999	4.6449	4.995023
Q_{c15} (Mvar)	2.86865	0.0109	4.08933	2.87864	3.0656	0.0198	0.0001	0.93368
Q_{c17} (Mvar)	3.67783	5	2.5313	4.99964	2,2085	4.99993	2.8502	4.59062
Q_{c20} (Mvar)	4.98223	0.0043	4.52856	0.82387	4.7414	1.50242	4.9985	4.999738
Q_{c21} (Mvar)	4.6327	5	3.29318	4.98853	4.8079	4.99998	4.412	4.019706
Q_{c23} (Mvar)	4.99987	4.9996	4.08493	4.99989	4.9987	4.99996	4.3356	4.998595
Q_{c24} (Mvar)	2.94032	5	4.422	4.99989	3.1112	4.9999	3.7032	2.84681
Q_{c29} (Mvar)	3.00097	1.801	2.21434	2.35222	0.8395	1.3373	2.7485	1.855214
Fuel cost ($/h)	799.2776	807.6408	646.2532	653.7671	823.1379	832.7090	799.3089	823.1673
Ploss (MW)	8.6959	10.7312	6.5982	8.3545	11.3341	14.2489	8.7113	11.3291
VD	1.8462	0.1273	1.6456	0.1286	1.7637	0.1288	1.8083	1.7768

- Case 11: Fuel cost + Valve-point effect

The effect of valve-point loading is also tested in this case. Therefore, the objective function in this case is expressed by Equation (25), the coefficients are given in Sinsupan et al. (2010). The obtained results to the aid of ISA are given in Table 3. In this case, the cost has slightly increased from 41676.9466 $/h to 41705.2941 $/h compared with Case 9.

4.3. Performance evaluation study

In order to evaluate the performance of the ISA, several optimization algorithms have been carried out including BBO, DE, HS and WDA to give equitable comparison. Control parameters of optimization algorithms are selected carefully after several trials that used in this investigation and are given in Table 4. From this table, we can see that all algorithms have more than two tuned parameters, while ISA has just one tuned parameter which is α, this property is mainly one of the ISA advantages.

Table 3. Optimal results for Case 9 through Case 11

Control variable (p.u.)	Case 9	Case 10	Case 11
P_1 (p.u)	140.297965	157.331371	143.284824
P_2 (p.u)	87.6032363	99.9998803	82.6737671
P_3 (p.u)	44.7563272	42.7420443	45.3545803
P_6 (p.u)	75.1485934	5.0982276	73.5050975
P_8 (p.u)	459.99558	456.982543	461.334812
P_9 (p.u)	95.9585096	96.472764	97.1573258
P_{13} (p.u)	362.149974	407.889603	362.561839
V_1 (p.u)	1.03776821	1.01256065	1.03893849
V_2 (p.u)	1.04215958	1.01401062	1.04252178
V_3 (p.u)	1.03201601	1.00277245	1.03216019
V_6 (p.u)	1.04688409	1.00529369	1.05108243
V_8 (p.u)	1.06311898	1.00706659	1.06558728
V_9 (p.u)	1.04143849	0.98281995	1.04086891
V_{13} (p.u)	1.0248601	1.00525444	1.02408603
$T_{4\text{–}18}$	0.92646197	0.99631524	1.00526435
$T_{4\text{–}18}$	0.93987275	0.98594086	0.96707268
$T_{21\text{–}20}$	0.98669423	1.0185223	1.04337586
$T_{24\text{–}25}$	0.99145917	0.94690761	0.97749794
$T_{24\text{–}25}$	1.00138985	0.9499337	0.97352597
$T_{24\text{–}26}$	1.01906685	1.03538627	0.96829392
$T_{7\text{–}29}$	1.02246784	0.9963351	1.0188147
$T_{34\text{–}32}$	0.99930487	1.0768902	1.06084883
$T_{11\text{–}41}$	1.0367947	1.01744033	0.93391552
$T_{15\text{–}45}$	1.03502693	0.91745731	0.96557376
$T_{14\text{–}46}$	1.02917518	0.94443448	0.90735034
$T_{10\text{-}51}$	0.9269382	1.06381535	0.95767122
$T_{13\text{–}49}$	0.98087975	1.03201152	0.98827504
$T_{11\text{–}43}$	0.92916229	0.95763177	0.96139549
$T_{40\text{-}56}$	1.07384495	1.00908972	0.98846446
$T_{39\text{–}57}$	0.95416665	0.9638778	1.06407405
$T_{9\text{–}55}$	0.92324215	1.02381055	0.97052031
Q_{C18}	10.2325736	7.2915012	10.0362723
Q_{C25}	12.8902838	15.520675	12.8718025
Q_{C53}	11.197684	10.4411106	11.5559707
Fuel cost ($/h)	41,676.9466	41,939.7706	41,705.2941
Ploss (MW)	15.1102	15.7164	15.0722
VD	2.3863	0.9931	2.4171

The results of this study are achieved after twenty different runs. Table 5 indicates that algorithm offers the minimum values of best, worst, median values of fuel cost and the standard deviation values. Also, we calculated the infeasibility rate that shows the success rate or not of the proposed algorithms. The OPF problem may become infeasible under certain operating conditions. It may not be possible to satisfy all the constraints, under these circumstances, some of the soft constraints must be relaxed to obtain a feasible solution. Detection of infeasibility is important because a

Table 4. Control parameters of the related algorithms used in the tests

Algorithm	Value	Description	Description of method
BBO	pmutate = 0	Mutation probability	Biogeography-based optimization
	pmodify = 1	Habitat modification probability	
	pmutate = 0.005	Initial mutation probability	
DE	*F* = 0.8	Differential weight	Differential evolution
	Cr = 0.8905	Crossover probability	
HS	HMS = 25	Harmony memory size	Harmony Search
	nNew = 20	Number of new harmonies	
	HMCR = 0.9	Harmony memory consideration rate	
	PAR = 0.1	Pitch adjustment rate	
	FW_damp = 0.995	Fret width damp ratio	
WDO	RT = 3	RT coefficient.	Wind driven optimization
	g = 0.2	Gravitational constant	
	al*p* = 0.4	Constants in the update equation	
	c = 0.4	Coriolis effect	
	maxV = 0.3	Maximum allowed speed	
ISA	α = 0.25		Interior search algorithm
Common parameters	Population size = 40 for each cases		
	Maximum number of iterations = 300		

solution with small violations is better than no solution at all. Infeasibility rate is calculated in this case as follows:

$$\text{Infeasibility rate} = \frac{\text{Number of run not converged}}{11\,(\text{Cases}) \times 20\,(\text{runs per case})} \times 100\,\% \tag{27}$$

We note from Table 5 that the difference between the minimum and the worst is very close, we can say from this table that the proposed method is robust. This is also demonstrated by the low values of the standard deviations calculated compared to other algorithms. ISA is the only algorithm that has a infeasibility rate of 0% compared with the remaining algorithms. This comparison can be seen clearly when the resolution of the OPF for larger systems has not even been able to converge with the BBO, HS and WDA as in the Cases of 9, 10 and 11.

On the other hand, the fast converge of an optimization algorithm to the optimal solution is an important issue in this domain. Figure 2 shows the trend for minimizing the generation fuel cost based objective function using the different algorithms for Cases (1, 3, 5 and 9). The algorithms were tested using same different initial solutions.

It is obvious that ISA increased the convergence speed and obtained better final results with less improvisation compared with the remaining algorithms.

For more demonstrating the convergence speed, the curve of convergence has been cut at four cut points which are 20, 40, 60 and 80% of the iterations maximum number.

Table 5. Comparison of the ISA with BBO, DE, HS and WDA for solving different OPF problems

Cases		ISA	BBO	DE	HS	WDA
Case 1	Best	799.2775	802.6928	799.310915	799.5643	799.6444
	Mean	799.2845	803.6076	799.5600	799.9286	799.8434
	Worst	799.3042	804.3370	799.8091	800.8656	800.1186
	Std.	0.0092	0.6950	0.3523	0.6259	0.1889
Case 2	Best	934.9252	974.3532	936.5690	961.0799	944.7191
	Mean	936.8553	989.0107	937.6885	964.2001	961.5295
	Worst	938.5149	999.0186	939.5740	967.0019	971.4390
	Std.	1.2327	9.6062	1.3193	2.4810	10.3380
Case 3	Best	646.2532	652.5699	647.4818	649.1088	650.6093
	Mean	649.3892	658.5834	648.0807	650.6525	652.0016
	Worst	651.6315	670.1644	648.8991	653.3658	654.3714
	Std.	2.2964	6.8418	0.7338	1.6863	1.7328
Case 4	Best	787.3417	846.4735	795.3084	818.3512	815.5853
	Mean	790.9931	860.2433	799.5446	821.7694	824.2396
	Worst	794.5488	870.3910	804.2022	826.0212	838.2845
	Std.	2.9132	9.9976	4.4619	3.3621	8.9931
Case 5	Best	823.1379	831.8466	823.9152	824.1363	824.0153
	Mean	823.2163	834.0031	825.5778	824.5204	825.1159
	Worst	823.4845	835.9777	827.9075	824.8977	827.2251
	Std.	0.1503	1.7894	2.0781	0.3854	1.2406
Case 6	Best	961.4677	1,023.68943	962.5552	991.9074	967.322769
	Mean	963.7583	1,036.19247	963.6568	992.9245	–
	Worst	967.9412	1,053.55143	964.5832	994.3956	–
	Std.	1.9093	11.0394	0.8379	1.0680	–
Case 7	Best	799.3089	802.1666	799.5587	799.7293	799.7948
	Mean	799.3232	803.6673	799.8470	800.0358	800.0453
	Worst	799.3389	804.5599	799.9946	800.5751	800.6144
	Std.	0.0115	1.0633	0.2497	0.3739	0.3377
Case 8	Best	823.1673	832.5560	823.9212	824.7796	824.3581
	Mean	823.2726	835.9265	825.4197	825.6587	825.0148
	Worst	823.4481	840.7193	826.9149	826.3168	826.3213
	Std.	0.1277	3.1244	1.2885	0.6697	0.8115
Case 9	Best	41,676.9466	41,721.2464	41,681.295	41,693.35813	–
	Mean	41,677.4271	–	–	–	–
	Worst	41678,2442	–	–	–	–
	Std.	0.5527	–	–	–	–
Case 10	Best	160,946.6871	163,475.793	161,166.85	161,747.272	–
	Mean	160,949.508	–	161,196.302	–	–
	Worst	160,953.5771	–	161,284.657	–	–
	Std.	3.6104	–	58.9034	–	–
Case 11	Best	41,705.2941	41,764.8019	41,708.4914	41,758.4312	–
	Mean	41,706.0507	–	–	–	–
	Worst	41,710.2813	–	–	–	–
	Std.	1.5435	–	–	–	–
No. of NC*		0	53	26	56	72
Infeasibility rate (%)		0	24	12	25.5	33

Note: NC* refers to how many times this algorithm has not converge for this case.

- 20% represents iterations number 60;
- 40% represents iterations number 120;
- 60% represents iterations number 180;
- 80% represents iterations number 240.

Then, we take the value of the objective function that corresponds to the number of iterations, and calculates the percentage of change value with the final value. These percentages are reported in Table 6. The final row is the average percentage for the cases of study. From this table, it can be concluded that the ISA is more robust and effective in solving the considered OPF problem. The ISA has reached more than 99% of the final objective value for each cut point. From this result, we can note that the ISA gives the best convergence speed.

For a comprehensive comparison, some published results have been traced. In Table 7, a comparison between the results is obtained by the proposed algorithm with those found in the literature,

Table 6. Convergence speed of the ISA algorithm

Case	Cut-point 1	Cut-point 2	Cut-point 3	Cut-point 4
Case 1	99.95	99.99	99.99	99.99
Case 2	98.88	99.69	99.93	99.99
Case 3	99.43	99.65	99.84	99.98
Case 4	97.45	99.31	99.97	99.99
Case 5	99.87	99.95	99.98	99.99
Case 6	97.43	99.5	99.88	99.99
Case 7	99.96	99.99	99.99	99.99
Case 8	99.92	99.97	99.99	99.99
Case 9	99.94	99.98	99.99	99.99
Case 10	99.61	99.98	99.99	99.99
Case 11	99.98	99.98	99.99	99.99
Average	99.3109	99.8173	99.9582	99.9891

Table 7. Some of results obtained by different algorithms

Methods	Case 1	Case 3	Case 5	Case 9	Method description
ISA	799.2776	646.2532	823.1379	41,676.947	Interior search algorithm
ICA (Ghasemi, Ghavidel, Rahmani, Roosta, & Falah, 2014)	801.7784	647.5687	826.1275	41,710.4933	Imperialist competitive algorithm
TLA (Ghasemi et al., 2014)	801.6524	647.4413	825.6608	41,686.7915	Teaching learning algorithm
MICA (Ghasemi et al., 2014)	801.0794	647.134	824.6749	41,683.048	Modified imperialist competitive algorithm
SFLA (Niknam, Narimani, & Azizipanah-Abarghooee, 2012)	801.97	–	825.9906		Shuffle frog leaping algorithm
GSA (Duman, Guvenc, Sonmez, & Yorukeren, 2012)	–	–	–	41,695.8717	Gravitational search algorithm
PSO (Abido, 2002b)	–	647.69	–	–	Particle swarm optimization
PSO (Narimani, Azizipanah-Abarghooee, Zoghdar-Moghadam-Shahrekohne, & Gholami, 2013)	801.89	649.41	–	–	Particle swarm optimization
PSOGSA (Taylor, Radosavljevi, Klimenta, & Jevti, 2015)	800.4985	–	824.7096	–	Hybrid PSO/GSA

which is made in this case for minimization of non-smooth cost functions. The results validated the proposed method and proved its performance in terms of solution quality.

5. Conclusion

This article has been dealt with the traditional optimal power flow problem which was formulated by considering equality and inequality constraints such as machine limits, allowable voltage and loading constraints. The proposed ISA based approach was applied to solve several cases studies using two power systems which are IEEE 30-bus and IEEE 57-bus test systems.

The conclusions and findings of this work can be summarized as follows:

- The proposed ISA based approach is found to be simple, robust and easy to understand. The most advantage of ISA is that it has just one control parameter which makes the simplicity in the main algorithm. Also, it is an effective technique to improve the solution diversity without losing solutions.
- A comparison of the feasibility and convergence with other well-known optimization algorithms such as BBO, DE, HS and WDA shows the robustness of the proposed method.
- The simulation results were compared to the available results in the literature.
- The results have indicated the effectiveness of the proposed approach in the optimization process.

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