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Performance assessment of multi-input-single-output (MISO) production process using transfer function and fuzzy logic: A case study of soap production

Chidozie Chukwuemeka Nwobi-Okoye and Stanley Okiy

Abstract: In this research an improved and novel method of assessing the performance of multi input single output (MISO) processes, as typified by soap production process was studied. The method involves the combination of transfer function and fuzzy logic and was used in assessing the three years performance of a soap factory. A comparison of the years studied shows that the year 2011 with a performance rating $\lambda$ of 0.761 which corresponds to the linguistic variable “Good” recorded the best performance, while the year 2012 with a performance rating $\lambda$ of 0.250 which corresponds to the linguistic variable “Poor” recorded the worst performance. The result of this study will help to improve maintenance effectiveness, quality, utilization of raw materials and efficiency of MISO production processes.

Subjects: Operations Research; Production Systems; Manufacturing & Processing

Keywords: transfer function; fuzzy logic; soap production; multi input single output process; modelling

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PUBLIC INTEREST STATEMENT

Production processes ought to be very efficient so as to boost the profitability of businesses and enhance better utilization of scarce raw materials. Efficiency also helps conserve energy and protect the environment. In order to ensure that the production processes, such as soap production process, remains efficient there must be continuous monitoring and control of the process. Better monitoring and control technique measure the efficiency better and if religiously adopted and implemented by production managers would lead to improved processes. This work successfully introduced a better method for performance assessment and monitoring of soap production process, a typical multi-input-single-output process. The method uses transfer function and fuzzy logic to better measure efficiency, monitor and control process performance. This promising method would help ensure better utilization of raw materials in soap production, provide better diagnostic tool to monitor and control process performance in soap production, as well as act as an excellent quality control tool.
1. Introduction

Wastes, losses, work-in-progress, poor quality raw materials are some problems often encountered in a typical soap production plant. If the machines are not functioning well, losses and work-in-process build up. On the other hand if the quality of raw materials is poor the quantity of additives added to the raw materials to produce the finished soap has to increase. Hence, the quantity of additives added to produce the finished soap depends on the quality of the input raw materials used to make the soap chips. Consequently, the production manager aims to transform as many raw materials and soap chips as possible to finished soap while minimizing as much as possible the consumption of additives.

It is absolutely necessary that organizations monitor and control their production processes effectively. By so doing wastes and losses are minimized, productivity increases and output quality improves. A very good tool used to monitor process performance is the transfer function (Box, Jenkins, & Reinsel, 2008; Igboanugo & Nwobi-Okoye, 2011, 2012; Lai, 1979; Nwobi-Okoye & Igboanugo, 2012, 2015).

Transfer function modeling of single input single output (SISO) processes are quite complex and the modeling complexity increases when the input or output is more than one. Soap production process as depicted in Figure 1 is a typical multi input single output (MISO) process. As Figure 1 depicts, there is variability between the inputs namely additives and soap chips raw materials and the output which is the finished soap. The soap chips are produced by the chemical process known as saponification, where vegetable oil reacts with caustic soda to produce the base soap. The base soap undergoes further processing to produce the soap chips. The additives are added to soap chips to produce the finished soap suitable for domestic use. The production process involves mixing the soap with the additive; the mixture is blended in a rolling mill, extruded and stamped to produce the finished soap.

As stated in the literature, transfer functions relate or models the causal relationship between the input(s) and output of processes (Box et al., 2008; Lai, 1979; Nwobi-Okoye & Igboanugo, 2012, 2015). The use of transfer functions to model performance was reported by Nwobi-Okoye and Igboanugo (2012, 2015). In the research, they studied only the SISO systems, as typified by power generation systems. The complexity of evaluating the performance of MISO Processes using transfer function modeling makes it imperative to introduce fuzzy logic to handle the vagueness involved in the analysis.

Zadeh (1965) introduced fuzzy logic which is often used as the state of the art method by engineers for analyzing and modeling uncertainty. Uncertainty and vagueness have not always been embraced by the scientific community, because engineers and scientists have always sought for precision in measurements and design (Klir & Yuan, 1995; Ross, 2004). The advent of fuzzy logic has a profound effect in our understanding and management of uncertainty and vagueness by engineers and scientists.

Plant and production process performance irrespective of whether it is SISO, MISO, single input multi output (SIMO) or multi input multi output (MIMO) depends on the five M’s of production...
namely: men, machines, material, method and money. As stated by Nwobi-Okoye and Igboanugo (2015) performance assessment which involves all the five Ms. of production are called Macro level analysis and evaluations. On the other hand, performance assessment that considers one of the five Ms. was termed micro level performance assessment. Macro level performance assessment methods include, data envelopment analysis (DEA), stochastic frontier analysis (SFA) and analytic network process (ANP) (Atmaca & Basar, 2012; Jha & Shrestha, 2006). Performance evaluation methods of production processes at the micro level include but not limited to: reliability modeling and analysis, availability studies, output quality assessment, queuing modeling and analysis, production time measurements and efficiency measurements. At the micro level, quality control indices and metrics like process capability indices, six sigma etc. have been used to access the performance of productions processes (Wu, Pearn, & Kotz, 2009). In terms of performance assessment by production time or productivity measurements the works of Kumar, Duhan, and Haleem (2016) and Ali Naqvi, Fahad, Atir, Zubair, and Shehzad (2016) provides deep insight to this important method of assessment. Reliability and availability modeling is an important micro level performance evaluation method. The works of Bhuiyan and Yazdani (2010), Lo Prete et al. (2012), Bourouni (2013), Garg and Sharma (2012), Sharma and Garg (2011), Sawle, Gupta, and Kumar Bohre (2016) amply demonstrates the application of this state of the art method. Efficiency measurements one of the oldest performance assessment methods has found its use in plant performance evaluations over the years. Efficiency measurements in all its variants such as Exergy analysis (Oyedepo, Fagbenle, Adefila, & Alam, 2015), coefficient of performance (COP) (Anand, Gupta, Anand, & Tyagi, 2016; Nwobi-Okoye & Igboanugo, 2012, 2015; Ricardo Costa & Garcia, 2015) and conventional efficiency analysis (Habert, Billard, Rossi, Chen, & Roussel, 2010; Higton, 1999; Ozbilen, Dincer, & Rosen, 2013 etc.).

The method presented in this work belongs to the micro level. The unique feature of transfer function modeling for performance assessment is that in some situations it measures only efficiency, while in other situations when combined with fuzzy logic could measure efficiency, access quality and production time simultaneously.

The literature is replete with the use of fuzzy logic for condition monitoring and performance assessment. For example civil engineers use it to assess the conditions of bridges and other civil engineering structures (Ross, 2004). Seçme, Bayrakdaroğlu, and Kahraman (2009) used a fuzzy multi-criteria decision model to evaluate the performances of banks using fuzzy analytic hierarchy process. Ertuğrul and Karakaşoğlu (2009) used the method to assess the performance of the fifteen Turkish cement firms in the Istanbul Stock Exchange. Chen (1996) evaluated the performance of weapon systems using fuzzy arithmetic operations which performed better than previous methods which used entropy weight calculations. Yang and Chen (2004) introduced an evaluation method that integrates triangular fuzzy numbers and the analytical hierarchy process to develop a fuzzy multiple-attribute decision-making model for key quality-performance evaluation. Sadiq, Al-Zahrani, Sheikh, and Husain (2004) used fuzzy logic to evaluate and predict the performance of slow sand filters used for wastewater treatment and obtained results which compared favourably with results obtained with a multiple regression model. Yeh, Deng, and Chang (2000) introduced an effective fuzzy multicriteria analysis approach to performance evaluation for urban public transport systems involving multiple criteria of multilevel hierarchies and subjective assessments of decision alternatives. The approach was found to be computationally efficient, and its underlying concepts simple and comprehensible. The integrated approach developed by Wu (2009) which uses an integrated approach to rate decision alternatives using DEA and fuzzy preference relations is another exemplification of the use of fuzzy in performance evaluation and decision-making. Other works which used fuzzy for performance assessment and decision-making include: Cheng and Lin (2002), Lin, Chiu, and Tseng (2006) etc.

In production systems specifically, fuzzy logic has been applied widely. According to Azadegan, Porobic, Ghazinoory, Samouei, and Soman Kheirkhah (2011), there is inherent uncertainty and imprecision in manufacturing. Three reasons why fuzzy logic is imperative in manufacturing, production systems and production management as stated by Karwowski and Evans (1986) are:
Imprecision and vagueness are inherent to the decision maker’s mental model. In the production management environment, the information required to formulate a model’s objective, decision variables, constraints and parameters may be vague or not precisely measurable. Imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information.

Hence, as a consequence many researchers have applied fuzzy logic to production systems. For example Ip (1998) suggested the use of fuzzy logic for optimization of the efficiency in cutting sculptured surface using numerically controlled machine tool. Babuška, Verbruggen, and van Can (1999) reported the use of fuzzy logic in Penicillin-G conversion process. Lee, Piramuthu, and Tsai (1998) developed a method for optimizing machining operations using fuzzy non linear programming approach. Shivathaya and Fang (1999) reported the application fuzzy logic in steel making where it was used to generate about 15–30 different target compositions in order to meet different customer orders. As stated by Meziane, Vadera, Kobbacy, and Proudlove (2000), a lot of factors affecting facility layout and location problems are imprecise and requiring a considerable amount of human judgement. For example, Guiffrida and Nagi (1998) by taking into consideration the subjectivity in the model parameters used fuzzy set theory to effectively model facility layout and location. Fuzzy logic has been applied to quality control in designing quality control charts and in modeling quality decisions (Meziane et al., 2000; Turanoğlua, Kayab, & Kahramanc, 2012).

Bozdağ, Kahraman, and Ruan (2003) applied fuzzy group decision-making for selection among computer integrated manufacturing systems. Karsak (2002) used fuzzy approach for evaluating flexible manufacturing system alternatives. Ayağ and Özdemir (2006) used fuzzy AHP approach for evaluating machine tool alternatives. Kahraman, Çevik, Ates, and Gülbay (2007) used fuzzy logic for multi criteria evaluation of industrial robotic systems. These are typical cases of use of fuzzy for manufacturing systems evaluation and selection. Fuzzy logic could be used in optimization of manufacturing processes such as electrical discharge machining process, submerged arc welding process etc. (Lin & Lin, 2005; Mohd Adnan, Sarkheyli, Mohd Zain, & Haron, 2013; Tarrng, Yang, & Juang, 2000). Fuzzy logic has been applied to production process planning and product mix prioritization. For example, Singh and Mohanty (1991) applied fuzzy logic to process planning, while Azadegan et al. (2011) and Ghazinoory, Fattahi, and Samouei (2013) used fuzzy logic and linear programming to develop an algorithm to solve the product mix prioritization problem. It has also been applied to supply chain management. For applications of fuzzy to supplier selection see Kahraman, Cebeci, and Ulukan (2003), Chan and Kumar (2007), Ohdar and Ray (2004), Sari, Ugurlu, and Kahraman (2014) etc. The literature cited above, while quite not exhaustive, is replete with salient applications of fuzzy logic to production and quite in line with the assertions of Karwowski and Evans (1986) on the imperative of applications of fuzzy to production systems. Ours is to extend this frontier of applications of fuzzy to productions systems by bringing in the concept of fuzzy transfer function modeling.

Fuzzy logic and transfer functions, as the literature shows, are good performance assessment tools. Hence, combining the two tools would result to better process monitoring and control. This work is an attempt to take advantage of the accuracy and precision of transfer function modeling and fuzzy logic’s excellence in dealing with vagueness and uncertainty to assess the performance of a MISO process as typified by the soap production process. Our case study is a local soap production company known as Promotex Industrial and Chemical Company Limited located at Umudim, Nnewi, Anambra State Nigeria. Promotex, a subsidiary of Chicason Group of Companies, was incorporated in 1984 for manufacturing and marketing of a wide variety of soaps ranging from toilet, medicated and laundry soaps.
2. Theoretical brief

2.1 Multiple input transfer function models

In terms of the impulse response weights \( \nu(B) \) the transfer function can be represented as (Box et al., 2008):

\[
Y_t = \nu(B)X_{t-b} + N_t
\]

But \( \nu(B) = \delta^{-1}(B)\alpha(B)^{s} \) (Box et al., 2008), hence:

\[
Y_t = \delta^{-1}(B)\alpha(B)^{s}X_{t-b} + N_t
\]

In the case of several inputs, \( X_{1,t}, X_{m,t-r}, \ldots \), the following is obtained:

\[
Y_t = \nu_1(B)X_{1,t} + \cdots + \nu_m(B)X_{m,t} + N_t
\]

\[
Y_t = \delta^{-1}(B)\alpha(B)^{s}X_{1,t-b} + \cdots + \delta^{-1}(B)\alpha(B)^{s}X_{m,t-b} + N_t
\]

\( \nu_j(B) \) is the generating function of the impulse response weights relating to \( X_{j,t} \) to the output. Applying differencing to the input and output series the following is obtained:

\[
y_j = \nu_1(B)x_{1,t} + \cdots + \nu_m(B)x_{m,t} + n_t
\]

Multiplying throughout by \( X_{1,t-b}, X_{2,t-b}, \ldots, X_{m,t-b} \) in turn and taking expectations and forming the generating functions, the following is obtained:

\[
\gamma^{s,y}(B) = \nu_1(B)\gamma^{s,y_{1,t}}(B) + \nu_2(B)\gamma^{s,y_{2,t}}(B) + \cdots + \nu_m(B)\gamma^{s,y_{m,t}}(B)
\]

\[
\gamma^{s,y}(B) = \nu_1(B)\gamma^{s,y_{1,t}}(B) + \nu_2(B)\gamma^{s,y_{2,t}}(B) + \cdots + \nu_m(B)\gamma^{s,y_{m,t}}(B)
\]

\[
\vdots
\]

\[
\gamma^{s,y}(B) = \nu_1(B)\gamma^{s,y_{1,t}}(B) + \nu_2(B)\gamma^{s,y_{2,t}}(B) + \cdots + \nu_m(B)\gamma^{s,y_{m,t}}(B)
\]

Substituting \( B = e^{-2\pi if} \) the spectral equations are obtained. For the case of \( m = 2 \), the spectral equations are:

\[
p_{1,y}(f) = H_1(f)p_{1,s_1}(f) + H_2(f)p_{1,s_2}(f)
\]

\[
p_{1,y}(f) = H_1(f)p_{1,s_1}(f) + H_2(f)p_{1,s_2}(f)
\]

The frequency response functions \( H_1(f) = \nu_1(e^{-2\pi if}), H_2(f) = \nu_2(e^{-2\pi if}) \) can be calculated through methods outlined in the literature on spectral analysis such as Koopmans (2003), Jenkins and Watts (1968) etc. The impulse response weights can be obtained by the inverse transformation thus:

\[
\nu_k = \int_{-\frac{1}{2}}^{\frac{1}{2}} \nu(e^{-2\pi if})e^{2\pi if} df
\]

2.2. Transfer function-fuzzy logic modelling

It has been successfully proposed and demonstrated that transfer functions could be used as the predictor tool, with the variables \( \delta, \omega, b \) and \( N_t \) in Equations (2) and (4) serving as maintenance status and operation’s efficiency indicators (Nwobi-Okoye & Igboanugo, 2012, 2015). The simplest case of the models shown in Equations (2) and (4) occurs when \( r, s \) and \( N_t \) are zero and \( b \) is a constant. In other cases, \( \delta, \omega, b \) and \( N_t \) are regarded as fuzzy numbers. Hence for the parameter, \( \omega \), we can define a fuzzy set such that:
Here $A_i$ denotes membership function $i$ of $\omega$.

$\delta, \omega, b$ and $N_t$ being fuzzy variables could be used as inputs to a fuzzy inference system using either the MAMDANI, SUGENO or any other suitable fuzzy inference model to generate an output, $\lambda$, which measures the efficiency and evaluates performance of the process or system.

The transfer function parameters $\delta, \omega, b$ and $N_t$ are regarded as minor coefficients of performance ($COP_{min}$), while the parameter, $\lambda$, is regarded as the major coefficient of performance ($COP_{maj}$).

3. Methodology

Exploratory data analysis using Box Plots and other methods was used on the three year data obtained from our investigation hub. Subsequently, the input output data for the years 2011, 2012 and 2013 was used to determine the transfer function models, based on Equation (4), for each year. Following this, a plot of the three year input-output data was obtained. As a follow-up to the plot, stationarity of the series obtained from each plot was investigated. This was done using the plots of autocorrelation functions (ACF) and partial autocorrelation functions (PACF). The series that were found to be unstationary were differenced to achieve stationarity.

The input $X_1$ and output $Y_t$, and input $X_2$ and output $Y_t$ for each of the years was fitted with an autoregressive integrated moving average model in order to respectively estimate the prewhitened input series $\alpha_1t$ and $\alpha_2t$, and pretreated output series $\beta_1t$ and $\beta_2t$ respectively. Cross correlation functions, CCF($k$) of $\beta_1t\alpha_1t-k$ and $\beta_2t\alpha_2t-k$ was used to identify $r$, $s$ and $b$ parameters of the transfer function model. Following this, the impulse response weights $v_k$, estimated with spectral analysis, were used to estimate the transfer function parameters. Subsequently, the transfer function parameters were combined with fuzzy logic for the plant’s performance assessment.

In the fuzzy logic analysis, membership functions were developed for the input variables and the performance ratings of the output. The development of the membership function was based on intuition. On fuzzification of the input and output variables, based on the result of a sensitivity analysis, the Mamdani fuzzy logic inference system (Ross, 2004) was used to model the effects of the input variables on plant performance. The defuzzification was done using the centroid method in favour of other methods based on sensitivity analysis results.

4. Results

4.1. Transfer function modelling

Figure 2 shows the weekly raw materials consumption and the corresponding output (soap production) in the year 2012 for Promotex Nigeria Limited. The raw material $X_1$ is soap chips while the raw material $X_2$ is the additive. The results of the transfer function modeling are presented herein.

4.1.1. Analysis of the relationship between input 1 ($X_1$) and output (Y)

After the plots shown in Figure 2, the data ($X_1$ series) was investigated for stationarity, using the plots of the autocorrelation functions and PACF. The plots of the ACF and PACF which is shown in Figures 3 and 4 shows that series $X_1$ is stationary, hence differencing was not used to achieve stochastic stationarity. Examination of the ACF shown in Figure 3, the ACF at lags 1, 2, and 4 are significant. But examination of the PACF shown in Figure 4, only the PACF at lag 1 repeated its significance, and this is indicative that auto regression one (AR (1)) model is the appropriate model to use (see Box et al., 2008; DeLurgio, 1998; Nwobi-Okoye & Igboanugo, 2012, 2015 etc.).

Since $X_1$ series is AR (1), the formula for AR (1) models (Box et al., 2008; DeLurgio, 1998) is given by Equation (11):
But for AR (1) models, we have:

\[
X_{1t} = \theta_0 + \theta_1 X_{1t-1} + e_t \tag{11}
\]

But for AR (1) models, we have:

\[
ACF (1) = \theta_1 = 0.395 \tag{12}
\]

\[
\theta_0 = (1 - \theta_1) \mu \tag{13}
\]

\[
\theta_0 = (1 - 0.395) \times 5811.84
\]
Fitting the coefficients $\theta_0$ and $\theta_1$ to the formula for AR (1) models, Equation (14) is obtained.

$X_{1t} = 3516.1632 + 0.395X_{1t-1} + \epsilon_t$  \hspace{1cm} (14)

But

$e_t = \alpha_t$  \hspace{1cm} (15)

In forecasting form Equation (14) is transformed to Equation (16):

$\hat{X}_{1t} = 3516.1632 + 0.395X_{1t-1}$  \hspace{1cm} (16)

Pre-treating the output in the same way the input was transformed, we obtain:

$Y_t = 4146.0045 + 0.395Y_{t-1} + \epsilon_t$  \hspace{1cm} (17)

But

$e_t = \beta_t$  \hspace{1cm} (18)

In forecasting form Equation (17) is transformed to Equation (19):

$\hat{Y} = 4146.0045 + 0.395Y_{t-1}$  \hspace{1cm} (19)
The CCF between $\beta_t$ and $\alpha_t$ is shown in Figure 5. It has one significant CCF at lag zero (0). Hence, according to Box et al. (2008), Nwobi-Okoye and Igboanugo (2012, 2015) and DeLurgio (1998), the parameters $r$, $s$ and $b$ of the transfer function that supports such CCF pattern are 0, 0 and 0 respectively. In view of this fact, the CCF supports the following transfer function model:

$$y_t = \omega_1 x_{1t} + N_t$$

(20)

The Ljung-Box statistics shown in Table 1 which has a low value of 6.373 with a significance of 0.990 suggested that the residual may be white noise (see Box et al., 2008; DeLurgio, 1998; Nwobi-Okoye & Igboanugo, 2012, 2015 etc.), and upon further analysis of the residuals using ACF and PACF plots, the transfer function was found to have white noise residuals, hence we disregarded the noise term $N_t$ to obtain Equation (21).

$$y_t = \omega_1 x_{1t}$$

(21)

As shown by Box et al. (2008) and DeLurgio (1998),

$$\nu_{10} = \omega_{10}$$

(22)
v10 = impulse response for X1

But

\[ X_{1t} - \mu_1 = x_{1t} \]  

(23)

And

\[ Y_t - \mu_y = y_t \]  

(24)

Substituting Equation (24) into Equations (21) and (25) is obtained.

\[ Y_t = \mu_y + \omega 1_0 x_{1t} \]  

(25)

4.1.2. Analysis of the relationship between input 2 (X2) and output (Y)

The ACF and PACF of series X2 are shown in Figures 6 and 7 respectively. The plots of the ACF and PACF which is shown in Figures 6 and 7 show that series X2 is stationary. Inspection of the ACF shown in Figure 6 indicates that only the ACF at lag 1 is significant. But inspection of the PACF in Figure 7 indicates that the PACF at lag 1 repeated its significance, and this is indicative that auto regression one (AR (1)) model is the appropriate model to use.

The formula for AR (1) models (Box et al., 2008; DeLurgio, 1998) is given by Equation (26):

\[ X_{2t} = \theta_0 + \beta_1 X_{2t-1} + e_t \]  

(26)

But for AR (1) models, we have:
Figure 7. PACF of the input series.

Figure 8. CCF of the pre-whitened series.
Fitting the coefficients $\theta_0$ and $\theta_1$ into the formula for AR (1) models, Equation (29) is obtained. But in forecasting form Equation (29) is transformed to Equation (31):

$$ X_{2t} = 659.54091 + 0.391X_{2t-1} + \varepsilon_t $$

(29)

But

$$ \varepsilon_t = a_t $$

(30)

In forecasting form Equation (29) is transformed to Equation (31):

$$ \hat{X}_{2t} = 659.54091 + 0.391X_{2t-1} $$

(31)

Pre-treating the output in the same way the input was transformed, we obtain:

$$ Y_t = 4173.4161 + 0.391Y_{t-1} + \varepsilon_t $$

(32)

But

$$ \varepsilon_t = \beta_t $$

(33)

In forecasting form Equation (32) is transformed to Equation (34):

$$ \hat{Y}_t = 4173.4161 + 0.391Y_{t-1} $$

(34)

The fact that the CCF between $\beta_i$ and $\alpha_t$ is shown in Figure 8 has one significant CCF at lag zero (0) indicates that the parameters $r$, $s$ and $b$ of the transfer function that supports such CCF pattern are 0, 0 and 0 respectively. The transfer function model that corresponds to this is given by:

$$ y_t = \omega^2 \varepsilon_{2t} + N_t $$

(35)

The Ljung-Box statistics shown in Table 2 which has a low value of 6.568 with a significance of 0.993 suggested that the residual may be white noise. Consequent to a further analysis of the residuals using ACF and PACF plots, the transfer function was found to have white noise residuals, hence Equation (36) was obtained.

$$ y_t = \omega^2 \varepsilon_{2t} $$

(36)
As shown by Box et al. (2008), Nwobi-Okoye and Igboanugo (2012, 2015) and DeLurgio (1998),

\[ v_{20} = \omega_{10} \]

(37)

\[ v_{20} = \text{impulse response for } X_2 \]

But

\[ X_{2t} - \mu_2 = x_{2t} \]

(38)

And

\[ Y_t - \mu_y = y_t \]

(39)

Substituting Equation (39) into Equations (36) and (40) is obtained.

\[ Y_t = \mu_y + \omega_{20} x_{2t} \]

(40)

4.1.3. Obtaining the transfer function models

The analysis above of the relationship between \( X_1 \) and \( Y \), as well as \( X_2 \) and \( Y \) shows that the transfer function relating \( Y \) with \( X_1 \) and \( X_2 \) is of the form:

\[ Y_t = \omega_{10} X_{1t} + \omega_{20} X_{2t} \]

(41)

Since \( y_t = Y_t - \mu_y \), \( x_{1t} = X_{1t} - \mu_1 \) and \( x_{2t} = X_{2t} - \mu_2 \),

\[ Y_t = \mu_y + \omega_{10}(X_{1t} - \mu_1) + \omega_{20}(X_{2t} - \mu_2) \]

(42)

Based on the fact that

\[ v_{10} = \omega_{10} \text{ and } v_{20} = \omega_{20} \]

where

\[ v_{10} = \text{impulse response for } X_1 \text{ and } v_{20} = \text{impulse response for } X_2. \]

\[ Y_t = \mu_y + v_{10}(X_{1t} - \mu_1) + v_{20}(X_{2t} - \mu_2) \]

(43)

\( v_{10} \) and \( v_{20} \) can be obtained by spectral analysis.

From spectral analysis, \( v_{10} = 0.77586 \) and \( v_{20} = 3.4413 \). Therefore for 2012 operation of the soap plant, the transfer function is given by:

\[ Y_t = \mu_y + 0.77586(X_{1t} - \mu_1) + 3.4413(X_{2t} - \mu_2) \]

(44)
Following the procedures presented in the analysis above, the transfer function models for 2011 and 2013 were obtained. Table 3 shows the transfer function models for the three years operation of the plant.

Table 4 shows the minor coefficients of performance (COP) of the soap plant for three years. From the table, the used of additives was least in the year 2011 while the rate of conversion of the soap chips raw material to finished soap was highest in the same year. The overall plant performance evaluation was later calculated by fuzzy inference process.

4.2. Fuzzy logic

A typical fuzzy set $A_h$ for low raw material consumption is given by:

$$A_h = \left\{ \frac{0}{0} + \frac{1}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} \right\}$$  \hspace{1cm} (45)

The triangular membership functions developed by intuition from the fuzzy sets, for the input variables, which model the COP on Table 4, and the output variable (plant performance) are shown in Figures 9–12.

The lag variable, $b$, is zero in all the transfer function models shown in Table 4 necessitating the consideration of only the parameters $\omega_{1o}$ and $\omega_{2o}$ in the analysis. This led to the development of a two dimensional performance evaluation matrix as depicted in Table 5 and linguistic variables for performance rating depicted in Table 6.

![Figure 10. Membership function for coefficient of input 1 (soap chips raw material).](image)

![Figure 11. Membership function for coefficient of input 2 (additive).](image)

### Table 3. Transfer function models of the soap plant

<table>
<thead>
<tr>
<th>Year</th>
<th>Transfer function model ($v(B)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>$Y_t = \mu_x + 1.143727(X_{1t} - \mu_x) + 0.7043(X_{2t} - \mu_x)$</td>
</tr>
<tr>
<td>2012</td>
<td>$Y_t = \mu_x + 0.77586(X_{1t} - \mu_x) + 3.4413(X_{2t} - \mu_x)$</td>
</tr>
<tr>
<td>2013</td>
<td>$Y_t = \mu_x + 0.799972(X_{1t} - \mu_x) + 2.219733(X_{2t} - \mu_x)$</td>
</tr>
</tbody>
</table>

### Table 4. COP of the soap plant

<table>
<thead>
<tr>
<th>Year</th>
<th>COP (soap chips raw material) $\omega_{1o}$</th>
<th>COP (additive) $\omega_{2o}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1.143727</td>
<td>0.7043</td>
</tr>
<tr>
<td>2012</td>
<td>0.77586</td>
<td>3.4413</td>
</tr>
<tr>
<td>2013</td>
<td>0.799972</td>
<td>2.219733</td>
</tr>
</tbody>
</table>
4.2.1. Fuzzy rules and inference system

From Tables 5 and 6, the fuzzy logic rules were developed. Some of the rules are:

If Soap chips raw material is Low and additive is Very high then performance rating is Poor.

If Soap chips raw material is Low and additive is Low then performance rating is Fair.

In total we developed sixteen rules. The rules were aggregated to a single fuzzy output and defuzzified using centroid method after sensitivity analysis of other methods to obtain the performance ratings shown in Table 7. To get an insight into how the performance rating was obtained, let us examine the fuzzy inference process shown in Figure 13.

Examining Figure 13, if we assume that the two rules earlier stated are the only two rules and the coefficients $\omega_1$ and $\omega_2$ are 0.5, then the inputs to the fuzzy inference system are 0.5 for soap chips and 0.5 for additive. The inference system consists of three stages. Each of the first and second stages of the process consists of three steps namely: the inputs, logical operation and implication method, while the third stage is known as aggregation. After aggregation the centroid method was used to determine the output from the result of the aggregation shown in Figure 13. Thus since the centroid of the shaded trapezium is 0.5 as shown in Figure 13, the output is 0.5.

Applying this principle to the sixteen rules to the fuzzy inference system similar to the one in Figure 13, the results shown in Table 7 were obtained. As shown in Table 7, the best performance
rating denoted by \( \lambda \) is 0.761 which occurred in the year 2011. The linguistic variable corresponding to 0.761 as shown in the table is “Good”. Hence, the best performance over the three years is rated “Good”. Similarly, the worst performance occurred in the year 2012 with \( \lambda \) value equal to 0.250. The linguistic variable corresponding to 0.250 as shown in the table is “Poor” and the worst performance is rated “Poor”.

5. Discussion
The stochastic nature of input and output to production systems as stated by Nwobi-Okoye and Igboanugo (2012, 2015) is confirmed by Figure 4. The utility of this result is ever important considering the fact that the concept of using COP, a superior metric for evaluating the performance of processes, introduced by Nwobi-Okoye and Igboanugo (2012, 2015) is simple to use in SISO processes. When this concept is extended to MISO processes it becomes problematic because and as the inputs increases the complexity increases. The problem of handling the complexity of assessing the performance of MISO processes using the concept of COP as this study demonstrated is solved by the use of fuzzy logic.

### Table 7. COP of the soap plant

<table>
<thead>
<tr>
<th>Year</th>
<th>COP (soap chips) ( \omega_1 )</th>
<th>COP (additive) ( \omega_2 )</th>
<th>Performance rating (overall) ( \lambda )</th>
<th>Linguistic variable of performance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td>1.143727</td>
<td>0.7043</td>
<td>0.761</td>
<td>Good</td>
</tr>
<tr>
<td>2012</td>
<td>0.77586</td>
<td>3.4413</td>
<td>0.250</td>
<td>Poor</td>
</tr>
<tr>
<td>2013</td>
<td>0.799972</td>
<td>2.219733</td>
<td>0.395</td>
<td>Fair</td>
</tr>
</tbody>
</table>

Figure 13. The fuzzy inference system.

![Fuzzy Inference System Diagram](image-url)
The level of maintenance and quality of machines and equipment used in the production processes, as well as the quality of raw materials used in the production process determines the COP and overall performance rating of the plant studied here. Equation (42) in all its practicality, shows that given autonomous values $\mu_y$, $\mu_1$ and $\mu_2$, for every unit increase in $X_1$, the output $Y$ is increased by $\omega_10$ the first minor COP and for every unit increase in $X_2$, the output $Y$ is increased by $\omega_20$ the second minor COP.

Comparative examination of the year 2012 and 2013 performances shows that whereas the rate of transformation of soap chips to finished soap is slightly lower in 2013, as indicted by lower value of $\omega_10$, the quality of the soap chips used in the year was much lower, as indicted by higher value of $\omega_20$, thus making the overall performance rating $\lambda$ lower than in 2013. Higher values of $\omega_10$ indicates that the machines functioned better during the period.

The parameter $\omega_10$ measures how effective the process is in transforming base/raw material into finished soap without losses, while $\omega_20$ is a measure of the additional material requirements to improve soap quality which depends on the quality of the base materials and resultant soap chips.

Consider Table 8 an extract from the soap production data for the year 2011.

The mass balance for the soap production process shown in Table 1 is given by:

\[
\text{Mass of base materials} + \text{Losses/wastes} = \text{Mass of chips}
\]

\[
\text{Mass of base materials} + \text{additives} + \text{Losses/wastes} = \text{Mass of produced soap}
\]

Table 8. Extract from soap production data for 2011

<table>
<thead>
<tr>
<th>Period</th>
<th>Input (base raw material) (kg)</th>
<th>Chips (kg)</th>
<th>Additive (kg)</th>
<th>Output (produced soap) (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 2 (June 2011)</td>
<td>4,054</td>
<td>3,012</td>
<td>600</td>
<td>3,601</td>
</tr>
<tr>
<td>Week 3 (June 2011)</td>
<td>4,060</td>
<td>2,905</td>
<td>580</td>
<td>3,477</td>
</tr>
<tr>
<td>Week 1 (January 2011)</td>
<td>4,004</td>
<td>3,112</td>
<td>778</td>
<td>3,841</td>
</tr>
</tbody>
</table>

Practically, the production manager desires $\omega_10$ to be very high indicating high efficiency in raw material transformation to finished product and $\omega_20$ to be low indicating low requirements for additives due to high quality of raw materials. This fact guided the establishment of the fuzzy rules which is peculiar to the process under consideration.

The fuzzy rules and membership function boundaries are not sacrosanct. Fuzzy systems are evolving and evolutionary systems. Based on experience and availability of new facts, adjustments could be made to the system to reflect new realities.
Generally, the major advantages of this method as stated by Nwobi-Okoye and Igboanugo (2015) are:

(a) Greater accuracy in efficiency measurement over a given period.
(b) Statistically robust efficiency measurements.
(c) Better plant fault diagnosis and superior aid to predictive and preventive maintenance.

In addition to the ones mentioned above, another major advantage of this method is that fuzzy transfer function modeling can simultaneously access in one platform quality, production efficiency and production time of a plant or machine.

What could be regarded as the major drawback of this method is the complexity of computations required to implement the method.

5.1. General performance modelling of systems and processes using transfer function

Generally, a system is defined as a set of interacting elements designed to perform a particular function. When a system receives an input stimulus, it will respond to the input signals according to the internal structure of the system (Lai, 1979). The system’s internal structure, which governs the input–output relationship, filters the input signal into the output response. The input signals at time \( t-b \) is received by the output at time \( t \). The variable \( b \) is known as the time delay or response lag.

The mechanism of filtration of the input(s) could be modeled by a deterministic differential equation, on the assumption that the filtration process is perfect without disturbances. This is the ideal case which hardly occurs in practice. Thus, in practice the filter is affected by disturbances, hence the filtration process is stochastic, and can only be effectively modeled by using stochastic or probabilistic equations. The discrete transfer function equations are one of the most accurate and powerful stochastic modeling tools used to model the filtration process.

The case study of this work, soap production process/system, is an exemplification of a typical system. Hence, what obtains in soap production system is obtainable in every other system in the sense that every system has its characteristic transfer function as depicted in Figure 14. For example, power generation systems, such as hydropower generation systems, gas based power generation system etc. power distributions systems, production processes/systems, chemical production processes/systems etc. have their characteristic transfer functions. The nature of the transfer function characteristics helps to determine the benchmarks for performance bounds. It is noteworthy that the system depicted in Figure 14 could be SISO system, SIMO system, MISO system or MIMO system.

The conceptual model of performance modeling using transfer function and fuzzy transformations is depicted in Figure 15. In reference to Figure 15, the general procedure for performance modeling with transfer function and fuzzy transformations begins with the system’s input/output relationship.
plots. The input/output plots consist of the graphs of the input(s) and output(s), plot of the ACFs and PACFs of the input(s) and output(s), the plots of the CCFs etc. These plots aid the identification of the transfer function model and some details of the methodology were shown in Section 4 of this work. After model identification, the tentative model undergoes some diagnostic checks to confirm its adequacy. During the adequacy check, if the model diagnostics check detects some errors in the modeling, the process moves back to the identification stage in order to redo the model identification. On the other hand, if the model diagnostic check detects no error, the transfer function parameters’ efficiency are improved by optimization, and the process moves to the next stage. If as stated in Section 3, \( r, s \) and \( N_t \) are zero and \( b \) is constant, the single unique parameter is used as the system COP. Alternatively, if more than one parameter is needed to model performance, the parameters are used as minor COPs. The minor coefficients of performance undergo fuzzification. In order words fuzzy membership functions are created for the minor COPs. Using the fuzzified variables as an input to a fuzzy inference process, the defuzzified output becomes the major coefficient of performance (\( \lambda \)).

6. Conclusion
Modelling multivariate processes as typified by this research is quite challenging. The complexity increases as the number of inputs increases. This research has succeeded in furthering the works of Nwobi-Okoye and Igboanugo (2012, 2015) by extending the concept of coefficient of performance to multi input single output (MISO) processes. This work considered only the two input single output process as typified by the soap production process, but could be extended to other MISO processes.
such as soft drink production, cement production etc. The complexity of interaction between raw material quality and operations efficiency in determining plant’s performance is a major triumph of this work. This work is even more imperative as the world moves towards greater energy and raw materials conversion efficiency to reduce the carbon footprint and improve the environment.

7. Future work
For future work as fallout of this new method, it is suggested as follows:

(1) It is suggested that the modelling of MISO processes with more than two inputs, SIMO production processes as well as MIMO processes should be investigated and modeled. It is equally suggested that the concept of COP for MISO processes introduced in this work, which was used in analysis of one type of MISO production processes (soap plant) be further applied to other types of production processes/systems such as: Refinery operations, cement production, etc.

(2) Secondly, the concept of COP for MISO processes should be applied extensively to oil and gas industries, power transmission and distribution systems, chemical and communication industries, econometric modeling, as well as every system in general.

(3) Furthermore, the concept of transfer function modelling could be applied to the prediction of pipeline failure/fracture/rupture as well as failure of critical parts of machineries or equipment. Even pipeline leaks could be detected by discrete transfer function modelling. These should be the subject of further investigation.

Nomenclature, symbols and notations

- $k$: lag variable
- $\beta_t$: pretreated output series
- $\alpha_t$: prewhitened input series
- $v(B)$: transfer function
- $B$: backshift operator
- $Y_t$: process output at time $t$
- $X_t$: process input at time $t$
- $y_t$: differenced output series
- $x_t$: differenced input series
- $\hat{Y}_t$: output forecast
- $\hat{X}_t$: input forecast
- $a_t$: error term/white noise
- $\nu_k$: impulse response weight at lag $k$
- $h$: ACF/PACF lag
- $q$: order of moving average operator
- $p$: order of autoregressive operator
- $d$: number of differencing
- $\theta$: autoregressive operator
- $\phi$: autoregressive operator
- $\Xi$: coefficient of output variable of differential equation
- $H$: coefficient of input variable of differential equation
- $\chi$: covariance function
- $b$: transfer function lag
- $\omega$: difference equation variable for input
- $\delta$: difference equation variable for output
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Author’s Contributions
Nwobi-Okoye conceived the study and participated in its design and coordination. Okiy gathered the data and helped in the data analysis and interpretation. All authors read and approved the final manuscript.

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References


