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MECHANICAL ENGINEERING | SHORT COMMUNICATION

Liquid-liquid-gas stratified fractional model flow: I. Model in two parallel plates

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Abstract: Three phase fluid flow is a crucial subject in many industries such as chemical, oil and gas. Actually, software and empirical predictions exist for pressure drop in liquid-liquid or liquid-gas stratified flow but liquid-liquid-gas flow is not well known. Simple prediction of pressure gradient in three phase flow will lead to decisions and the use or not of robust methods for a better design of an energy efficient transportation system. This paper presents a solution for the velocity profile and pressure drop for liquid-liquid-gas flow applied in parallel plates. Also, by using fractional calculus, we show a method to determine the pressure drop in liquid-liquid-gas systems, which allows us to consider the effect of flow regime on velocity profile and pressure drop. We also propose a relationship to determine the order of fractional derivatives. This is related to the Reynolds number corresponding to each phase, which is found flowing at the specified volumetric speed. The solved unified model is useful for laminar, transient of turbulent flow.

Subjects: Mechanical Engineering; Energy & Fuels; Control Engineering

Keywords: stratified flow; three phase pressure gradient; fractional flow model

1. Introduction

In the petroleum industry, fluid transport challenges are increasing due to characteristics of the fluid produced, particularly its viscosity and compositional variation, or phase separation (Hosseini-Dastgerdi, Tabatabaei-Nejad, Sahraei & Nowroozi 2014; Leontaritis & Mansoori, 1987).

Sometimes it is necessary to add other chemical products to the current for better transport (Adams, 2014). It is known that impact on stability means changes in viscosity, which affect the pressure drops (Becker, 1997; Bird, Stewart, & Lightfoot, 2002). Using viscosity reducers i.e. BRV, we can achieve fluid pressure drop in a pipeline (Suárez-Domínguez, Palacio-Pérez, Rodríguez Contreras, & Izquierdo-Kulich, 2014).

There are many advantages to multiphase flow such as a lower pressure drop when the higher viscosity fluid is not in contact with the walls of the pipe (Bensakhria, Peysson, & Antonini, 2004; He, Lin, Li, Sui, & Xu, 2015).

ABOUT THE AUTHORS

The authors are specialists in fluid flow phenomena from the University of Havana, Cuba, the State of Tamaulipas, and the UNAM (National University of Mexico) and Centro de Investigacion Aplicada y Tecnologica (CIAT) that are currently involved in proposing sustainable methods for the downstream transport of heavy oil.

PUBLIC INTEREST STATEMENT

The work presented strives to simplify very complex phenomena through the calculus of the so-called fractional derivatives, that is non-the-less straightforward. The proposed method is applied to the stratified flow between parallel plates of water, oil and air. The analytical results are encouraging within the limitations imposed in the modeling of the simple geometry.

Mathematical models used to predict behavior of the pressure drop in these systems under different flow regimes are fundamentally empirical. This is due to the complex flow patterns that may occur, which are determined by interfacial properties, flow regimes, and system geometry as well as chemical and physical properties of the fluid involved. In this sense, the deterministic models established based on phenomenological equations consider a total separation of phases and laminar flow regime.

The proposed phenomenological models are derived from a consideration of complete separation between the phases.

Historically important periods are recognized in model equations development of two-phase flow (Shippen & Bailey, 2012). Brauner (2013) has explained several models for different liquid-liquid flow types and Zhang, Wang, Sarica, and Brill (2003) made an empirical unified model for liquid-gas; Waqas et al. (2016) in the other hand, have studied about viscous dissipation, skin factions and mathematical models using Nusselt number. On the other hand, computational fluid dynamics (CFD) is used for prediction of multiphase flow but time and computational resources must be known to obtain results.

There is not much literature for liquid-liquid-gas three phase flow or models to explain this phenomenon.

We presented a velocity profile and pressure drop analytical model to explain three phase flow in parallel plates and offer a mathematical method to explain pressure drop. The model takes into account the influence of flow regime through an index associated with fluctuations of speed and the implementation of fractional differential calculus.

2. Method
Figures 1a and 1b show the concept used in mathematical model.

To obtain the three phase model the following considerations are established:

1. A total separation exists between the phases, for which it is supposed that the flow occurs between two parallel plates, where the thickness is much larger than the distance between them, so that transport of momentum takes place in a direction that is perpendicular to the plate walls.

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2. Plates are horizontal, fluids are immiscible and they arrange according to density.
3. The conditions of gas are estimated with ideal gas law.

From a system of continuity and momentum transport equations for a Cartesian coordinate system of differential equations describing behavior of velocity profile, we obtain:

$$
0 = \begin{cases}
-\frac{dP}{dl} + \frac{\mu_1}{\rho_1}\frac{d^2 w_2}{dx^2} & \text{if} \quad 0 < x < h_1 \\
-\frac{dP}{dl} + \frac{\mu_2}{\rho_2}\frac{d^2 w_2}{dx^2} & \text{if} \quad h_1 < x < h_2 \\
-\frac{dP}{dl} + \frac{\mu_3}{\rho_3}\frac{d^2 w_3}{dx^2} & \text{if} \quad h_2 < x < H
\end{cases}
$$

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Figure 1a. Parallel plates used in development of mathematical model.
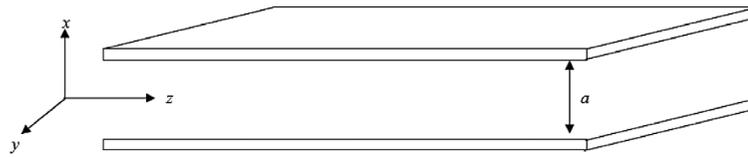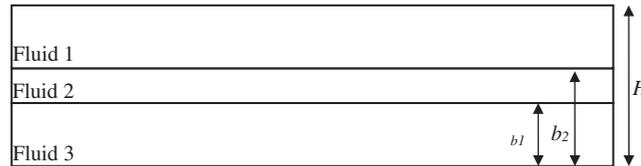

Figure 1b. Stratified flow is developed with two interfaces between the fluids.

where P is pressure (Pa), l the plate length, μ is viscosity, ρ is density, w is the mass velocity, H is total separation between plates, x is the position between plates and h is the interphase position between fluids, in which case it is established:

$$\rho_1 > \rho_2 > \rho_3$$

$$\rho_3 = \frac{P_M \times 10^{-3} P}{8.314510 T}$$

We define the dimensionless variables: $\varphi = \frac{w}{W}, l = \frac{(2x-H)}{H}, b = \frac{(2h-H)}{H}$ and the dimensionless parameters $\Phi = -\frac{dP}{4dl} \frac{H^2 \rho_1}{W \mu_1}, k_2 = \frac{\mu_1 \rho_2}{\mu_2 \rho_1}$ and $k_3 = \frac{\mu_1 \rho_3}{\mu_3 \rho_1}$

so that the equation is rewritten as:

$$0 = \begin{cases} \Phi + \frac{d^2 \varphi_1}{dl^2} & -1 < l < b_1 \\ \Phi k_2 + \frac{d^2 \varphi_2}{dl^2} & \text{if} \quad b_1 < l < b_2 \\ \Phi k_3 + \frac{d^2 \varphi_3}{dl^2} & b_2 < l < 1 \end{cases}$$

It is taken into account that gas is flowing in a turbulent manner. The effect of turbulence occurs in a nonlinear behavior on fluid shear tensor with respect to the distance between the plates, which will be described through the use of fractional differential equations (Tarasov & Vasily, 2010). According to this formalism, each differential equation order is related to the magnitude of fluctuations in the corresponding fluid and, therefore, is identified as the turbulence index, which is equal to 1 for laminar regime. From this consideration, it is convenient to express the non-dimensional system of differential equations in the form:

$$0 = \Phi + \frac{d^\alpha \tau_1}{dl^\alpha} \text{ and } \tau_1 = \frac{d\varphi_1}{dl} \text{ if } -1 < l < b_1$$

$$0 = \Phi k_2 + \frac{d^\beta \tau_2}{dl^\beta} \text{ and } \tau_2 = \frac{d\varphi_2}{dl} \text{ if } b_1 < l < b_2$$

$$0 = \Phi k_3 + \frac{d^\gamma \varphi_3}{dl^\gamma} \text{ and } \tau_3 = \frac{d\varphi_3}{dl} \text{ if } b_2 < l < 1$$

where α, β and γ are the turbulence indices corresponding to each fluid. The turbulence index (I) for each fluid is determined in the following manner: for each fluid, the corresponding Reynolds number is calculated corresponding to the specified volumetric velocity:

$$R_e = 2\frac{HV\rho}{\mu}$$

And the value of (I) is calculated through the numerical solution of the corresponding equation in accordance with its Reynolds number value:

$$I = \begin{cases} 1.9775 \times 10^{-2}(R_e)^{\frac{3}{4}} - I\Gamma(I)(I+2) & \text{si} \quad 2.1 \times 10^3 < R_e < 4 \times 10^4 \\ I\Gamma(I)(I+2) - \dfrac{0.019\,29R_e}{\ln^2\frac{1}{R_e}\left(0.233\,7R_e^{\frac{10}{9}}R_e+6.9\right)} & \text{si} \quad 4 \times 10^4 < R_e < 10^8 \\ 1 & \text{si} \quad R_e < 2.1 \times 10^3 \end{cases}$$

where Γ is the Gamma function.

The solution to the fractional differential equation system is:

$$\varphi_1 = C_{12} + C_{11}l - |l|^{\alpha+1}\frac{\Phi_1}{(\alpha+1)\Gamma(\alpha+1)} \quad \text{with } \tau_1 = C_{11} - \frac{1}{\alpha}\frac{\Phi_1}{\Gamma(\alpha)}|l|^\alpha\frac{l}{|l|}$$

$$\varphi_2 = C_{22} + C_{21}l - |l|^{\beta+1}\frac{\Phi_2}{(\beta+1)\Gamma(\beta+1)} \quad \text{with } \tau_2 = C_{21} - \frac{\Phi_2}{\beta\Gamma(\beta)}|l|^\beta\frac{l}{|l|}$$

$$\varphi_3 = C_{32} + C_{31}l - |l|^{\gamma+1}\frac{\Phi_3}{(\gamma+1)\Gamma(\gamma+1)} \quad \text{with } \tau_3 = C_{31} - \frac{\Phi_3}{\gamma\Gamma(\gamma)}|l|^\gamma\frac{l}{|l|}$$

To determine integer constants, the following boundary conditions are taken:

$\varphi_1(-1) = 0$
$\varphi_3(1) = 0$
$\varphi_1(b_1) = \varphi_2(b_1)$
$\tau_1(b_1) = \tau_2(b_1)$
$\varphi_2(b_2) = \varphi_3(b_2)$
$\tau_2(b_2) = \tau_3(b_2)$

so that integer constants are determined from the solution of the differential equation:

$$0 = C_{12} + C_{11}(-1) - |(-1)|^{\alpha+1}\frac{\Phi}{(\alpha+1)\Gamma(\alpha+1)}$$

$$0 = C_{32} + C_{31}(1) - |(1)|^{\gamma+1}\frac{\Phi k_3}{(\gamma+1)\Gamma(\gamma+1)}$$

$$C_{12} + C_{11}(b_1) - |(b_1)|^{\alpha+1}\frac{\Phi}{(\alpha+1)\Gamma(\alpha+1)} = C_{22} + C_{21}(b_1) - |(b_1)|^{\beta+1}\frac{\Phi k_2}{(\beta+1)\Gamma(\beta+1)}$$

$$C_{11} - \frac{1}{\alpha}\frac{\Phi}{\Gamma(\alpha)}|(b_1)|^\alpha\frac{b_1}{|b_1|} = C_{21} - \frac{\Phi k_2}{\beta\Gamma(\beta)}|(b_1)|^\beta\frac{b_1}{|b_1|}$$

$$C_{22} + C_{21}(b_2) - |(b_2)|^{\beta+1}\frac{\Phi k_2}{(\beta+1)\Gamma(\beta+1)} = C_{32} + C_{31}(b_2) - |(b_2)|^{\gamma+1}\frac{\Phi k_3}{(\gamma+1)\Gamma(\gamma+1)}$$

$$C_{21} - \frac{\Phi k_2}{\beta\Gamma(\beta)}|(b_2)|^\beta\frac{b_2}{|b_2|} = C_{31} - \frac{\Phi k_3}{\gamma\Gamma(\gamma)}|(b_2)|^\gamma\frac{b_2}{|b_2|}$$

Defining the functions $f_1(l) = \frac{\varphi_1(l)}{\Phi}$, $f_2(l) = \frac{\varphi_2(l)}{\Phi}$ and $f_3(l) = \frac{\varphi_3(l)}{\Phi}$ the average dimensionless mass velocity with respect to the flow area is determined as $\varphi = \Phi f$, where:

$$f = \frac{\left[\int_0^Z\int_{-1}^{b_1}f_1(l)\,dldz + \int_0^Z\int_{b_1}^{b_2}f_2(l)\,dldz + \int_0^Z\int_{b_2}^{1}f_2(l)\,dldz\right]}{\left[\int_0^Z\int_{-1}^{1}dldz\right]}$$

knowing for definition $\varphi = 1$ then $\Phi f = 1$.

Substituting the dimensionless parameters, a differential equation is obtained which must be solved to find the pressure gradient in the plates:

$$-\frac{dP}{dl} = \frac{4}{H^2 f}\frac{\mu_1}{\rho_1}W$$

$$P(0) = P_0$$

where the average mass velocity depends on average volumetric velocity established at the point $l = 0$ and the composition of the systems is expressed by the mass fraction:

$$W = \frac{V}{\frac{1}{\rho_1}x_1 - \frac{1}{\rho_2}(x_1 + x_3 - 1) + \frac{8314.5}{P}\frac{T}{P_M}x_3}$$

Interphase positions are related with the system composition according to:

$$x_1 = \frac{\left[\int_0^Z \int_{-1}^{b_1} f_1(l)\,dldz\right]}{\left[\int_0^Z \int_{-1}^{b_1} f_1(l)\,dldz + \int_0^Z \int_{b_1}^{b_2} f_2(l)\,dldz + \int_0^Z \int_{b_2}^{1} f_2(l)\,dldz\right]}$$

$$x_3 = \frac{\left[\int_0^Z \int_{b_2}^{1} f_2(l)\,dldz\right]}{\left[\int_0^Z \int_{-1}^{b_1} f_1(l)\,dldz + \int_0^Z \int_{b_1}^{b_2} f_2(l)\,dldz + \int_0^Z \int_{b_2}^{1} f_2(l)\,dldz\right]}$$

$$x_2 = 1 - x_1 - x_3$$

Mass composition of the systems is constant and functions f_1, f_2 and f_3 are system dependent so that the interphase position changes with pressure.

A condensed method to obtain pressure drop (see annex 1).

3. Results and discussion

To analyze the proposed model, a water-oil-gas system was studied, the properties of which are presented in Table 1.

The system parameters are shown in Table 2.

A system with a constant mass fraction of oil (0.94) and the following water and gas mass fractions:
  Case 1. $(x_1, x_3) = (0.01, 0.05)$
  Case 2. $(x_1, x_3) = (0.05, 0.01)$

were considered.

For the previously established operational conditions and applying the proposed methodology, the turbulence indices for water = 3.0704, crude oil = 1 and gas = 1 were obtained.

Solving the equation according to the proposed method, the interface behavior was obtained with respect to pressure and system composition. The corresponding equations were linealized using statistical techniques, obtaining, in this case:

$$b_1 = 1.7493 \times 10^{-8}P + 1.5634x_1 + 0.67994x_3 - 1.0025$$
$$b_2 = 1.1454 \times 10^{-7}P - 0.82384x_1 - 3.7713x_3 + 0.97655$$

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Table 1. Characteristics of the considered three phase fluid flow		
Fluid	**Viscosity (Pa s)**	**Density (kg m^{-3})**
1. Water	10^{-3}	1,000
2. Oil	35	980
3. Gas PM = 20	2×10^{-5}	$\frac{20\times10^{-3}(P)}{(8.314510)(T)}$

Table 2. Parameters for the system analyzed in this work	
Variable	**Value**
Distance between plates	0.1 m
Plate length	1,000 m
Volumetric velocity	0.1 m s^{-1}
Wall rugosity	0 m
Input pressure	4×10^5 Pa
Temperature	298.15 K

Results are shown in Figure 2, where we observe that pressure drops with increasing gas volume. This result is expected if we take into account that gas is the phase which has a lower viscosity and density than crude alone. When pressure drop for oil in monophasic flow is calculated and it is compared with liquid-liquid-gas model, a 95% lower pressure drop was found compared to monophasic

Figure 2. Pressure behavior in the system with respect to pipe length for two different compositions in a triphasic system.
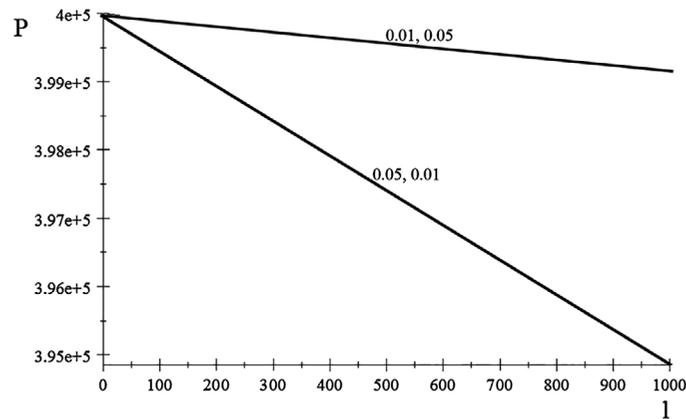

Figure 3. Interphase position of water-oil (bottom) and oil-gas (top) in 1,000 m of the plates.
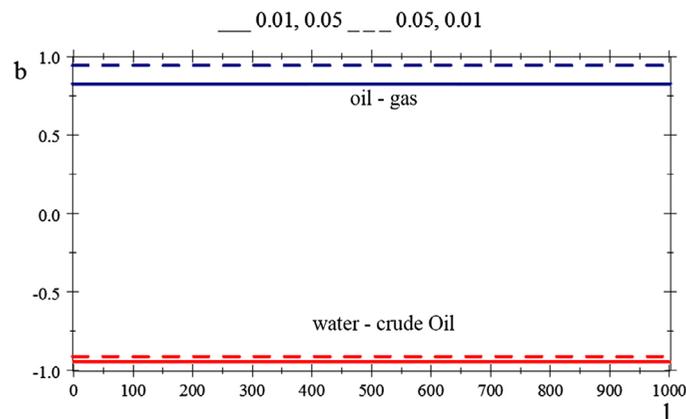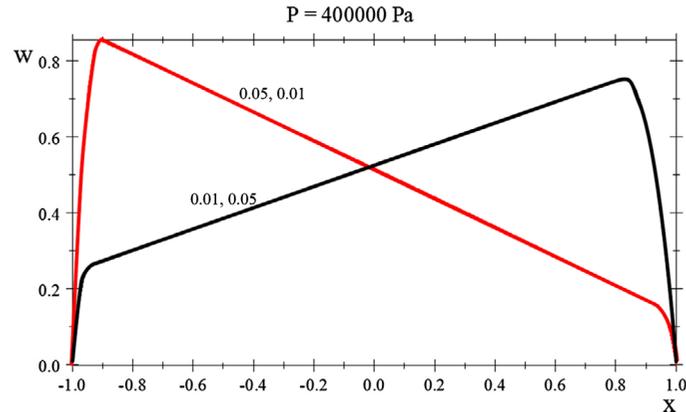
Figure 4. Dimensionless velocity (w) profiles according to the thickness of separation (x).

model. Thus, although it is better to transport a fluid in multiphasic flow as we have found, or in biphasic flow according to Bensakharia, it is necessary to probe the solution with more experimentation.

It is possible that this model may be applied to high viscosity fluid such as heavy or extraheavy crude oil, to provide greater convenience in transport by this method.

Figure 3 shows interphase position along the plates. It is shown that interphases don`t change significantly over distance, and therefore their position can be considered practically constant along the length of the pipe. This is a limiting of the model because it only happens in some real cases, mainly with low velocities.

Non dimensionless velocity profiles obtained for each composition for the initial pressure is shown in Figure 4.

4. Conclusions

We present a model and method to find pressure drop in two parallel plates. Results indicate that the movement of three phase flow where gas and higher density, less viscous liquid surround a high viscosity liquid, with interphases well defined, is better than the one-phase, higher viscosity flow.

The model has as limitation that it cannot set the conditions under which the formation of two interfaces is given, although a similar result may reasonably be expected if movement of fluids occurs in conditions explained previously. It is recommended to expand further work considering some concentration profiles as described by Hayat, Khan, Waqas, Alsaedi, and Yasmeen (2016) as well as performing experimental part and comparing both results.

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Annex 1

For the solution of the equation system we need to apply numerical techniques. We explain those briefly in three steps with a calculation algorithm:

Step 1. It is necessary to specify viscosity and density of each one of the liquid-liquid-gas fluids and the geometrical and operation parameters of the system such as separation of the plates, plate lengths, volumetric velocity, wall roughness, inlet pressure and temperature.

Step 2. To determine the positions behavior of each interphase with respect to pressure and system composition, it is necessary to make a simulation to determine behavior of the composition value for different values of both interphases and pressure. Once results are obtained, a multiple regression statistical adjustment is made from which an algebraic equations system can be obtained to determine change of interphase composition with respect to pressure for a specified composition.

For this an mathematical design will be considered with three variables (b1, b2, b3) and pressure at two levels which need eight calculations, for optimal multiple regression application. For every variable and pressure we calculate

(a) Reynolds number to the corresponding volumetric velocity.

$$R_e = 2\frac{HV\rho}{\mu}$$

(b) Turbulence index for each fluid according to:.

$$
I = \begin{cases}
1.977\,5 \times 10^{-2}\left(R_e\right)^{\frac{3}{4}} - I.\Gamma(I)\left(I+2\right) & \text{si} \quad 2.1 \times 10^3 < R_e < 4 \times 10^4 \\[2mm]
I.\Gamma(I)\left(I+2\right) - \dfrac{0.019\,29 R_e}{\ln^2 \frac{1}{R_e}\left(0.233\,7 R_e^{\frac{10}{9}} R_e + 6.9\right)} & \text{si} \quad 4 \times 10^4 < R_e < 10^8 \\[4mm]
1 & \text{si} \quad R_e < 2.1 \times 10^3
\end{cases}
$$

(c) Integer constants given by:

$$C_{11} = \tfrac{1}{2}B - \tfrac{1}{2}A + \tfrac{1}{2}x_{11} - \tfrac{1}{2}x_{12} + \tfrac{1}{2}x_{21} - \tfrac{1}{2}x_{22} + \tfrac{1}{2}x_{31} - \tfrac{1}{2}x_{32} + \tfrac{1}{2}x_{41} - \tfrac{1}{2}x_{42} - \tfrac{1}{2}b_1 x_{21} + \tfrac{1}{2}b_1 x_{22} - \tfrac{1}{2}b_2 x_{41} + \tfrac{1}{2}b_2 x_{42}$$

$$C_{12} = \tfrac{1}{2}A + \tfrac{1}{2}B + \tfrac{1}{2}x_{11} - \tfrac{1}{2}x_{12} + \tfrac{1}{2}x_{21} - \tfrac{1}{2}x_{22} + \tfrac{1}{2}x_{31} - \tfrac{1}{2}x_{32} + \tfrac{1}{2}x_{41} - \tfrac{1}{2}x_{42} - \tfrac{1}{2}b_1 x_{21} + \tfrac{1}{2}b_1 x_{22} - \tfrac{1}{2}b_2 x_{41} + \tfrac{1}{2}b_2 x_{42}$$

$$C_{21} = \tfrac{1}{2}B - \tfrac{1}{2}A + \tfrac{1}{2}x_{11} - \tfrac{1}{2}x_{12} - \tfrac{1}{2}x_{21} + \tfrac{1}{2}x_{22} + \tfrac{1}{2}x_{31} - \tfrac{1}{2}x_{32} + \tfrac{1}{2}x_{41} - \tfrac{1}{2}x_{42} - \tfrac{1}{2}b_1 x_{21} + \tfrac{1}{2}b_1 x_{22} - \tfrac{1}{2}b_2 x_{41} + \tfrac{1}{2}b_2 x_{42}$$

$$C_{22} = \tfrac{1}{2}A + \tfrac{1}{2}B - \tfrac{1}{2}x_{11} + \tfrac{1}{2}x_{12} + \tfrac{1}{2}x_{21} - \tfrac{1}{2}x_{22} + \tfrac{1}{2}x_{31} - \tfrac{1}{2}x_{32} + \tfrac{1}{2}x_{41} - \tfrac{1}{2}x_{42} + \tfrac{1}{2}b_1 x_{21} - \tfrac{1}{2}b_1 x_{22} - \tfrac{1}{2}b_2 x_{41} + \tfrac{1}{2}b_2 x_{42}$$

$$C_{31} = \tfrac{1}{2}B - \tfrac{1}{2}A + \tfrac{1}{2}x_{11} - \tfrac{1}{2}x_{12} - \tfrac{1}{2}x_{21} + \tfrac{1}{2}x_{22} + \tfrac{1}{2}x_{31} - \tfrac{1}{2}x_{32} - \tfrac{1}{2}x_{41} + \tfrac{1}{2}x_{42} - \tfrac{1}{2}b_1 x_{21} + \tfrac{1}{2}b_1 x_{22} - \tfrac{1}{2}b_2 x_{41} + \tfrac{1}{2}b_2 x_{42}$$

$$C_{32} = \tfrac{1}{2}A + \tfrac{1}{2}B - \tfrac{1}{2}x_{11} + \tfrac{1}{2}x_{12} + \tfrac{1}{2}x_{21} - \tfrac{1}{2}x_{22} - \tfrac{1}{2}x_{31} + \tfrac{1}{2}x_{32} + \tfrac{1}{2}x_{41} - \tfrac{1}{2}x_{42} + \tfrac{1}{2}b_1 x_{21} - \tfrac{1}{2}b_1 x_{22} + \tfrac{1}{2}b_2 x_{41} - \tfrac{1}{2}b_2 x_{42}$$

where:

$$x_{11} = \left|\left(b_1\right)\right|^{\alpha+1} \frac{1}{(\alpha+1)\,\Gamma(\alpha+1)}$$

$$x_{12} = \left|\left(b_1\right)\right|^{\beta+1} \frac{k_2}{(\beta+1)\,\Gamma(\beta+1)}$$

$$x_{21} = \frac{1}{\alpha}\frac{1}{\Gamma(\alpha)}\left|\left(b_1\right)\right|^{\alpha} \frac{b_1}{|b_1|}$$

$$x_{22} = \frac{k_2}{\beta\Gamma(\beta)}\left|\left(b_1\right)\right|^{\beta} \frac{b_1}{|b_1|}$$

$$x_{31} = \left|\left(b_2\right)\right|^{\beta+1} \frac{k_2}{(\beta+1)\,\Gamma(\beta+1)}$$

$$x_{32} = \left|\left(b_2\right)\right|^{\gamma+1} \frac{k_3}{(\gamma+1)\,\Gamma(\gamma+1)}$$

$$x_{41} = \frac{k_2}{\beta\Gamma(\beta)}\left|\left(b_2\right)\right|^{\beta} \frac{b_2}{|b_2|}$$

$$x_{42} = \frac{k_3}{\gamma\Gamma(\gamma)}\left|\left(b_2\right)\right|^{\gamma} \frac{b_2}{|b_2|}$$

$$A = \frac{1}{(\alpha+1)\,\Gamma(\alpha+1)}$$

$$B = \frac{k_3}{(\gamma+1)\,\Gamma(\gamma+1)}$$

(d) System composition from equations:.

$$x_1 = \frac{\int_{-1}^{b_1}\left(C_{12} + C_{11}l - |l|^{\alpha+1}\frac{1}{(\alpha+1)\Gamma(\alpha+1)}\right)dl}{f}$$

$$x_2 = \frac{\int_{b_1}^{b_2}\left(C_{22} + C_{21}l - |l|^{\beta+1}\frac{k_2}{(\beta+1)\Gamma(\beta+1)}\right)dl}{f}$$

$$x_3 = \frac{\int_{b_2}^{1}\left(C_{32} + C_{31}l - |l|^{\gamma+1}\frac{k_3}{(\gamma+1)\Gamma(\gamma+1)}\right)dl}{f}$$

$$f = \int_{-1}^{b_1}\left(C_{12} + C_{11}l - |l|^{\alpha+1}\frac{1}{(\alpha+1)\,\Gamma(\alpha+1)}\right)dl + \int_{b_1}^{b_2}\left(C_{22} + C_{21}l - |l|^{\beta+1}\frac{k_2}{(\beta+1)\,\Gamma(\beta+1)}\right)dl$$

$$+ \int_{b_2}^{1}\left(C_{32} + C_{31}l - |l|^{\gamma+1}\frac{k_3}{(\gamma+1)\,\Gamma(\gamma+1)}\right)dl$$

(e) Multivariable linear regression statistical adjustment to find behavior of composition for two of the fluids (third then is determined) .

$$x_1 = A_1 b_1 + A_2 b_2 + A_3 P + A_0$$
$$x_2 = D_1 b_1 + D_2 b_2 + D_3 P + D_0$$

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From which is cleared:

$$b_1 = -\frac{(D_0 A_2 - D_2 A_0 + D_2 x_1 - A_2 x_2 - PD_2 A_3 + PD_3 A_2)}{(D_1 A_2 - D_2 A_1)} \text{ and}$$

$$b_2 = \frac{(D_0 A_1 - D_1 A_0 + D_1 x_1 - A_1 x_2 - PD_1 A_3 + PD_3 A_1)}{(D_1 A_2 - D_2 A_1)}$$

Step 3. Pressure drop determination.

Gas turbulence index and interphase position depends on pressure. It is necessary to apply a solution numerical method to this. *A* given value of composition for different pressure values which must be lower or equal to the input value is specified. For every pressure value, the turbulence index, interphase values and coefficients of velocity profile are found and the integer is calculated:

$$f = \int_{-1}^{b_1} \left(C_{12} + C_{11} l - |l|^{\alpha+1} \frac{1}{(\alpha+1)\,\Gamma(\alpha+1)} \right) dl + \int_{b_1}^{b_2} \left(C_{22} + C_{21} l - |l|^{\beta+1} \frac{k_2}{(\beta+1)\,\Gamma(\beta+1)} \right) dl$$
$$+ \int_{b_2}^{1} \left(C_{32} + C_{31} l - |l|^{\gamma+1} \frac{k_3}{(\gamma+1)\,\Gamma(\gamma+1)} \right) dl$$

It is adjusted with statistical techniques *f* with respect to *P*. Finally, pressure is determined solving equation:

$$-\frac{dP}{dl} = \frac{4}{H^2 f} \frac{\mu_1}{\rho_1} W$$
$$P(0) = P_0$$

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