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Robust preview tracking control for a class of uncertain discrete-time systems

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Abstract: The robust preview control problem of uncertain discrete-time systems satisfying matching conditions is considered. First, for the nominal system, we use the difference between a system state and its steady-state value, instead of the usual difference between system states, to derive an augmented error system that includes the future information on the reference signal and disturbance signal to transform the tracking problem into a regulator problem. Then, the robust controller design problem based on the optimal controller of the augmented error system is proposed for uncertain system. And the proposed robust preview control law is obtained in terms of the linear matrix inequality (LMI) technique and the Lyapunov stability theory. Bringing the resulting controller back to the original system, a controller with preview actions achieving robust tracking performance is presented. The numerical simulation example also illustrates the effectiveness of the results presented in the paper.

Subjects: Systems & Control Engineering; Automation Control; Systems Engineering; Control Engineering; Systems & Controls

Keywords: uncertain discrete-time system; augmented error system; robust control; preview control; LMI; integrator

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PUBLIC INTEREST STATEMENT

Preview control utilizes the future information of disturbances or references to improve the disturbance rejection or tracking quality. It can be applied in various control systems, such as electromechanical servo systems, robot and unmanned aerial vehicle.

In studying the robust preview control problem for uncertain system with time-varying uncertainties, the main difficulty is the construction of an augmented error system. This paper finds a method to derive the augmented error system and the robust controller with reference and disturbance preview compensations is proposed. The effects of the two kinds of preview compensation on the tracking and rejection performance are both considered.

1. Introduction

The research question of preview control theory is as follows: when the reference signal or exogenous disturbance can be previewable, how can we take full advantage of the known future reference signal or disturbance signal to improve the tracking performance of a closed-loop system? Sheridan (1966) proposed the concept of preview control via three models. After Masayoshi Tomizuka's groundbreaking work (Tomizuka, 1975; Tomizuka & Whitney, 1975), much scholarly attention has focused on this approach. For constant coefficient linear systems, by applying the difference operator to the state equation of the system and the error signal, Tsuchiya Takeshi's paper (Tsuchiya & Egami, 1994) constructed an augmented error system that is equivalent to the original system in some sense. The tracking problem was thus transformed into a regulator problem, and the optimal control law for the augmented error system was obtained using optimal control theory. Liao, Takaba, Katayama, and Katsuura (2003) investigated the optimal preview control problem for multirate systems via a research study on the working mechanism of the chemical fractionation tower control system, and solved the problem well. And Liu and Liao (2008) continued to take the research further. Research has made possible a breakthrough (Liao, Lu, & Liu, 2016; Wu, Liao, & Tomizuka, 2016; Zhang, Bae, & Tomizuka, 2015; Zhao, Sun, Ren, & Li, 2016) via preview control for multi-agent systems, descriptor systems, the variable coefficient of control systems, stochastic control systems, and so on.

In recent years, linear robust preview control problem has been vigorously studied, especially with respect to H_2/H_∞ preview control problems. An input delay approach to the disturbance rejection problem for linear continuous time systems has been discussed (Marro & Zattoni, 2005; Moelja & Meinsma, 2005, 2006), and the H_2 preview control problem was transformed into a linear quadratic problem, and it was solved using the principle of optimality. The H_∞ performance of preview control was investigated by applying game theoretic approach to finite time horizon problems (Cohen & Shaked, 1997; Shaked & De Souza, 1995) and the same approach was employed to solve robust H_∞ preview control problem for uncertain systems with norm bounded uncertainty (Cohen & Shaked, 1998; De Souza & Fu, 1995). Subsequently, Gershon and Shaked (2014) further solved the problem of infinite-horizon H_∞ state-feedback preview tracking control for linear continuous time-invariant retarded systems with stochastic parameter uncertainties. However, this approach could not be applied to systems with non-uniform distributed parameter and the reference signals were assumed to be L_2 signals, and the existence of a positive semi-definite solution of a Riccati equation was a prerequisite in Cohen and Shaked (1997), Shaked and De Souza (1995), De Souza and Fu (1995), Cohen and Shaked (1998), Gershon and Shaked (2014). For polytopic uncertain systems, Takaba (2000), Liao, Wang, and Yang (2003), Oya, Hagino, and Matsuoka (2007) used the error system method to construct the augmented error system to convert the preview tracking problem into robust LQ problem and designed the desired controller by LMI approach. Unfortunately, the difference operator method in Takaba (2000), Liao et al. (2003), Oya et al. (2007) was not applicable to uncertain systems with time-vary uncertainties. Kojima (2015), Kojima and Ishijima (2003) studied the H_∞ preview control problems and obtained the optimal controller with preview action based on the optimal solution for standard H_∞ algebraic Riccati equation.

In addition, preview control design witnessed the incorporation of other control theories, for instance, multi-model adaptive control was combined with preview control, sliding control and fuzzy logic concepts were added to preview control problems, frequency domain was investigated for the design of preview controller (Cheng et al., 2014; Moran Cardenas, Rázuri, Bonet, Rahmani, & Sundgren, 2014; Running & Martins, 2009; Wang, Liao, & Tomizuka, 2016; Williams, Loukianov, & Bayro-Corrochano, 2015; Zhen, 2016) and so on.

Up to now, it is rare to find published research that studies preview control combined with uncertain discrete-time system with time-varying uncertainties based on the error system method.

For uncertain discrete-time system with time-varying uncertainties, the main contribution of this paper is that the designation of robust preview controller via sufficient utilization of the future information on the reference signal and disturbance signal. Because the uncertainties are time-varying and unknown, uncertainties became the main roadblock in applying the method of augmented error systems in classical preview control. The difficulty faced by the promotion error system approach is that the usual differential operator is nonlinear for uncertain systems, and thus the error system becomes very complicated. Therefore, the classical method in Tsuchiya and Egami (1994), Takaba (2000), Liao et al. (2003), Oya et al. (2007) becomes less usable. In order to overcome the difficulties, we learn from Liao et al. (2003) to construct an augmented error system for the nominal system (Liao et al., 2003 does not use the difference in essence). And then, considering the uncertainty of the system, the robust controller for the uncertain system is designed such that the output tracks the reference signal without steady-state error in the presence of uncertainty and exogenous disturbance. It should also be noted that the tracking and disturbance rejection problems for discrete-time system with time-varying uncertainties are both considered. And the effects of reference preview compensation and disturbance preview compensation on the tracking performance are studied. Finally, we perform numerical simulations to show the superiority of the robust control law with integral action and preview action.

Notations. R^n, $R^{n\times n}$ denote the n-dimensional real vector space and $n \times n$ matrix space, respectively. $p > 0$ denotes that the matrix P is positive definite. $P > Q$ denotes $P - Q > 0$. I denotes the identity matrix, and its dimension can be known from the context of the narrative.

2. Problem statement

Consider the uncertain discrete-time system:

$$\begin{cases} x(k+1) = [A + \Delta A]x(k) + [B + \Delta B]u(k) + Ew(k), \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(t) \in R^m$ is the input control vector, $y(k) \in R^q$ is the output vector, $w(k) \in R^l$ is the disturbance vector, and $w(k) \in L_2$, and A, B, C and E are known real constant matrices with the appropriate dimensions. $\Delta A = \Delta A(k, x, \alpha)$ and $\Delta B = \Delta B(k, x, \alpha)$ are uncertain matrices, which depend on the time variable k, the state vector x, or some parameter vector α.

First of all the necessary assumptions are given as follows.

A1: (A, B) is stabilizable and $\begin{bmatrix} A - I & B \\ C & 0 \end{bmatrix}$ is of full row rank.

A2: (C, A) is detectable.

Remark 1 A1 and A2 together show that the nominal system of system (1) is both stabilizable and detectable. And the rank condition of A1 implies that the system (C, A, B) has no transmission zeros at $z = 1$.

A3: Assume the preview length of the reference signal $r(k)$ is M_R, that is, at each time k, the M_R future values of $r(k+1)$, $r(k+2)$,..., $r(k+M_R)$ as well as the present and past values of the reference signal are available. The future values of the reference signal beyond the $k + M_R$ are constant vector r:

$$r(k+j) = r,\ j = M_R + 1,\ M_R + 2,\ M_R + 3, \ldots .$$

We assume that the reference signal $r(k)$ converges to constant vector r as time k goes to infinity. That is,

$$\lim_{k\to\infty} r(k) = r.$$

A4: Assume the preview length of the exogenous disturbance $w(k)$ is M_d, that is, at each time k, the M_d future values of $w(k+1)$, $w(k+2)$, ..., $w(k+M_d)$ as well as the present and past values of the exogenous disturbance are available. The future values of the exogenous disturbance beyond the $k + M_d$ are zero:

$$w(k+i) = 0,\ i = M_d + 1,\ M_d + 2,\ M_d + 3, \ldots .$$

Remark 2 A3 and A4 are that the reference signal and disturbance signal are both previewable. According to the characteristic of the control system itself, the previewable signals have a significant effect on the performance of the control system only for a certain time period; therefore, we can ignore the reference signal and the exogenous disturbance when they exceed the preview length. The latter parts of A3 and A4 are solely to facilitate the construction of an augmented error system.

A5: There exist known real constant matrices with appropriate dimensions E_i, H_i and uncertain matrices $\sum_i = \sum_i(k, x, \alpha)$, $(i = 1, 2)$ such that

$$\Delta A = E_1 \sum\nolimits_1 H_1,\ \Delta B = E_2 \sum\nolimits_2 H_2, \tag{2}$$

$$\sum\nolimits_i^T \sum\nolimits_i \le I. \tag{3}$$

Remark 3 (2) shows that the uncertain matrices of system (1) satisfy matching conditions; (3) shows that uncertain matrices are norm bounded. It is easily seen from expressions $\Delta A = \Delta A(k, x, \alpha)$ and $\Delta B = \Delta B(k, x, \alpha)$ that the uncertainties are associated with the state vector or some unknown parameter, and may be time varying in nature. Therefore, the uncertainties referred to in this article are very general.

Let $r(k) \in R^q$ be the reference signal and define the error signal as

$$e(k) = y(k) - r(k). \tag{4}$$

In this paper, our objective is to design a robust preview controller in such a manner that the output $y(k)$ of the closed-loop system tracks the reference signal $r(k)$ even in the presence of uncertainty or exogenous disturbances, that is,

$$\lim_{k\to\infty} e(k) = \lim_{k\to\infty}(y(k) - r(k)) = 0.$$

The paper requires the following lemmas.

Lemma 1 *Hu, Jiang, and Yang (2013) Let E and G be matrices of appropriate dimensions. Let $\Xi = \mathrm{diag}(\Xi_1, \Xi_2, \ldots, \Xi_s)$, where $\Xi_1, \Xi_2, \ldots, \Xi_s$ are uncertain matrices that satisfy $\Xi_i^T \Xi_i \le I$, i = 1,2,...,s. Then, for arbitrary positive scalars $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_s$, one has $E\Xi G + G^T\Xi^T E^T \le E\Lambda E^T + G^T\Lambda^{-1}G$, where $\Lambda = \mathrm{diag}(\varepsilon_1 I, \varepsilon_2 I, \ldots, \varepsilon_s I)$.*

To save space we directly give the following lemma which will be exploited for stability analysis of the system.

Lemma 2 *Benton and Smith (1998) The system $x(k + 1) = Ax(k)$ is asymptotically stable if and only if there is a positive definite matrix P such that*

$$A^TPA - P < 0.$$

3. Derivation of the augmented error system

First, we consider the optimal control law for the nominal system of system (1), and then design a robust controller for system (1) by reconstructing the optimal controller.

The nominal system of system (1) is

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k). \end{cases} \tag{5}$$

The method of preview control theory is utilized to derive an augmented error system to transform the tracking problem for system (5) into a regulator problem. And the optimal control law of the augmented error system is obtained using optimal control theory.

If the output of the closed-loop system for system (5) tracks the reference signal $r(k)$, there exist steady-state values $x(\infty) = x^*$, $u(\infty) = u^*$, and then

$$x(\infty) = Ax(\infty) + Bu(\infty), \tag{6}$$

$$Cx(\infty) = r. \tag{7}$$

That is

$$\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ u(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} r. \tag{8}$$

By A1, the coefficient matrix of Equation (8) and the augmented matrix have the same rank; thus, at least one solution exists. One pair here can be selected.

Define new vectors

$$\tilde{x}(k) = x(k) - x(\infty),\ \tilde{u}(k) = u(k) - u(\infty),\ \tilde{y}(k) = y(k) - y(\infty),\ \tilde{r}(k) = r(k) - r$$

Combining (5), (6), and (7) yields

$$\begin{cases} \tilde{x}(k+1) = A\tilde{x}(k) + B\tilde{u}(k), \\ \tilde{y}(k) = C\tilde{x}(k). \end{cases} \tag{9}$$

and

$$e(k) = C\tilde{x}(k) - \tilde{r}(k). \tag{10}$$

The quadratic performance index can be denoted as follows:

$$J = \sum_{k=0}^{\infty} \left[e(k)^T Q_e e(k) + \tilde{u}(k)^T H \tilde{u}(k) \right], \tag{11}$$

where the weighing matrices satisfy Qe > 0 and H > 0.

According to A3, $\tilde{r}(k)$ satisfies the following properties: at each time k, $\tilde{r}(k), \tilde{r}(k+1), \tilde{r}(k+2), \dots, \tilde{r}(k+M_R)$ are known and

$$\tilde{r}(k+j) = 0,\ j = M_R + 1,\ M_R + 2,\ M_R + 3, \dots.$$

According to Li and Liao (2015), the formal system can be obtained.

$$\begin{cases} \hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}\tilde{u}(k), \\ e(k) = \hat{C}\hat{x}(k), \end{cases} \tag{12}$$

where

$$\hat{x}(k) = \begin{bmatrix} e(k) \\ \tilde{x}(k) \\ X_R(k) \\ X_W(k) \end{bmatrix},\ \hat{A} = \begin{bmatrix} 0 & CA & G_{PR} & G_{PW} \\ 0 & A & 0 & G_W \\ 0 & 0 & A_R & 0 \\ 0 & 0 & 0 & A_W \end{bmatrix},\ \hat{B} = \begin{bmatrix} CB \\ B \\ 0 \\ 0 \end{bmatrix},\ \hat{C} = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix},$$

$$G_{PR} = \begin{bmatrix} 0 & -I & \cdots & 0 \end{bmatrix},\ G_W = \begin{bmatrix} E & 0 & \cdots & 0 \end{bmatrix},\ G_{PW} = \begin{bmatrix} CE & 0 & \cdots & 0 \end{bmatrix} = CG_W,$$

and

$$X_R(k) = \begin{bmatrix} \tilde{r}(k) \\ \tilde{r}(k+1) \\ \vdots \\ \vdots \\ \tilde{r}(k+M_R) \end{bmatrix},\ X_W(k) = \begin{bmatrix} w(k) \\ w(k+1) \\ \vdots \\ \vdots \\ w(k+M_d) \end{bmatrix},\ A_R = \begin{bmatrix} 0 & I & & & 0 \\ 0 & \ddots & \ddots & & \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & \cdots & 0 & I \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix}$$

$$A_W = \begin{bmatrix} 0 & I & & & 0 \\ 0 & \ddots & \ddots & & \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & \cdots & 0 & I \\ 0 & \cdots & \cdots & 0 & 0 \end{bmatrix},$$

where $X_R(k) \in R^{(M_R+1)q}$, $A_R \in R^{[(M_R+1)q]\times[(M_R+1)q]}$, $X_W(k) \in R^{(M_d+1)l}$ and $A_W \in R^{[(M_d+1)l]\times[(M_d+1)l]}$

In terms of the state vector and input vector of formal system (12), the performance index is expressed as

$$J = \sum_{k=0}^{\infty} [\hat{x}(k)^T \hat{Q} \hat{x}(k) + \tilde{u}(k)^T H \tilde{u}(k)], \tag{13}$$

where $\hat{Q} = \hat{C}^T Q_e \hat{C} = \begin{bmatrix} Q_e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and H is shown in (11).

Note that the main characteristics of system (12) are that both $X_R(k)$ and $X_W(k)$ with future information are part of the state vector. Therefore, the formal system (12) contains the future information of the reference signal and disturbance signal. However, in system (12), $\tilde{u}(k)$ is obtained rather than the difference of $u(k)$; as a result, the controller obtained from system (12) by the LMI approach does not include the integral of error $e(k)$; therefore, the final closed-loop system does not contain an integrator that helps to eliminate the static error. Due to this reason, the discrete integrator defined by Katayama, Ohki, Inoue, and Kato (1985)

$$v(k+1) = v(k) + e(k), \tag{14}$$

namely,

$$v(k) = \sum_{j=0}^{k-1} e(j) + v(0),$$

where $v(0)$ can be assigned as needed.

Define $X(k) = \begin{bmatrix} \hat{x}(k) \\ v(k) \end{bmatrix}$ again, and combining (12) and (14), one gets

$$X(k+1) = FX(k) + G\tilde{u}(k), \tag{15}$$

where

$$F=\left[\begin{array}{cccc|c} 0 & CA & G_{PR} & G_{PW} & 0 \\ 0 & A & 0 & G_W & 0 \\ 0 & 0 & A_R & 0 & 0 \\ 0 & 0 & 0 & A_W & 0 \\ \hline I & 0 & 0 & 0 & I \end{array}\right]=\begin{bmatrix} \hat{A} & 0 \\ \hat{C} & I \end{bmatrix},\ G=\left[\begin{array}{c} CB \\ B \\ 0 \\ 0 \\ \hline 0 \end{array}\right]=\begin{bmatrix} \hat{B} \\ 0 \end{bmatrix}.$$

If the performance index (13) is still used for system (15), it is difficult to guarantee the existence of the solution to the associated Riccati equation. Therefore, we introduce the penalty on the integrator as follows:

$$\begin{aligned} \bar{J} &= J+\sum_{k=0}^{\infty} v(k)^T Q_v v(k)=\sum_{k=0}^{\infty}[\hat{x}(k)^T\hat{Q}\hat{x}(k)+\tilde{u}(k)^T H\tilde{u}(k)+v(k)^T Q_v v(k)] \\ &=\sum_{k=0}^{\infty}[X(k)^T\bar{Q}X(k)+\tilde{u}(k)^T H\tilde{u}(k)], \end{aligned} \tag{16}$$

where $\bar{Q}=\mathrm{diag}(\hat{Q},Q_v)=\mathrm{diag}(\hat{C}^T Q_e\hat{C},Q_v)=\begin{bmatrix} \hat{Q} & \\ & Q_v \end{bmatrix}$.

Now the first theorem is obtained in the following.

THEOREM 1 *If (F, G) is stabilizable,* $\left(\bar{Q}^{1/2},F\right)$ *is detectable, and the optimal controller of system (15) minimizing the performance index (16) is given by*

$$\tilde{u}(k)=-[H+G^T\tilde{P}G]^{-1}G^T\tilde{P}FX(k)=KX(k),$$

where

$$K=-[H+G^T\tilde{P}G]^{-1}G^T\tilde{P}F, \tag{17}$$

and $\tilde{P}$ is the unique symmetric semi-positive definite solution of the algebraic Riccati equation,

$$\tilde{P}=\bar{Q}+F^T\tilde{P}F-F^T\tilde{P}G[H+G^T\tilde{P}G]^{-1}G^T\tilde{P}F. \tag{18}$$

In the following, the PBH rank test (Benton & Smith, 1998) will be employed to prove the stabilizability and dectectability of the augmented error system (15).

LEMMA 3 *The pair (F, G) is stabilizable if and only if (A, B) is stabilizable and* $\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}$ *has full row rank.*

Proof By the PBH criteria (Benton & Smith, 1998), the pair (F, G) is stabilizable if and only if the matrix

$$\begin{bmatrix} F-sI & G \end{bmatrix}$$

has full row rank for any complex s satisfying $|s|\geq 1$.

Note that it follows from the structure of A_R and A_W that both A_R-sI and A_W-sI are nonsingular for any s satisfying $|s|\geq 1$. From the expression of F and G, it can be seen that

$$\begin{bmatrix} F-sI & G \end{bmatrix}=\left[\begin{array}{ccccc|c} -sI & CA & G_{PR} & G_{PW} & 0 & CB \\ 0 & A-sI & 0 & G_W & 0 & B \\ 0 & 0 & A_R-sI & 0 & 0 & 0 \\ 0 & 0 & 0 & A_W-sI & 0 & 0 \\ I & 0 & 0 & 0 & (1-s)I & 0 \end{array}\right],$$

$(1-s)I$ is nonsingular for any complex s satisfying $|s|\geq 1$ and $s\neq 1$. By elementary matrix transformation, we have

$$\begin{bmatrix} F-sI & G \end{bmatrix} \to \left[\begin{array}{ccccc|c} -sI & 0 & 0 & 0 & 0 & 0 \\ 0 & A-sI & 0 & 0 & 0 & B \\ 0 & 0 & A_R-sI & 0 & 0 & 0 \\ 0 & 0 & 0 & A_W-sI & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-s)I & 0 \end{array}\right].$$

Therefore, the matrix $\begin{bmatrix} F-sI & G \end{bmatrix}$ is of full row rank if and only if $\begin{bmatrix} A-sI & B \end{bmatrix}$ is of full row rank.

When $s = 1$, one gets from elementary matrix transformation

$$\begin{bmatrix} F-sI & G \end{bmatrix}_{s=1} \to \left[\begin{array}{ccccc|c} 0 & CA & 0 & 0 & 0 & CB \\ 0 & A-I & 0 & 0 & 0 & B \\ 0 & 0 & A_R-I & 0 & 0 & 0 \\ 0 & 0 & 0 & A_W-I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccccc|c} 0 & C & 0 & 0 & 0 & 0 \\ 0 & A-I & 0 & 0 & 0 & B \\ 0 & 0 & A_R-I & 0 & 0 & 0 \\ 0 & 0 & 0 & A_W-I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Thus the matrix $\begin{bmatrix} F-sI & G \end{bmatrix}_{s=1}$ is of full row rank if and only if $\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}$ is of full row rank.

Lemma 3 holds.

LEMMA 4 *$\left(\bar{Q}^{1/2}, F\right)$ is detectable if and only if (C, A) is detectable.*

Proof By the PBH criteria (Benton & Smith, 1998), the pair $\left(\bar{Q}^{1/2}, F\right)$ is detectable if and only if the matrix

$$\begin{bmatrix} F-sI \\ \bar{Q}^{1/2} \end{bmatrix}$$

has full column rank for any complex s satisfying $|s| \geq 1$. From the expression of F and $\bar{Q}$, one can have

$$\begin{bmatrix} F-sI \\ \bar{Q}^{1/2} \end{bmatrix} = \left[\begin{array}{ccccc} -sI & CA & G_{PR} & G_{PW} & 0 \\ 0 & A-sI & 0 & G_W & 0 \\ 0 & 0 & A_R-sI & 0 & 0 \\ 0 & 0 & 0 & A_W-sI & 0 \\ I & 0 & 0 & 0 & (1-s)I \\ \hline Q_e^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_v^{1/2} \end{array}\right].$$

Similarly, it follows from the structure of A_R and A_W that both $A_R - sI$ and $A_W - sI$ are nonsingular for any s satisfying $|s| \geq 1$. By elementary matrix transformation, one has

$$\begin{bmatrix} F-sI \\ \bar{Q}^{1/2} \end{bmatrix} \to \left[\begin{array}{ccccc} sI & CA & 0 & 0 & 0 \\ 0 & A-sI & 0 & 0 & 0 \\ 0 & 0 & A_R-sI & 0 & 0 \\ 0 & 0 & 0 & A_W-sI & 0 \\ I & 0 & 0 & 0 & (1-s)I \\ \hline Q_e^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_v^{1/2} \end{array}\right] \to \left[\begin{array}{ccccc} 0 & C & 0 & 0 & 0 \\ 0 & A-sI & 0 & 0 & 0 \\ 0 & 0 & A_R-sI & 0 & 0 \\ 0 & 0 & 0 & A_W-sI & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline Q_e^{1/2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_v^{1/2} \end{array}\right].$$

Thus the matrix $\begin{bmatrix} F-sI \\ \bar{Q}^{1/2} \end{bmatrix}$ is of full column rank if and only if $\begin{bmatrix} A-sI \\ C \end{bmatrix}$ is of full column rank, that is, (C, A) is detectable.

Lemma 4 holds.

Remark 4 If A1 holds, that is, (A, B) is stabilizable and $\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}$ has full row rank, then (F, G) is stabilizable. If A2 holds, that is, (C, A) is detectable, then $\left(\bar{Q}^{1/2}, F\right)$ is detectable. Therefore, A1 and A2 imply that the augmented error system (15) is both stabilizable and detectable. Consequently, the existence of the optimal controller minimizing the performance index can be guaranteed, and the Riccati equation has a unique symmetric semi-positive definite solution.

4. Design of a robust preview controller

Now the problem of a robust preview control for uncertain system (1) is considered. First, the augmented error system is upgraded to uncertain system. The method is as follows: replace A with $A+\Delta A$ and B with $B+\Delta B$ respectively, and the rest remains the same. And the following will be obtained:

$$X(k+1) = [F+\Delta F]X(k) + [G+\Delta G]\tilde{u}(k), \tag{19}$$

where,

$$F = \begin{bmatrix} \hat{A} & 0 \\ \hat{C} & I \end{bmatrix},\ G = \begin{bmatrix} \hat{B} \\ 0 \end{bmatrix},\ \Delta F = \begin{bmatrix} 0 & C\Delta A & 0 & 0 & 0 \\ 0 & \Delta A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},\ \Delta G = \begin{bmatrix} C\Delta B \\ \Delta B \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

System (15) can be regarded as the nominal system of system (19). Noticing (2) and (3), the following equations are given by

$$\Delta F = \left[\begin{array}{cccc|c} 0 & CE_1\sum_1 H_1 & 0 & 0 & 0 \\ 0 & E_1\sum_1 H_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right] = \left[\begin{array}{c} CE_1 \\ E_1 \\ 0 \\ 0 \\ \hline 0 \end{array}\right] \sum\nolimits_1 \left[\begin{array}{cccc|c} 0 & H_1 & 0 & 0 & 0 \end{array}\right] = \bar{E}_1 \bar{\sum_1} \bar{H}_1, \tag{20}$$

$$\Delta G = \begin{bmatrix} CE_2 \sum_2 H_2 \\ E_2 \sum_2 H_2 \\ 0 \\ 0 \\ \hline 0 \end{bmatrix} = \begin{bmatrix} CE_2 \\ E_2 \\ 0 \\ 0 \\ \hline 0 \end{bmatrix} \sum_2 H_2 = \bar{E}_2 \bar{\sum_2} \bar{H}_2, \tag{21}$$

This result shows that the uncertain matrices in system (19) still satisfy the matching conditions.

Based on (17), we seek a state feedback for system (19)

$$\tilde{u}(k) = [K + K_D]X(k) = \hat{K}X(k) \tag{22}$$

to make the closed-loop system of system (19) asymptotically stable, where K is shown in (17) and K_D is an adjustable gain matrix. K_D is time-invariant matrix which will be determined by LMI approach in the following.

Considering (19) and (22), the closed-loop system is inferred as

$$X(k+1) = \left[F + \Delta F + G\hat{K} + (\Delta G)\hat{K}\right]X(k). \tag{23}$$

THEOREM 2 *Suppose that A1–A5 are satisfied. If there exist a positive definite matrix M and a matrix Y*; and two scalars $\varepsilon_i > 0$, $i = 1, 2$, such that

$$\begin{bmatrix} -M & M^T(F+GK)^T + Y^T G^T & M^T \bar{H}_1^T & (M^T K^T + Y^T)\bar{H}_2^T \\ (F+GK)M + GY & -M + \sum_{i=1}^{2} \varepsilon_i \bar{E}_i \bar{E}_i^T & 0 & 0 \\ \bar{H}_1 M & 0 & -\varepsilon_1 I & 0 \\ \bar{H}_2(KM+Y) & 0 & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{24}$$

then the closed-loop system (23) is asymptotically stable, where the adjustable gain

matrix is $K_D = YM^{-1}$ and the control law is given by

$$\tilde{u}(k) = \left[K + YM^{-1}\right]X(k). \tag{25}$$

Proof From Lemma 2, the closed-loop system (23) is asymptotically stable if and only if there is a positive definite matrix P such that

$$\left[F + \Delta F + (G + \Delta G)(K + K_D)\right]^T P \left[F + \Delta F + (G + \Delta G)(K + K_D)\right] - p < 0. \tag{26}$$

Applying the Schur complement lemma (Kheloufi, Zemouche, Bedouhene, & Souley-Ali, 2016) and $p > 0$, (26) is equivalent to

$$\begin{bmatrix} -P & (F+\Delta F)^T + (K+K_D)^T(G+\Delta G)^T \\ F + \Delta F + (G+\Delta G)(K+K_D) & -P^{-1} \end{bmatrix} < 0. \tag{27}$$

For the sake of simplicity of narrative, Φ will be used to denote the left matrix of (27) and separate the uncertainties from Φ. And it follows from (20) and (21) that

$$\Phi = \Theta + \Xi_1 \Upsilon \Xi_2 + \Xi_2^T \Upsilon^T \Xi_1^T, \tag{28}$$

where

$$\Theta = \begin{bmatrix} -P & F^T + (K+K_D)^T G^T \\ F + G(K+K_D) & -P^{-1} \end{bmatrix},\ \Xi_1 = \begin{bmatrix} 0 & 0 \\ \bar{E}_1 & \bar{E}_2 \end{bmatrix},\ \Upsilon = \begin{bmatrix} \bar{\Sigma}_1 & \\ & \bar{\Sigma}_2 \end{bmatrix},\ \Xi_2 = \begin{bmatrix} \bar{H}_1 & 0 \\ \bar{H}_2(K+K_D) & 0 \end{bmatrix}.$$

It can be seen from (3) that $\Upsilon^T \Upsilon \le I$. Applying Lemma 1 to (28), for $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\Lambda = \begin{bmatrix} \varepsilon_1 I & \\ & \varepsilon_2 I \end{bmatrix}$, one can gets

$$\Phi < \Theta + \Xi_1 \Lambda \Xi_1^T + \Xi_2^T \Lambda^{-1} \Xi_2. \tag{29}$$

Therefore, if $\Theta + \Xi_1 \Lambda \Xi_1^T + \Xi_2^T \Lambda^{-1} \Xi_2 < 0$, then $\Phi < 0$. Now the necessary and sufficient conditions for (29) will be discussed. Using Schur complement lemma, $\Theta + \Xi_1 \Lambda \Xi_1^T + \Xi_2^T \Lambda^{-1} \Xi_2 < 0$ is equivalent to

$$\begin{bmatrix} \Theta + \Xi_1 \Lambda \Xi_1^T & \Xi_2^T \\ \Xi_2 & -\Lambda \end{bmatrix} < 0. \tag{30}$$

By performing a congruence transformation an invertible symmetric matrix diag$\{P^{-1}, I\}$ to (30) and denoting $M = P^{-1}$, $Y = K_D M$, one can obtain (24). (24) is satisfied if and only if (30) is satisfied. Therefore, (24) can imply that (26) holds, that is, system (23) is asymptotically stable. Theorem 2 holds.

Now based on Theorems 1 and 2, we will discuss the control input of system (1).

When A1–A5 are satisfied, the control input (22) of system (19) is obtained. The gain matrix $\hat{K} = K + K_D$ will be decomposed into:

$$\hat{K} = \begin{bmatrix} K_e & K_x & K_R(0) & K_R(1) & \cdots & K_R(M_R) & K_d(0) & K_d(1) & \cdots & K_d(M_d) & K_v \end{bmatrix}, \tag{31}$$

and then (22) can be written as

$$\tilde{u}(k) = K_e e(k) + K_x \tilde{x}(k) + \sum_{i=0}^{M_R} K_R(i) \tilde{r}(k+i) + \sum_{i=0}^{M_d} K_d(i) w(k+i) + K_v v(k).$$

Substituting $\tilde{x}(k) = x(k) - x^*$, $\tilde{u}(k) = u(k) - u^*$, $\tilde{r}(k) = r(k) - r$ into the above expression, the following is obtained.

THEOREM 3 *Suppose that A1–A5 are satisfied. The controller of system (1) can be taken as*

$$u(k) = K_e e(k) + K_x x(k) + \sum_{i=0}^{M_R} K_R(i)(r(k+i) - r) + \sum_{i=0}^{M_d} K_d(i) w(k+i) + K_v(\sum_{s=0}^{k-1} e(s) + v(0)) + \begin{bmatrix} -K_x & I \end{bmatrix} \begin{bmatrix} x^* \\ u^* \end{bmatrix}, \tag{32}$$

where K is shown in (17) and $K_D = YM^{-1}$, $M > 0$ and Y can be determined by (24). (31) determines the relationship between K_e, K_x, $K_R(0)$, $K_R(1)$, $\cdots$, $K_R(M_R)$ $K_d(0)$, $K_d(1)$,..., $K_d(M_d)$, K_v and $\hat{K}$. x^* and u^* can be obtained by (8). Under the controller, the output $y(k)$ of the closed-loop system of system (1) tracks the reference signal $r(k)$ accurately.

Remark 5 From the above equation, the preview controller of system (1) consists of six terms. The first term represents tracking error compensation, the second term represents the state feedback, the third term represents the feedforward or preview action based on the future information of the reference signal, the fourth term represents the feedforward or preview action based on the future information of the exogenous disturbance, the fifth term represents the integral action of the tracking error, and the sixth term represents the compensation by the initial and final values.

Remark 6 In the paper, the error system method and discrete lifting technique are employed to construct the augmented error system including previewed information and the robust preview controller is designed based on robust linear quadratic and LMI approach. Marro and Zattoni (2005), Moelja and Meinsma (2005, 2006), Cohen and Shaked (1997), Shaked and De Souza (1995), De Souza and Fu (1995), Cohen and Shaked (1998), Gershon and Shaked (2014), Kojima (2015), Kojima and Ishijima (2003) utilize neither of these techniques. The methods in Takaba (2000), Liao et al. (2003), Oya et al. (2007), Cheng et al. (2014), Ryu, Kim, and Park (2008) for constructing the augmented error system were inapplicable for uncertain systems with norm bounded uncertainty because the difference operator cannot be applied to the time-varying uncertain terms directly. Tomizuka (1975), Tomizuka and Whitney (1975), Tsuchiya and Egami (1994), Liao et al. (2003), Liu and Liao (2008), Wu et al. (2016), Zhang et al. (2015), Liao et al. (2016), Zhao et al. (2016) did not involve robustness of preview control systems.

5. Numerical example

In system (1), $A = \begin{bmatrix} 0.4 & 0.7 & 0.3 & -0.3 \\ 0.05 & 1 & 0.02 & 0.2 \\ -0.55 & 0.2 & 1 & 0.02 \\ 0.3 & -0.5 & 0.01 & 1 \end{bmatrix}, B = \begin{bmatrix} -0.12 \\ 0.04 \\ -0.55 \\ 0.03 \end{bmatrix}, E = \begin{bmatrix} 0.2 \\ 0 \\ -0.1 \\ 0 \end{bmatrix},$

$$C = \begin{bmatrix} 0.3 & 0.9 & 0 & -0.06 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, H_2 = \begin{bmatrix} 0.01 \\ 0 \\ 0 \\ 0.02 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.02 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & -0.03 & 0 \\ 0 & 0 & 0 & 0.04 \end{bmatrix}, E_2 = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix},$$

$$\sum_1 = \begin{bmatrix} 0.5\cos(0.3k\pi) + a_1 & 0 & 0 & 0 \\ 0 & 0.4\sin(0.1k\pi) + a_2 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & -0.4 \end{bmatrix},$$

$$\sum_2 = \begin{bmatrix} 0.5\sin(0.1k\pi) + a_3 & 0 & 0 & 0 \\ 0 & 0.4\cos(0.5k\pi) + a_4 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & -0.2 \end{bmatrix}.$$

$|a_i| \le 0.5$ ($i = 1, 2, 3, 4$) are uncertain parameters. In the numerical simulation, 4 random sequences, with the absolute values no more than 0.5, are generated to simulate a_1, a_2, a_3 and a_4, respectively.

Through verifying, (A, B) is stabilizable and $\begin{bmatrix} A-I & B \\ C & 0 \end{bmatrix}$ satisfies full row rank, and (C, A) is detectable and $\sum_1$ and $\sum_2$ satisfy (3) for all k. Therefore, the system satisfies the basic assumptions.

The exogenous disturbance is taken as

$$w(k) = \begin{cases} 1.5\sin(1/20\pi k), & 20 \le k \le 60, \\ 0, & \text{other.} \end{cases} \tag{33}$$

The reference signal is taken as

$$r(k) = \begin{cases} 2, & k \ge 30 \\ 0, & k < 30 \end{cases} \tag{34}$$

Let $Q_e = 10$, $H = 100$, $Q_v = 10$ and perform simulations for the following situations, that is, (1) $M_d = 2, M_R = 9$, $M_R = 4$; (2) $M_R = 2$, $M_d = 7$, $M_d = 2$; (3) $M_R = M_d = 0$.

First, the optimal control gain matrix K can be obtained by solving the algebraic Riccati Equation (18) using MATLAB. Then, according to Theorem 2, matrix variables $M$and Y in LMI (24) can be determined; then the adjustable gain matrix $K_D = YM^{-1}$ is obtained naturally. Therefore, the control input for system (19) is obtained.

When $M_R = 9$, $M_d = 2$, the following will be obtained:

$$K = [0.24568 \mid 0.33548 \quad 1.58624 \quad 1.06120 \quad 2.12365 \mid 0 \quad -0.24579 \quad 0.24735 \\ -0.24562 \quad -0.24181 \quad -0.23652 \quad -0.22925 \quad -0.21886 \quad -0.20417 \quad -0.184626 \\ -0.00359 \quad 0.01291 \quad 0.02488 \mid 0.24568],$$

$$K_D = [1.01944 \mid 1.41679 \quad 7.25767 \quad 2.08035 \quad 3.96836 \mid 0.02596 \quad -0.96277 \\ -1.03539 \quad -0.97202 \quad -0.92119 \quad -0.87425 \quad -0.79370 \quad -0.66326 \quad -0.49919 \\ 0.32515 \mid 0.28610 \quad 0.24745 \quad 0.13505 \quad 0.98792],$$

and

$$K_e = 1.26513,\ K_x = [\ 1.75227 \quad 8.84392 \quad 3.14154 \quad 6.09201\ ],$$

$$K_R = [0.02596 \quad -1.20856 \quad -1.28274 \quad -1.21763 \quad -1.16301 \quad -1.11077 \\ -1.02295 \quad -0.88212 \quad -0.70336 \quad -0.50978],$$

$$K_d = [\ 0.28251 \quad 0.26037 \quad 0.15993\ ],$$

$$K_v = 1.23360.$$

When $M_R = 4$, $M_d = 2$, the gain matrices will be obtained as follows:

$$K = [0.24568 \mid 0.33548 \quad 1.58624 \quad 1.06120 \quad 2.12365 \mid 0 \quad -0.24579 \quad -0.24735 \\ -0.24562 - 0.24181 \mid -0.00359 \quad 0.01291 \quad 0.02488 \mid 0.24568],$$

$$K_D = [0.99032 \mid 1.35712 \quad 7.12228 \quad 2.05393 \quad 3.84643 \mid 0.01863 \quad -0.93714 \\ -1.00813 \quad -0.96581 -0.91803 \mid 0.27839 \quad 0.24327 \quad 0.13126 \mid 0.97062],$$

and

$$K_e = 1.23600,$$

$$K_x = [\ 1.69260 \quad 8.70852 \quad 3.11512 \quad 5.97007\ ],$$

$$K_R = [0.01863 \quad -1.18293 \quad -1.25548 \quad -1.21143 \quad -1.15985],$$
$$K_d = [0.27480 \quad 0.25618 \quad 0.15614],\ K_v = 1.21631.$$

When $M_R = M_d = 0$, the following will be obtained:

$$K = [\ 0.24568 \mid 0.33548 \quad 1.58624 \quad 1.06120 \quad 2.12365 \mid 0.24568\ ],$$

$$K_D = [\ 0.97588 \mid 1.29736 \quad 7.01623 \quad 2.03748 \quad 3.68029 \mid 0.95542\ ],$$

and

$$\hat{K} = K + K_D = [\ K_e \quad K_x \quad K_v\ ] = [\ 1.22156 \mid 1.63284 \quad 8.60247 \quad 3.09868 \quad 5.80394 \mid 1.20110\ ].$$

In the above three situations, let the initial conditions of $x(k)$ be $x(0) = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$ and the initial conditions of $v(k)$ be $v(0) = 0.53256$, $v(0) = 7.32404$ and $v(0) = 15.18051$, respectively.

Figure 1 shows the output response of the closed-loop system for system (1). Figure 2 shows the tracking error. It can be seen from the above three situations that the outputs can all track the reference signal accurately. However, both the tracking error and the input peak decrease with the increase of the preview length of the reference signal when the preview length of the exogenous disturbance is identical, namely, $M_d = 2$, and the settling time can be shortened. Compared with no preview, in addition to the advantages mentioned above, the disturbance attenuation performance is improved by virtue of preview compensation. This is exactly how preview control achieves its goal.

Note that from the simulation results, it can be seen that the dramatic changes in output caused by a disturbance signal can be eliminated by choosing $v(0)$ properly.

In addition, the steady-state values of state variables $x(k)$ and input variables $u(k)$ are given by

$$u(\infty) = \lim_{k\to\infty} u(k) \approx -1.98317,\ x(\infty) = \lim_{k\to\infty} x(k) \approx \begin{bmatrix} 2.47398 \\ 1.37843 \\ 0.65137 \\ -0.28700 \end{bmatrix}.$$

The simulation shows that the steady-state values of $x(k)$ and $u(k)$ tend toward the theoretical values. Figure 3 plots the curve of the control input changing in time and it is given here as an example.

Furthermore, it follows from Figure 1 that the closed-loop system has desirable steady-state response characteristics. In the following, the output response curve can be further analyzed by making use of the dynamic characteristics to further analyze. The following will be obtained:

$$\text{The rise time: } k = 36,\ k = 38,\ k = 42;$$

$$\text{The peak time: } k = 38,\ k = 41,\ k = 45;$$

$$\text{The settling time: } k = 40,\ k = 44,\ k = 67;$$

$$\text{The peak value: } y_{textmax} = 2.12284,\ y_{\max} = 2.21607,\ y_{\max} = 2.36242.$$

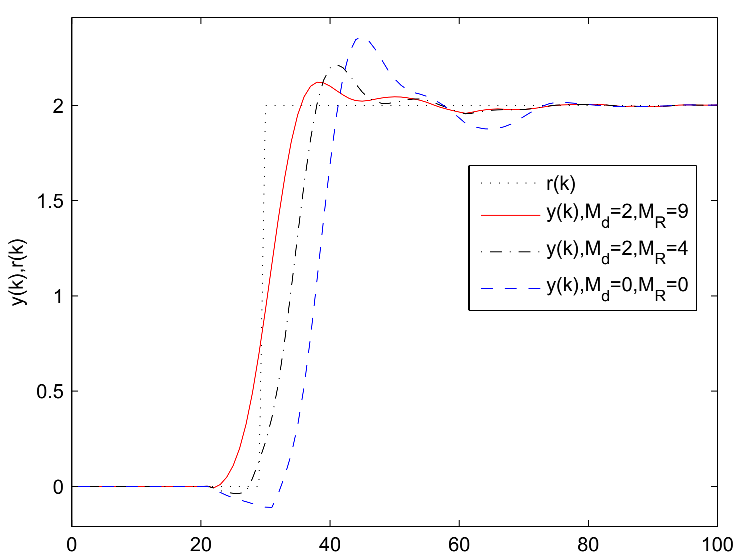


Figure 1. Comparison of the output response of the closed-loop system with the same M_d over a different M_R and no preview.

Figure 2. The tracking error of the closed-loop system with the same M_d over a different M_R and no preview.

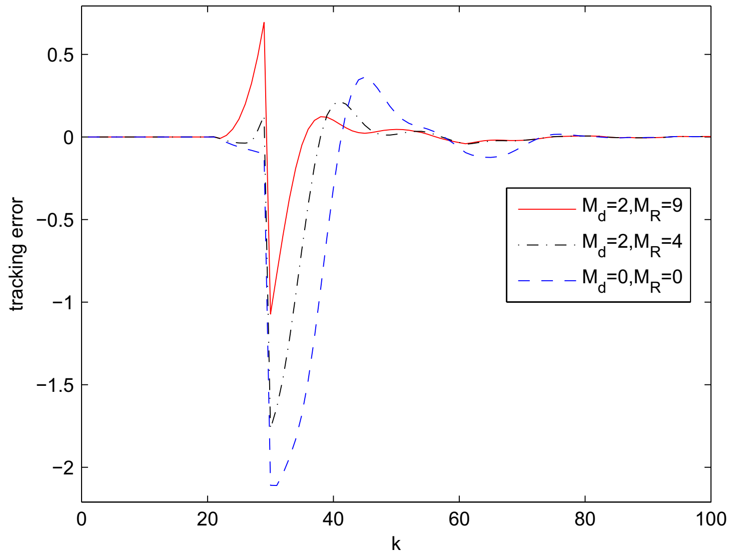
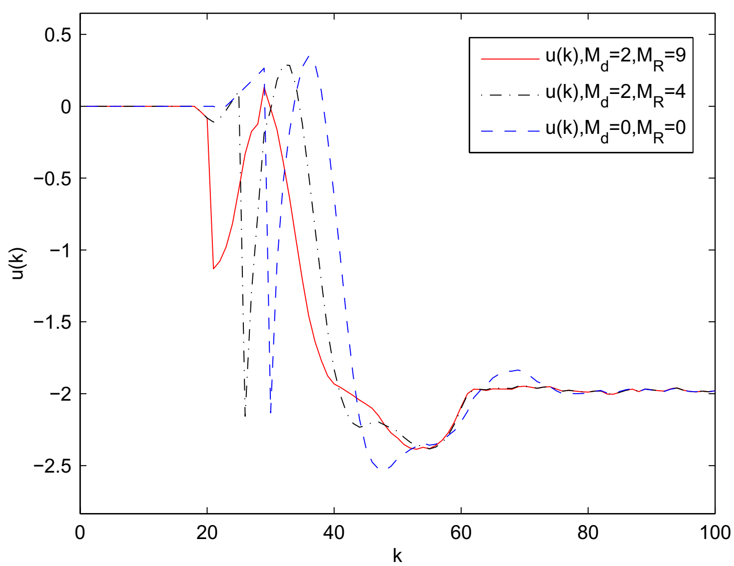


Figure 3. The control input of the closed-loop system with the same M_d over a different M_R and no preview.



The above simulation results verify the preview controller can provide better dynamic characteristics.

Next, we study the effect of the preview length M_d on the tracking performance which follows the condition that the preview length of the reference signal is identical and without preview,(i.e. $M_R = 2$, $M_d = 7$ and $M_d = 2$ and $M_R = M_d = 0$). The output response of the closed-loop system is depicted in Figure 4. Figure 5 shows the tracking error And Figure 6 indicates the control input. It can be seen from Figures 4–6 that the overshoot of the output response decreases and the oscillation caused by external disturbance can effectively be suppressed by increasing the preview length of the disturbance signal.

From the simulations for various forms of the reference signals and the disturbance signals, we find that the closed-loop system still has remarkably good target tracking performance only if the

Figure 4. Comparison of the output response of the closed-loop system with the same M_R over a different M_d and no preview.

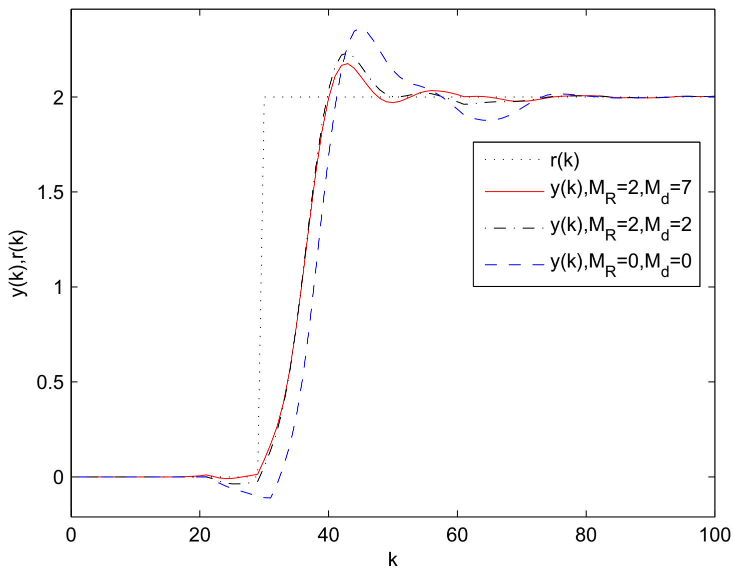
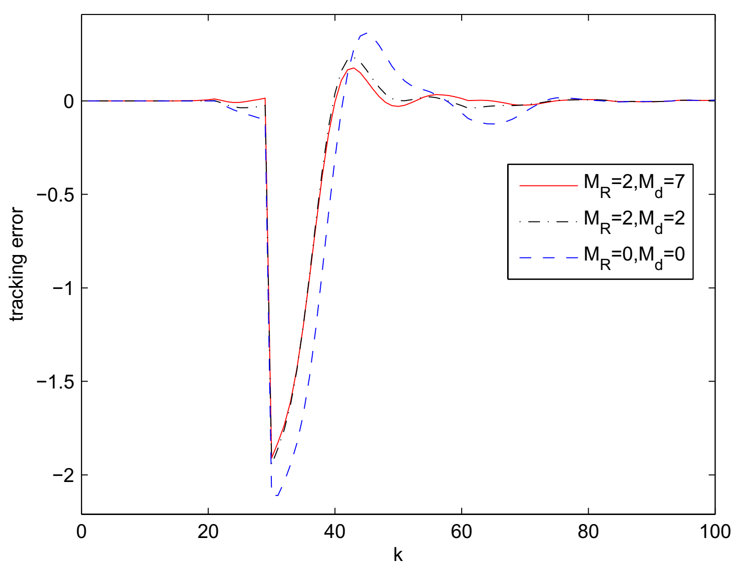


Figure 5. The tracking error of the closed-loop system with the same M_R over a different M_d and no preview.



reference signals or only if the disturbance signals are previewable. In the following, the simulation results for $M_R = 0$, $M_d = 0, 2, 7$ and $M_d = 0$, $M_R = 0, 4, 9$ will be presented, respectively.

Figure 7 shows the output response of the closed-loop system with $M_R = 0$, $M_d = 0, 2, 7$. Figure 8 plots the tracking error and Figure 9 depicts the control input. We can see from the figures that the closed-loop system has good interference suppression. From Figures 1, 4, and 7, we can find that the output for the system with $M_R = M_d = 0$ has a larger fluctuation when the disturbance signal appears or disappears. However, disturbance preview compensation can effectively inhibit the interference caused by the external disturbance.

Figure 6. The control input of the closed-loop system with the same M_R over a different M_d and no preview.

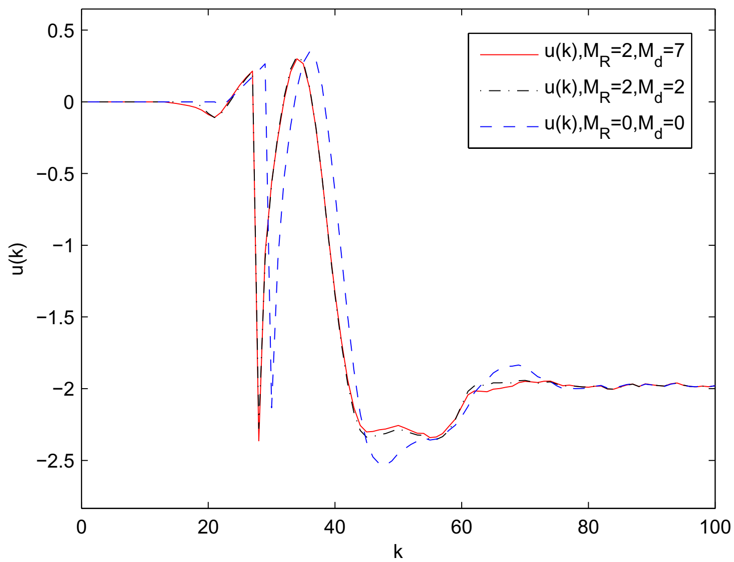


Figure 7. Output response of the closed-loop system with M_R = 0 over a different M_d.

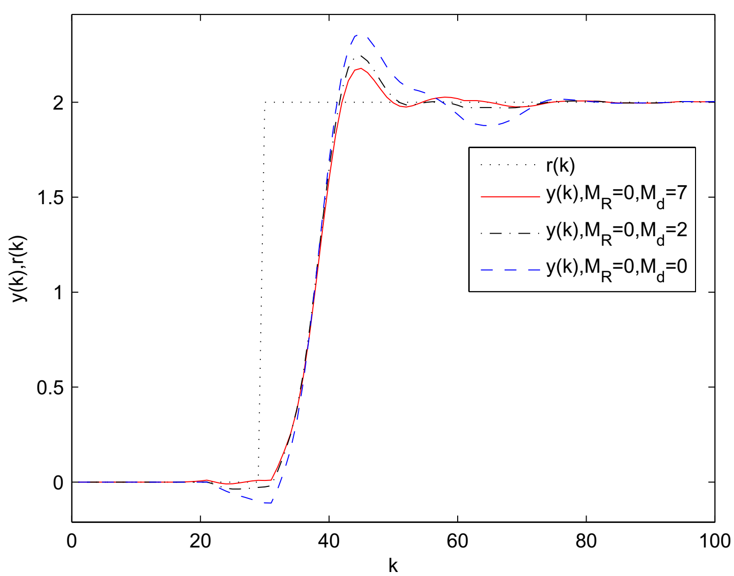


Figure 10 shows the output response of the closed-loop system with $M_d = 0$, $M_R = 0, 4, 9$. Figures 11 and 12 show the tracking error and the control input, respectively. Similarly, preview actions can shorten adjusting time and reduce the overshoot. But the output curves are obviously affected by the external disturbance when the preview length $M_d = 0$.

We also perform simulations for periodic interference signals. The simulation reveals that the output response of the closed-loop system with $M_R = M_d = 0$ produces fluctuations during the survival of periodic interference signals. However, the case in which preview action is used can eliminate

Figure 8. The tracking error of the closed-loop system with $M_R = 0$ over a different M_d.

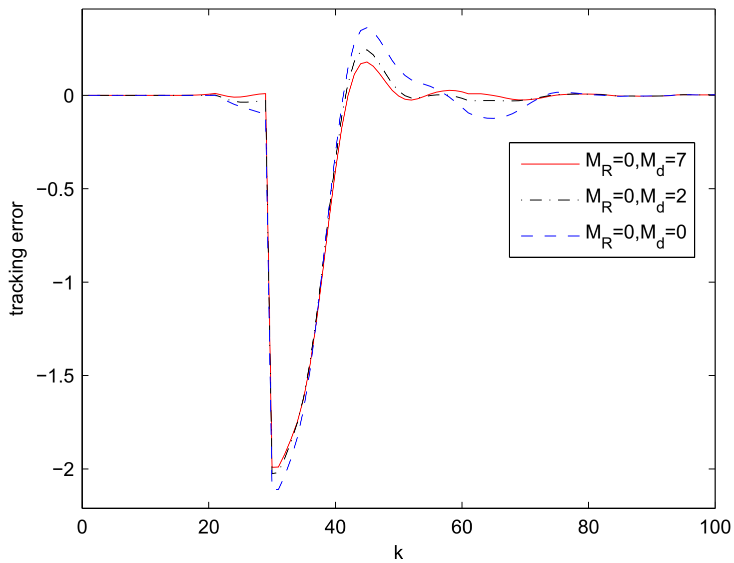
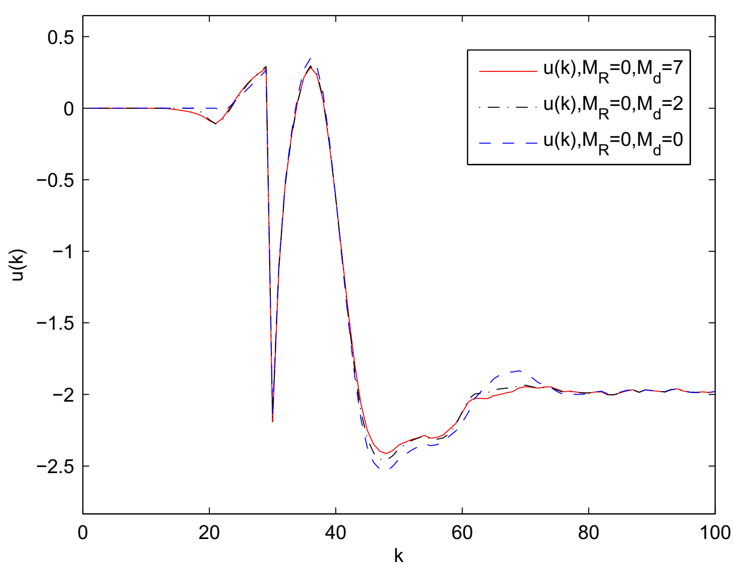


Figure 9. The control input of the closed-loop system with $M_R = 0$ over a different M_d.



them. The simulation for the cases without integrator is also performed; the results show that the output response produces steady-state error. Therefore, it is necessary to add an integrator into the control input. Considering the length of this paper, the figures for these results will no longer be given.

Figure 10. Output response of the closed-loop system with $M_d = 0$ over a different M_R.

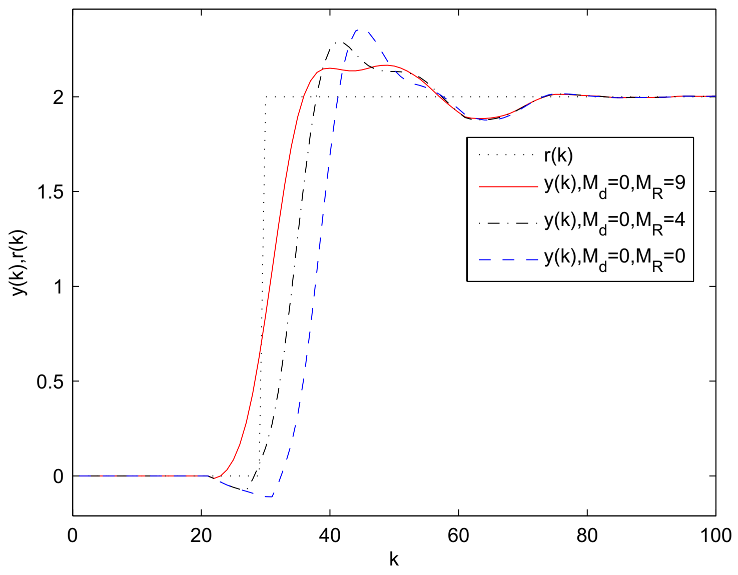
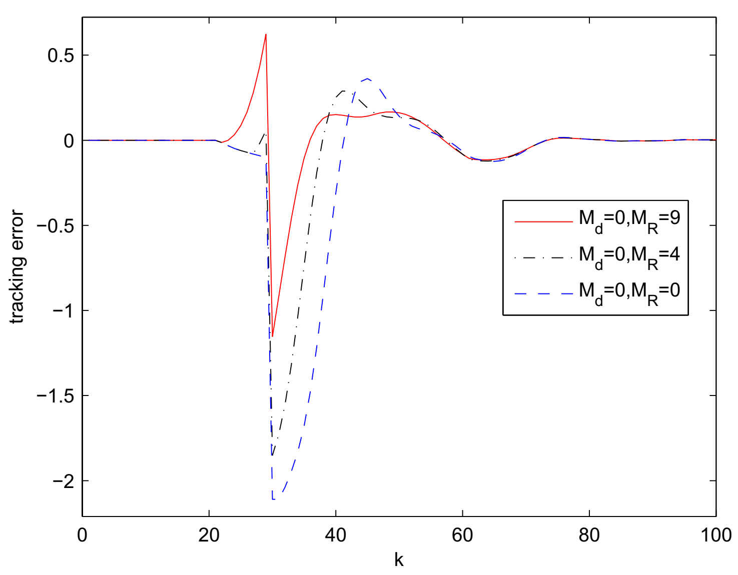


Figure 11. The tracking error of the closed-loop system with $M_d = 0$ over a different M_R.



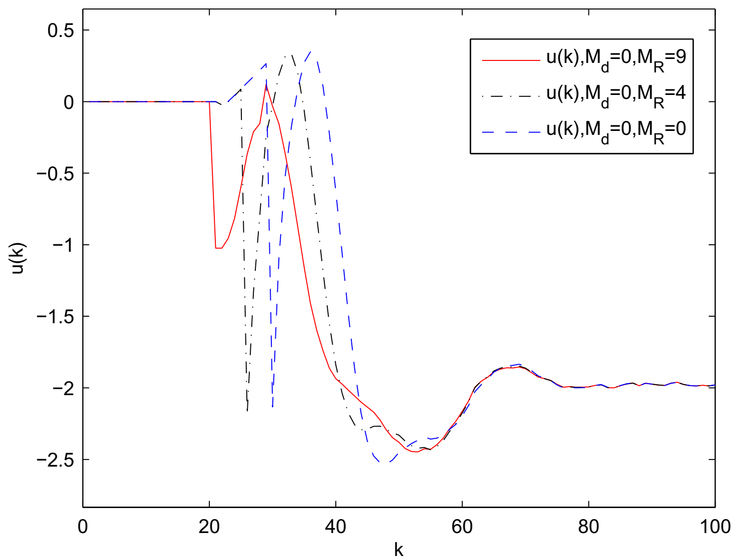


Figure 12. The control input of the closed-loop system with M_d = 0 over a different M_R.

6. Conclusion

In this paper, the robust preview control problem has been proposed for a class of uncertain discrete-time systems. First, the steady-state values of the nominal system were determined and a method was employed to derive an augmented error system with previewable reference signal and disturbance signal. Then, the optimal controller is obtained for an augmented error system (i.e. system (15)) applying optimal control theory. Based on the optimal controller, a robust control input was designed for uncertain systems (i.e. system (19)). Finally, the condition of the existence and the design method of the controller were presented using Lyapunov's second method. The state feedback gain matrix could be obtained by solving an LMI problem. By putting the obtained controller into the original system, the robust preview controller with preview action for the original system was given. The numerical simulation results showed that it was necessary to introduce the integrator in this paper. If we did not, the output of the closed-loop system tracks the reference signal with steady-state error. The numerical simulation example also has illustrated the effectiveness of the preview controller.

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