

**PRODUCTION & MANUFACTURING | RESEARCH ARTICLE**

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# Production inventory model for two-level trade credit financing under the effect of preservation technology and learning in supply chain

Sunil Kumar<sup>1</sup>, Nidhi Handa<sup>1</sup>, S.R. Singh<sup>2</sup> and Dharmendra Yadav<sup>3\*</sup>

**Abstract:** The present study investigated the inventory model for a retailer under two levels of trade credit to reflect the supply chain management. Supplier offers trade credit period of  $M$  to the retailer while in turn retailer provides a trade credit period of  $N$  to his/her customers. The supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the partial trade credit period to his/her customers. Here, selling items are considered as perishable items such as fruits, fresh fishes, gasoline, photographic films, etc. so that its potential worth decreases. It is assumed that decay in potential worth of items can be increased by using preservation technology. The demand is considered as the function of selling price and trade credit. Ordering cost can be reducing due to learning by doing phenomenon. By applying convex fractional programming results, we obtain necessary and sufficient conditions of an optimal solution. Some theorems are developed to determine retailer's optimal ordering policies and numerical examples are given to illustrate these theorems. In addition, some managerial insights from the numerical examples are also concluded.

**Subjects:** Mathematical Modeling; Operations Management; Operations Research

**Keywords:** production inventory model; two-level trade credit; preservation technology; learning phenomenon

### 1. Introduction

The managing of inventories is one of the most significant tasks that every manager must do efficiently and effectively in any organization so that their organization can grow. Now, all the organizations are involved in a global competitive market and then these organizations are taking seriously the activities



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### ABOUT THE AUTHOR

Dharmendra Yadav is an assistant professor in Vardhaman (P.G.) College, MJP Rohilkhand University, Bijnor, U.P. (India). He received his PhD in Inventory Control from the CCS University. His interests are in operations and supply chain management and optimization. He has had his articles published in journals, such as *Opsearch*, *Asia-Pacific Journal of Operational Research*, *International Journal of Systems Science*, *International Journal of Operational Research*, and others.

### PUBLIC INTEREST STATEMENT

The aim of this paper is to develop production inventory model. We assume that supplier offers full trade credit period of  $M$  to the manufacturer whereas manufacturer provides partial trade credit period of  $N$  to his/her wholesaler. Here, concept of preservation technology has been used to increase the potential worth of the product. We have determined the optimal cycle time so that the total inventory cost is minimum. We found that Huang (2003), Shah (1993), and Goyal (1985) are special case of our proposed model. In addition, some managerial insights on the basis of numerical examples are also concluded.

related to manage inventories. Thus, recently the practitioners and researchers have been increasing their interest in optimizing the inventory decisions in a holistic way. Thus, the manager try to find out different way through which the cost associated with inventory can reduce and the profit of the organization can increase.

The economic order quantity (EOQ) model is widely used by practitioners as a decision-making tool for the control of inventory. The basic EOQ model is based on the implicit assumption that the retailer must pay for the items as soon as he receives them from a supplier. However, this may not be true. In today's business transactions, it is more and more common to see that the supplier will allow a certain fixed time period (say, 30 days) for settling the amount that the supplier owes to retailer for the items supplied. We term this period as trade credit period. Usually, interest is not charged for the outstanding amount if it is paid within the permissible delay period. This credit term in financial management is denoted as "net 30". Therefore, the retailer can sell the goods and earn the interest on the accumulated revenue received, and delay the payment up to the last moment of the permissible period allowed by the supplier. However, if the payment is not paid within the permissible delay period, then interest is charged on the outstanding amount under the previously agreed terms and conditions. This brings some economic advantage to the retailers as they may earn some interest from the revenue realized during the period of permissible delay. The trade credit financing produces two benefits to the supplier:

- (1) It should attract new customers who consider it to be a type of price reduction, and
- (2) It should cause a reduction in the sales outstanding, since some established customers will pay more promptly in order to take advantage of permissible delay more frequently.

In a real world, the supplier often makes use of this policy to promote his commodities. In India, gas stations adopted a price policy that charged less money per gallon to the customer who paid by cash, instead of by a credit card. Likewise, a store owner in many Chinatowns around the world usually charges a customer 5% more if the customer pays by a credit card, instead of by cash. As a result, the customer must decide which alternative to take when the supplier provides not only a cash discount but also a permissible delay. In this regard, a number of research papers appeared which deal with the EOQ problem under the condition of permissible delay in payments. Goyal (1985) is the first person to consider the EOQ inventory model under the condition of trade credit. Chand and Ward (1987) analyzed Goyal's model under assumptions of the classical EOQ model, obtaining different results. Shinn, Hwang, and Sung (1996) extended Goyal (1985) model and considered quantity discount for freight cost. Huang (2004) investigated that the unit selling price and the unit purchasing price are not necessarily equal within the EPQ framework under a supplier's trade credit policy. Teng, Chang, and Goyal (2005) developed the optimal pricing and lot sizing under permissible delay in payments by considering the difference between the selling price and the purchase cost and demand is a function of price. There are several interesting and relevant papers related to trade credit such as Chung, Goyal, and Huang (2005), Mahata and Goswami (2007), Bera, Kar, Chakraborti, and Sinha (2014), Yadav, Singh, and Kumari (2015) and Rao and Rao (2015), and their references. All the above inventory models implicitly assumed one-level trade credit financing, i.e. it is assumed that the supplier would offer the retailer a delay period and the retailer could sell the goods and accumulate revenue and earn interest within the trade credit period. They implicitly assumed that the customer would pay for the items as soon as the items are received from the retailer. That is, they assumed that the supplier would offer the retailer a delay period but the retailer would not offer any delay period to his/her customer. In most business transactions, this assumption is unrealistic. Usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his/her customers. For example, in India, the TATA Company can delay the payment of purchasing cost until the end of the delay period offered by his supplier. The TATA Company also offers permissible delay payment period to his dealership. Huang (2003) presented an inventory model assuming that the retailer also permits a credit period to its customer which is shorter than the credit period offered by the supplier, in order to stimulate the demand. Huang (2006) extended Huang's (2003) model to investigate the

retailers inventory policy under two levels of trade credit and limited storage space. Mahata and Goswami (2007) developed an inventory model to determine an optimal ordering policy for deteriorating items under two-level trade credit policy in the fuzzy sense. Huang (2007) incorporated Huang's (2003) model to investigate the two-level trade credit policy in the EPQ frame work. Ho, Ouyang, and Su (2008) developed an integrated supplier-buyer inventory model with the assumption that demand is sensitive to retail price and the supplier adopts a two-part trade credit policy. Liao (2008) developed an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit financing. Tsao (2009) developed an EOQ model under advance sales discount and two-echelon trade credits. Teng and Chang (2009) extended the Huang's (2007) model by relaxing the assumption  $N < M$ . Kreng and Tan (2010) modify Huang's (2003) model by developing optimal wholesaler's replenishment decisions in the EOQ model under two levels of trade credit policy depending on the order quantity. Min, Zhou, and Zhao (2010) developed an inventory model for deteriorating items under stock-dependent demand and two-level trade credit. The inventory models under two levels of trade credit policy using the viewpoint of Huang (2003) can be found in several research papers such as Kreng and Tan (2010), Feng, Li, and Zhao (2013), Zhou, Zhong, and Wahab (2013), and Chung (2013), Shastri, Singh, Yadav, and Gupta (2014).

Deterioration of items present in inventory had been studied in the past decades (Dye, Ouyang, & Hsieh, 2007; Lin & Lin, 2004; Lin, Tan, & Lee, 2000; Mahata, 2012; Min et al., 2010; Soni, 2013; Wee, 1995; Yadav et al., 2012a). In that research, researchers mainly focused on: (1) considering different pattern of deterioration rate which may be time dependent or constant, (2) quantity discount, and (3) supply chain coordination. However, investigation on preservation technology to reduce deterioration rate has received little attention in the past years. The consideration of preservation technology is important due to the fact that preservation technology can reduce the deterioration rate significantly. The higher rate of deterioration would result in a higher total annual relevant cost and a lower demand rate. Murr and Morris (1975) showed that a lower temperature will increase the storage life and decrease decay. Moreover, drying or vacuum technology is introduced to reduce the deterioration rate of medicine and foodstuff. Zauberman, Ronen, Akerman, and Fuchs (1990) developed a method for color retention of Litchi fruits with  $SO_2$  fumigation. The tradeoff between the increased cost of investment and the increased profit due to decreased deterioration rate is the focus of our study. Ouyang, Wu, and Yang (2006) found that if the retailer can reduce effectively the deteriorating rate of item by improving the storage facility, the total annual relevant inventory cost will be reduced. Wang, Wang, and Yang (2007) focused on deciding on resources portfolio and allocating resources to various orders in each production period. Many enterprises invest on equipment to reduce the deterioration rate and extend the product expiration date. For example, refrigeration equipment is used to reduce the deterioration rate of fruits, flowers, and sea foods in the supermarket. Shastri et al. (2014) developed an EOQ inventory model for a retailer under two-levels of trade credit to reflect the supply chain management (SCM) by using preservation technology to increase the potential worth of the deteriorated items.

Arrow (1962) and Rosen (1972) observed that the cost associated with production system especially for a new product declines by a factor 10–50% each time the accumulated production volumes doubles, due to learning by doing. Many researchers have applied this learning-by-doing phenomenon into production-marketing model to obtain optimal pricing, advertising, quality, and other strategies. Salameh and Jaber (2000) assumed that the defective items could be sold as a single batch at a discounted price prior to receiving the next shipment, and found that the economic lot size tends to increase as the average percentage of imperfect quality items decreases. Jaber, Goyal, and Imran (2008) extended the work of Salameh and Jaber (2000) by assuming that the defective percentage of goods within a shipment was reduced as the learning curve increased. Tsai (2012) presented two models for determining an optimal integrated EOQ and economic production quantity policy in a recoverable manufacturing environment. Tsai assumed that the unit production time of the recovery process decreases with the increase in total units produced as a result of learning. Yadav et al. (2012b) developed an EOQ model by incorporating the effect of learning in holding cost, ordering cost, and on the number of defective items present in each lot. Teng, Lou, and Wang (2013) proposed an economic

production quantity model from seller's prospective to determine his/her optimal trade credit period and production lot size simultaneous in which (1) trade credit increases not only sales but also opportunity cost and default risk, and (2) production cost declines and obeys a learning curve phenomenon. Khan, Jaber, and Ahmad (2014) developed an integrated mathematical model for determining an optimal vendor-buyer inventory policy by accounting for quality inspection errors of type-I and type-II at the buyer's end and learning in production follows the Wright (1936) learning curve at the vendor's end. Goyal, Singh, and Yadav (2015) developed an EOQ model for imperfect lot with partial backordering. They also considered the effect of learning while obtaining the optimal solution.

The present study investigated the inventory model especially for a retailer under two levels of trade credit to reflect the SCM. Here, we are taking into account the following factors:

- (1) the selling items are perishable such as fruits, fresh fishes, gasoline, photographic films, etc. It is assumed that decay in potential worth of items can be increased by using preservation technology;
- (2) supplier offers trade credit period of  $M$  to the retailer while in turn retailer provides a trade credit period of  $N$  to his/her customers;
- (3) the replenishment rate is finite;
- (4) the supplier is willing to provide the retailer a full trade credit period for payments and the retailer offers the partial trade credit period to his/her customers;
- (5) the demand is the function of selling price and trade credit;
- (6) ordering cost reduces due to learning by doing phenomenon.

Under these conditions, we model inventory system as a cost minimization problem. By applying convex fractional programming results, we obtain necessary and sufficient conditions of an optimal solution. Some theorems are developed to determine retailer's optimal ordering policies and numerical examples are given to illustrate these theorems. In addition, some managerial insights from the numerical examples are also concluded.

## 2. Assumptions and notations

The following assumptions and notation are considered to develop the model.

### 2.1. Assumptions

- (1) Replenishments rate,  $P$ , is known and constant.
- (2) Credit period reduces the retailer's inventory holding cost, and hence a positive impact on demand. Similarly selling price of the items also produces positive impact on the demand. So, we consider demand as a function of trade credit and selling price.
- (3) Preservation technology is used to reduce the decay rate of items and there is no replacement or repair of deteriorated items during a given cycle.
- (4) It is a well-known learning by doing phenomenon that the total ordering cost declines by a factor of 10–50% each time the accumulative production run especially during the introduction phase of a new product. Mathematically, it can be represented as

$$A(n) = A_0/n^\delta$$

where  $A_0$  is the ordering cost in first production run and  $\delta > 0$ .

- (5) The supplier offers the full trade credit to the retailer, when  $T \geq M$ , the account settled at  $T = M$ , the retailers pay off all units sold and keep his/her profits, and start paying for the interest charges on the items in stock with rate  $I_c$ . When  $T \leq M$ , the account is settled at  $T = M$  and the retailer no need to pay any interest on the stock.

- (6) The retailer just offers the partial trade credit to his/her customers. Hence, the customer must make a partial payment to the retailer when the items are sold. Then the customer must pay off the remaining balance at the end of the trade credit period offered by the retailer. That is, the retailer can accumulate interest from his/her customer payment with rate  $I_e$ .
- (7) Shortages are not allowed and lead time is zero.
- (8) Time horizon is infinite.

## 2.2. Notations

$P$	production rate per year
$D(s, M)$	( $=as^{-b}e^{cM}$ , $a, b, c > 0$ ) demand rate of the customers
$\epsilon$	preservation technology cost for reducing deterioration rate in order to preserve the products, $\epsilon \geq 0$
$m(\epsilon)$	reduce deterioration rate, a function of $\epsilon$
$k$	original deterioration rate, $k > 0$
$h$	inventory holding cost per item per year excluding interest charges
$A(n)$	( $=A_0/n^a$ ) ordering cost per order
$c$	the unit purchase cost per unit item
$s$	the unit selling price of items of good quality where $s \geq c$
$\alpha$	customer's fraction of the total amount owed payable at the time of placing an order offered by the retailer $0 \leq \alpha \leq 1$
$M$	retailer's trade credit period offered by the supplier in years
$N$	customer's trade credit period offered by the retailer in years
$I_c$	the interest charged per year per \$ in stocks by the supplier
$I_e$	the interest earned per \$ per year
$T_1$	the time at which the production stops in a cycle
$T$	cycle time in years

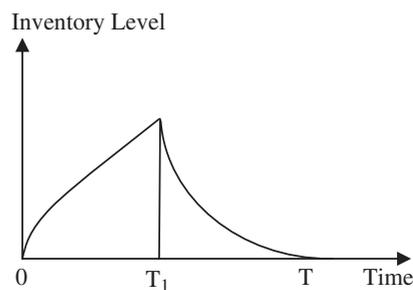
## 3. Mathematical modeling of supply chain under two level of trade credits

In this paper, we developed a retailer's EPQ-based inventory model under two levels of trade credit to reflect the SCM. It is assumed that the position of retailer is strong in the system and can obtain full trade credit offered by the supplier and the retailer just offered partial trade credit to his customers. We also considered that items deteriorate with deterioration rate  $k$  in the absence of preservation technology. However, reduced deterioration rate  $m(x)$  is assumed when the retailer invests "\$ $x$ " on the preservation technology cost.

A production process starts at  $t = 0$  at the rate  $P$  and continues up to  $t = T_1$  where the inventory level reaches the maximum level. During this period, inventory level rises due to production and decline due to demand and deterioration. At  $t = T_1$ , production process stops and afterward inventory level declines only due to demand and deterioration. Inventory level reaches to zero at  $t = T$ . This process continues as the planning horizon is infinite. The graphical representation of this inventory system is clearly depicted in Figure 1. Let  $I(t)$  be the inventory level at any time  $t$  ( $0 \leq t \leq T$ ). Initially, the stock level is  $Q$ . The inventory level decreases both due to demand and deterioration until it becomes zero at time  $t = T$ .

The differential equation governing the transition of the inventory system during the planning horizon is

**Figure 1. Pictorial representation of inventory level.**



$$\frac{dI_1(t)}{dt} + (k - m(\epsilon))I(t) = P - D(s, M), \quad 0 \leq t \leq T_1 \quad (1)$$

With the initial conditions  $I_1(0) = 0$ . The solution of the differential Equation 1 with the initial condition  $I_1(0) = 0$  is

$$I_1(t) = \frac{P - D(s, M)}{(k - m(\epsilon))} \left( 1 - e^{-(k-m(\epsilon))t} \right), \quad 0 \leq t \leq T_1 \quad (2)$$

In the time interval  $(T_1, T]$ , the inventory declines due to demand and deterioration. Hence, the change in the inventory level is governed by the following differential equations:

$$\frac{dI_2(t)}{dt} + (k - m(\epsilon))I(t) = -D(s, M), \quad T_1 \leq t \leq T \quad (3)$$

With the boundary conditions  $I_1(T) = 0$ . The solution of the differential Equation 3 with the boundary condition  $I_2(T) = 0$  is

$$I_2(t) = \frac{D(s, M)}{(k - m(\epsilon))} \left( e^{(k-m(\epsilon))(T-t)} - 1 \right) \quad (4)$$

In addition, using the boundary condition

$$I_1(T_1) = I_2(T_1)$$

We obtain the following equations:

$$P \left( 1 - e^{-(k-m(\epsilon))T_1} \right) = D(s, M) \left( e^{(k-m(\epsilon))(T-T_1)} - e^{-(k-m(\epsilon))T_1} \right) \quad (5)$$

$$T_1 = \frac{1}{(k - m(\epsilon))} \log \left\{ 1 + \frac{D(s, M)}{P} \left( e^{(k-m(\epsilon))T} - 1 \right) \right\} \quad (6)$$

Now, we derive the total relevant cost for the seller which is comprised of the annual ordering cost, annual holding cost, annual interest payable, annual interest earned, and annual deterioration cost.

Annual ordering cost =  $A(n)/T$

$$\begin{aligned} \text{Annual holding cost (excluding interest charges)} &= \frac{h}{T} \left[ \int_0^{T_1} I_1(t) dt + \int_{T_1}^T I_2(t) dt \right] = \frac{h}{(k-m(\epsilon))^2 T} \\ &\left( (k - m(\epsilon))T_1 + e^{-(k-m(\epsilon))T_1} - 1 \right) P + \frac{h}{(k-m(\epsilon))^2 T} \left( e^{(k-m(\epsilon))(T-T_1)} - (k - m(\epsilon))T - e^{-(k-m(\epsilon))T_1} \right) D(s, M) \\ &= \frac{h}{(k-m(\epsilon))^2 T} (PT_1 - D(s, M)T) \quad \text{using Equation 5.} \end{aligned}$$

Total annual cost due to deterioration of items during the cycle =  $\frac{h}{T} (PT_1 - D(s, M)T)$

From the values of  $N$  and  $M$ , there are two possible cases:

Case-1:  $M > N$     Case-2:  $N \geq M$

Now, we discuss these two cases separately one by one.

**Case-1:  $M > N$**

Based on the values of  $M$ ,  $T$ ,  $T + N$  (i.e. the time at which the seller receives the payment from the last customer), three sub-cases can occur.

*Sub-case-1.1:  $M \leq T$*

In this sub-case, the seller accumulates revenue and earns interest.

- (i) from the portion of instant payment starting time 0 through  $M$  and
- (ii) from the portion of delayed payment starting time  $N$  through  $M$ .

Hence, the interest earned per cycle is  $I_e$  times the total area of the triangle  $OMA$  and the triangle  $OMA'$  as shown in Figure 2. Therefore, annual interest earned is given by

$$\frac{sI_e D(s, M)}{2T} (\alpha M^2 + (1 - \alpha)(M - N)^2)$$

On the other hand, the seller grants its buyers a trade credit of  $N$  periods, and receives the money from its buyers from time  $N$  through  $T + N$ . Thus, at time  $M$  the seller receives  $\alpha sD(s, M)M$  dollars from the instant payment and  $(1 - \alpha)sD(s, M)(M - N)$  dollars from the delayed payment, and pays its supplier  $\alpha cD(s, M)M + (1 - \alpha)cD(s, M)(M - N)$  dollars. The retailers must finance

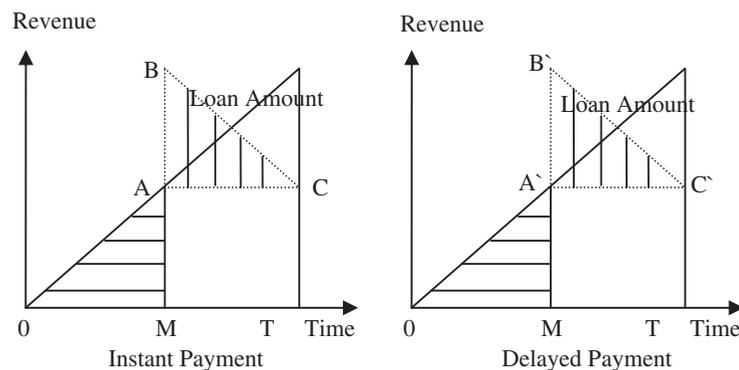
- (i) all items sold after  $M$  for the portion of instant payment, and
- (ii) all items sold after  $M - N$  for the portion of delayed payment at an interest charged  $I_c$  per dollar per year.

As a result, the interest payable per cycle is  $cI_c$  times the total area of the triangle  $ABC$  and the triangle  $A'B'C'$  as shown in Figure 2.

Therefore, the annual interest payable is given by

$$\frac{cI_c D(s, M)}{2T} (\alpha (T - M)^2 + (1 - \alpha)(T + N - M)^2)$$

Figure 2.  $M > N$  and  $M \leq T$ .



The seller's annual total relevant cost is

$$\begin{aligned} \text{TRC}_{1.1}(T) = & \frac{A(n)}{T} + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))T} (PT_1 - D(s, M)T) \\ & + \frac{cI_c D(s, M)}{2T} (\alpha(T - M)^2 + (1 - \alpha)(T + N - M)^2) \\ & - \frac{sI_e D(s, M)}{2T} (\alpha M^2 + (1 - \alpha)(M - N)^2) \end{aligned} \quad (7)$$

Sub-Case-1.2:  $T \leq M \leq T + N$

Seller accumulates revenue and earns interest from two accounts:

- (i) the portion of instant payment starting time 0 through  $M$ , and
- (ii) the portion of delayed payment starting from  $N$  through  $M$ .

Hence, the seller's annual interest earned as shown in Figure 3 as

$$\frac{sI_e D(s, M)}{2T} (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2)$$

The seller receives all instant payment by time  $T (\leq M)$  so that there is no interest payable for the portion of instant payment. However, the seller must finance all items sold during time interval  $[M - N, T]$ . Therefore, the annual interest payable is

$$\frac{cI_c D(s, M)}{2T} (1 - \alpha)(T + N - M)^2$$

The seller's annual total relevant cost by

$$\begin{aligned} \text{TRC}_{1.2}(T) = & \frac{A(n)}{T} + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))T} (PT_1 - D(s, M)T) \\ & + \frac{cI_c D(s, M)}{2T} (1 - \alpha)(T + N - M)^2 - \frac{sI_e D(s, M)}{2T} \\ & \times (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2) \end{aligned} \quad (8)$$

Sub-case-1.3:  $T + N \leq M$

The seller receives the total revenue before the trade credit period  $M$ , and hence there is no interest payable.

**Figure 3.  $M > N$  and  $T \leq M \leq T + N$ .**

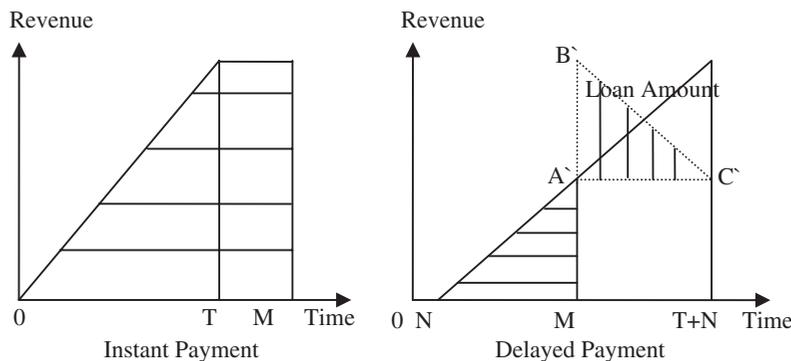
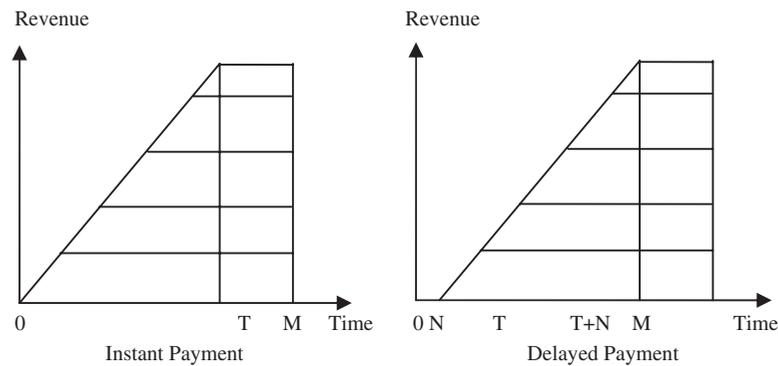


Figure 4.  $M > N$  and  $T + N \leq M$ .



From Figure 4, the annual interest earned is

$$\begin{aligned} & \frac{sI_e D(s, M)}{2T} (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)T^2 + 2(1 - \alpha)T(M - T - N)) \\ &= \frac{sI_e D(s, M)}{2} (2M - T - 2(1 - \alpha)N) \end{aligned}$$

The seller's annual total relevant cost by

$$TRC_{1.3}(T) = \frac{A(n)}{T} + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))T} (PT_1 - D(s, M)T) - \frac{sI_e D(s, M)}{2} (2M - T - 2(1 - \alpha)N) \quad (9)$$

The seller's annual total relevant cost is given by

$$TRC_1(T) = \begin{cases} TRC_{1.1}(T) & \text{if } M \leq T \\ TRC_{1.2}(T) & \text{if } T \leq M \leq T + N \\ TRC_{1.3}(T) & \text{if } T + N \leq M \end{cases} \quad (10)$$

Hence,  $TRC_1(T)$  is continuous in  $T$ , and has the following properties

$$TRC_{1.1}(M) = TRC_{1.2}(M)$$

$$TRC_{1.2}(M - N) = TRC_{1.3}(M - N)$$

#### Case-2: $M \leq N$

Now, on the bases of the values of  $M$  and  $T$ , the following two sub-cases can occur: (i)  $M \leq T$  and (ii)  $M \geq T$ . Now, we discuss them in length.

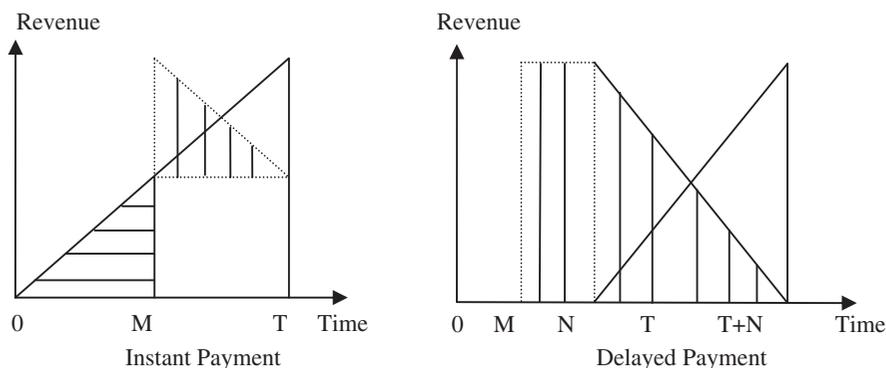
##### Sub-Case-2.1: $M \leq T$

From Figure 5, the annual interest earned from the instant payment is

$$\frac{sI_e D(s, M)}{2T} (\alpha M^2)$$

In this sub-case, for instant payment the seller must finance  $\alpha cD(s, M)(T - M)$  at time  $M$ , and pay off the loan at time  $T$ . As to delayed payment, the seller must finance  $(1 - \alpha)cD(s, M)T$  for delayed payment at time  $M$ , and pay off the loan at time  $T + N$ . Therefore, the annual interest payable is

Figure 5.  $M \leq N$  and  $M \leq T$ .



$$\frac{cI_c D(s, M)}{2T} (\alpha(T - M)^2 + (1 - \alpha)T (T + 2(N - M)))$$

The annual total relevant cost of the seller is

$$\begin{aligned} TRC_{2.1}(T) = & \frac{A(n)}{T} + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))T} (PT_1 - D(s, M)T) \\ & + \frac{cI_c D(s, M)}{2T} (\alpha(T - M)^2 + (1 - \alpha)T (T + 2(N - M))) - \frac{sI_e D(s, M)}{2T} (\alpha M^2) \end{aligned} \quad (11)$$

Sub-Case-2.2:  $M \geq T$

From Figure 6, the annual interest earned from the instant payment is

$$\frac{s\alpha I_e D(s, M)}{2} (T + 2(M - T))$$

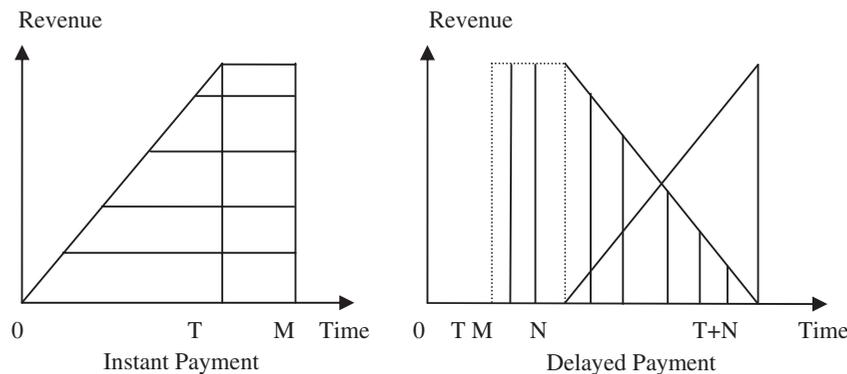
In this sub-case, there is no interest payable for instant payment. However, the seller must finance  $(1 - \alpha)cD(s, M)T$  for delayed payment at time  $M$ , and pay off the loan at time  $T + N$ . Therefore, the annual interest payable is

$$\frac{c\alpha I_c D(s, M)}{2} (1 - \alpha) (T + 2(N - M))$$

The seller's annual total relevant cost is

$$TRC_{2.2}(T) = \frac{A(n)}{T} + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))T} (PT_1 - D(s, M)T) + \frac{c\alpha I_c D(s, M)}{2} (1 - \alpha) (T + 2(N - M))$$

Figure 6.  $M \leq N$  and  $M \geq T$ .



$$(T + 2(M - T)) - \frac{s\alpha I_e D(s, M)}{2} (T + 2(M - T)) \tag{12}$$

The seller's annual total cost is

$$TRC_2(T) = \begin{cases} TRC_{2,1}(T) & \text{if } M \leq T \\ TRC_{2,1}(T) & \text{if } M \geq T \end{cases} \tag{13}$$

It is clear that  $TRC_2(T)$  is continuous in  $T$ , and has the following properties

$$TRC_{2,1}(M) = TRC_{2,2}(M)$$

Now, we have to determine the optimal replenishment cycle  $T$  for both cases of  $N < M$  and  $N \geq M$ .

#### 4. Determination of the optimal replenishment cycle

In this section, the necessary and sufficient conditions for the determination of the optimal solution, say  $(T^*, TRC^*(T))$  are presented here for the case  $N < M$  and  $N \geq M$ .

**Intermediate Value Theorem:** Let  $f(x)$  be a continuous function on  $[a, b]$  and  $f(a)f(b) < 0$ , then there exists a number  $c \in (a, b)$  such that  $f(c) = 0$ .

**Property-1:** For a function  $f: S \rightarrow R_1$  defined by  $f(x) = g(x)/h(x)$  where  $g: S \rightarrow R_1$ , and  $S$  is a non-empty convex set in  $E_n$ . The following discuss convexity and concavity functions:

- (a)  $g$  is convex on  $S$ , and  $g(x) \geq 0$  for each  $x \in S$ .
- (b)  $h$  is concave on  $S$ , and  $h(x) \geq 0$  for each  $x \in S$ , and
- (c) Both  $g$  and  $h$  are differentiable.

The  $f(x)$  is convex if  $g(x)$  is convex and  $h(x)$  is concave.

##### 4.1. Determination of optimal replenishment cycle for the case of $N < M$

By applying the above-mentioned results, we separately minimize each of  $TRC_{1,i}(T)$  for  $i = 1, 2, 3$  and obtain the following theoretical results.

Theorem 1.

1.  $TRC_{1,1}(T)$  is a strictly pseudo-convex function in  $T$ , and hence there exists a unique minimum solution  $T_\alpha^*$ .
2. If  $M \leq T_\alpha^*$  then  $TRC_{1,1}(T)$  subjected to  $M \leq T$  is minimized at  $T_\alpha^*$ .
3. If  $M \geq T_\alpha^*$  then  $(T)$  subjected to  $M \leq T$  is minimized at  $M$ .

*Proof.* Let

$$g(T) = A(n) + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} (PT_1 - D(s, M)T) + \frac{cI_c D(s, M)}{2} (\alpha(T - M)^2 + (1 - \alpha)(T + N - M)^2) - \frac{sI_e D(s, M)}{2} (\alpha M^2 + (1 - \alpha)(M - N)^2) \geq 0 \text{ for } T < 0$$

and

$$h(T) = T > 0$$

Taking first and second derivative of  $g(T)$ , we get

$$\frac{dg(T)}{dT} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{PD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - D(s, M) \right) + cI_c D(s, M) ((T - M) + (1 - \alpha)N)$$

$$\frac{d^2g(T)}{dT^2} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{P^2 D(s, M)(k - m(\epsilon))e^{(k-m(\epsilon))T}}{(P + D(s, M)(e^{(k-m(\epsilon))T} - 1))^2} (1 - D(s, M)/P) \right) + cI_c D(s, M) > 0$$

Therefore,  $TRC_{1,1}(T) = g(T)/h(T)$  is a strictly pseudo-convex function in  $T$ , which completes the proof of part (1).

Part (2) and (3) are the direct consequences of part (1).

To find  $T_a^*$ , we set  $\frac{dTRC_{1,1}(T)}{dT} = 0$

$$\frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{D(s, M) [\alpha M^2 + (1 - \alpha)(M - N)^2] [sI_e - cI_c]}{2} + \frac{cI_c D(s, M)T^2}{2} - A(n) = 0$$

**THEOREM 2**

1.  $TRC_{1,2}(T)$  is a strictly pseudo-convex function in  $T$ , and hence there exists a unique minimum solution  $T_b^*$ .
2. If  $M - N \leq T_b^* \leq M$  then  $TRC_{1,2}(T)$  subjected to  $T \leq M \leq T + N$  is minimized at  $T_b^*$ .
3. If  $T_b^* \leq M - N$  then  $TRC_{1,2}(T)$  subjected to  $T \leq M \leq T + N$  is minimized at  $M - N$ .
4. If  $T_b^* \geq M$  then  $TRC_{1,2}(T)$  subjected to  $T \leq M \leq T + N$  is minimized at  $M$ .

*Proof* Let

$$g(T) = A(n) + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} (PT_1 - D(s, M)T) + \frac{cI_c D(s, M)}{2} (1 - \alpha)(T + N - M)^2 - \frac{sI_e D(s, M)}{2} (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2) \geq 0 \text{ for } T > 0$$

and

$$h(T) = T > 0$$

Taking first and second derivative of  $g(T)$ , we get

$$\frac{dg(T)}{dT} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{PD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - D(s, M) \right) + cI_c D(s, M)(1 - \alpha)(T - M + N) - sI_e D(s, M)\alpha(M - T)$$

$$\frac{d^2g(T)}{dT^2} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{P^2 D(s, M)(k - m(\epsilon))e^{(k-m(\epsilon))T}}{(P + D(s, M)(e^{(k-m(\epsilon))T} - 1))^2} (1 - D(s, M)/P) \right) + D(s, M) (c(1 - \alpha)I_c + \alpha sI_e) > 0$$

Therefore,  $TRC_{1,2}(T) = g(T)/h(T)$  is a strictly pseudo-convex function in  $T$ , which completes the proof of part (1).

Part (2), (3), and (4) are the direct consequences of part (1).

To find  $T_b^*$ , we set  $\frac{dTRC_{1,2}(T)}{dT} = 0$

$$\frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{D(s, M)(1 - \alpha)(M - N)^2 [sI_e - cI_c]}{2} + \frac{D(s, M)T^2 [\alpha sI_e + (1 - \alpha)cI_c]}{2} - A(n) = 0$$

**THEOREM 3**

1.  $TRC_{1,3}(T)$  is a strictly pseudo-convex function in  $T$ , and hence there exists a unique minimum solution  $T_c^*$ .
2. If  $T_c^* \leq M - N$  then  $TRC_{1,3}(T)$  subjected to  $T + N \leq M$  is minimized at  $T_c^*$ .
3. If  $T_c^* \geq M - N$  then  $TRC_{1,3}(T)$  subjected to  $T + N \leq M$  is minimized at  $M - N$ .

*Proof* Let

$$g(T) = A(n) + \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} (PT_1 - D(s, M)T) - \frac{sI_e D(s, M)}{2} (2MT - T^2 - 2(1 - \alpha)NT) \geq 0 \text{ for } T > 0$$

and

$$h(T) = T > 0$$

Taking first and second derivative of  $g(T)$ , we get

$$\frac{dg(T)}{dT} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{PD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - D(s, M) \right) - sI_e D(s, M)(M - T - (1 - \alpha)N)$$

$$\frac{d^2g(T)}{dT^2} = \frac{h + (k - m(\epsilon))c}{(k - m(\epsilon))} \left( \frac{P^2 D(s, M)(k - m(\epsilon))e^{(k-m(\epsilon))T}}{(P + D(s, M)(e^{(k-m(\epsilon))T} - 1))^2} (1 - D(s, M)/P) \right) + sI_e D(s, M) > 0$$

Therefore,  $TRC_{1,3}(T) = g(T)/h(T)$  is a strictly pseudo-convex function in  $T$ , which completes the proof of part (1).

Part (2) and (3) are the direct consequences of part (1).

To find  $T_c^*$ , we set  $\frac{dTRC_{1,3}(T)}{dT} = 0$

$$\frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{sI_e D(s, M)T^2}{2} - A(n) = 0$$

**LEMMA 1** Show that  $\Delta_1 < \Delta_2$ , where

$$\Delta_1 = \frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \times \left( \frac{(M - N)D(s, M)e^{(k-m(\epsilon))(M-N)}}{P + D(s, M)(e^{(k-m(\epsilon))(M-N)} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))(M-N)} - 1) \right) \right) + \frac{sI_e D(s, M)(M - N)^2}{2} - A(n)$$

and

$$\Delta_2 = \frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{MD(s, M)e^{(k-m(\epsilon))M}}{P + D(s, M)(e^{(k-m(\epsilon))M} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))M} - 1) \right) \right) + \frac{D(s, M) [\alpha M^2 + (1 - \alpha)(M - N)^2] [sI_e - cI_c]}{2} + \frac{cI_c D(s, M)T^2}{2} - A(n)$$

*Proof* Since  $TRC_{1,2}(T)$  is a strictly pseudo-convex function in  $T$ . We have

$$\frac{dTRC_{1,2}(T)}{dT} = \frac{1}{T^2} \left( \frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{D(s, M)(1 - \alpha)(M - N)^2 [sI_e - cI_c]}{2} + \frac{D(s, M)T^2 [\alpha sI_e + (1 - \alpha)cI_c]}{2} - A(n) \right)$$

is an increasing function in  $T$ , and hence we get

$$\frac{dTRC_{1,2}(M - N)}{dT} < \frac{dTRC_{1,2}(M)}{dT}$$

$$\Rightarrow \frac{\Delta_1}{(M - N)^2} < \frac{\Delta_2}{M^2}$$

$$\Rightarrow \Delta_1 < \Delta_2.$$

**THEOREM 4**

1. If  $\Delta_2 < 0$  then  $TRC_1(T)$  is minimized at  $T_\sigma^*$ .
2. If  $\Delta_2 = 0$  then  $TRC_1(T)$  is minimized at  $M$ .
3. If  $\Delta_1 < 0$  then  $TRC_1(T)$  is minimized at  $T_b^*$ .
4. If  $\Delta_1 = 0$  then  $TRC_1(T)$  is minimized at  $M - N$ .
5. If  $\Delta_1 > 0$  then  $TRC_1(T)$  is minimized at  $T_c^*$ .

*Proof* We have

$$\frac{dTRC_{1,1}(T)}{dT} = \frac{1}{T^2} \left( \frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{D(s, M) [\alpha M^2 + (1 - \alpha)(M - N)^2] [sI_e - cI_c]}{2} + \frac{cI_c D(s, M)T^2}{2} - A(n) \right)$$

If  $\Delta_2 < 0$ , then

$$\lim_{T \rightarrow \infty} \frac{dTRC_{1.1}(T)}{dT} = \frac{cI_c D(s, M)}{2} > 0$$

and

$$\left[ \frac{dTRC_{1.1}(T)}{dT} \right]_{T=M} = \frac{\Delta_2}{M^2} < 0$$

By applying the mean value theorem and Theorem 1, there exists a unique  $T_\alpha^* \in (M, \infty)$  such that  $\frac{dTRC_{1.1}(T_\alpha^*)}{dT} = 0$ . Hence,  $TRC_{1.1}(T)$  is minimizing at the unique point  $T_\alpha^*$ . By using the analogous argument, we have

$$\frac{dTRC_{1.2}(M-N)}{dT} = \frac{\Delta_1}{(M-N)^2} < \frac{dTRC_{1.2}(M)}{dT} = \frac{\Delta_2}{M^2} < 0$$

which implies that  $TRC_{1.2}(M)$  is minimizing at  $M$ . Likewise, we get

$$\lim_{\delta \rightarrow \infty} \frac{dTRC_{1.3}(\delta)}{dT} < \frac{dTRC_{1.3}(M-N)}{dT} = \frac{\Delta_1}{(M-N)^2} < 0$$

which implies that  $TRC_{1.3}(T)$  is minimizing at  $M-N$ . Consequently, we obtain

$$TRC_{1.1}(T_\alpha^*) < TRC_{1.1}(M) = TRC_{1.2}(M) < TRC_{1.2}(M-N) = TRC_{1.3}(M-N)$$

This shows the end of the first part of the theorem. Similarly, remaining part of the theorem can be obtained.

#### 4.2. Determination of optimal replenishment cycle for the case of $N \geq M$

By applying the above-mentioned results, we separately minimize each of  $TRC_{2.i}(T)$  for  $i = 1, 2$ , and obtain the following theoretical results.

##### THEOREM 5

1.  $TRC_{2.1}(T)$  is a strictly pseudo-convex function in  $T$ , and hence there exists a unique minimum solution  $T_d^*$
2. If  $M \leq T_d^*$  then  $TRC_{2.1}(T)$  subjected to  $M \leq T$  is minimized at  $T_d^*$
3. If  $M \geq T_d^*$  then  $TRC_{2.1}(T)$  subjected to  $M \leq T$  is minimized at  $M$ .

To find  $T_d^*$  we set  $\frac{dTRC_{2.1}(T)}{dT} = 0$

$$\frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{D(s, M)\alpha M^2 [sI_e - cI_c]}{2} + \frac{cI_c D(s, M)T^2}{2} - A(n) = 0$$

It is obvious from the above theorem that above expression has a unique solution  $T_d^*$ . If  $T_d^* \geq M$  then  $TRC_{2.1}(T)$  is minimized at  $T_d^*$ . Otherwise,  $TRC_{2.1}(T)$  is minimized at  $M$ .

**THEOREM 6**

1.  $TRC_{2,2}(T)$  is a strictly pseudo-convex function in  $T$ , and hence there exists a unique minimum solution  $T_e^*$ .
2. If  $M \geq T_e^* \leq T$  then  $TRC_{2,2}(T)$  subjected to  $M \geq T$  is minimized at  $T_e^*$ .
3. If  $M \geq T_e^*$  then  $TRC_{2,1}(T)$  subjected to  $M \geq T$  is minimized at  $M$ .

To find  $T_e^*$  we set  $\frac{dTRC_{2,2}(T)}{dT} = 0$

$$\frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{TD(s, M)e^{(k-m(\epsilon))T}}{P + D(s, M)(e^{(k-m(\epsilon))T} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))T} - 1) \right) \right) + \frac{\alpha s I_e D(s, M) T^2}{2} + \frac{c I_c (1 - \alpha) D(s, M) T^2}{2} - A(n) = 0$$

It is obvious from the above theorem that above expression has a unique solution  $T_e^*$ . If  $T_e^* \leq M$  then  $TRC_{2,2}(T)$  is minimized at  $T_e^*$ . Otherwise,  $TRC_{2,2}(T)$  is minimized at  $M$ .

**THEOREM 7**

1. If  $\Delta_3 < 0$  then  $TRC_2(T)$  is minimized at  $T_e^*$ .
2. If  $\Delta_3 = 0$  then  $TRC_2(T)$  is minimized at  $M$ .
3. If  $\Delta_3 > 0$  then  $TRC_2(T)$  is minimized at  $T_e^*$ .

where

$$\Delta_3 = \frac{P(h + (k - m(\epsilon))c)}{(k - m(\epsilon))} \left( \frac{MD(s, M)e^{(k-m(\epsilon))M}}{P + D(s, M)(e^{(k-m(\epsilon))M} - 1)} - \frac{1}{(k - m(\epsilon))} \log \left( 1 + \frac{D(s, M)}{P} (e^{(k-m(\epsilon))M} - 1) \right) \right) + \frac{\alpha s I_e D(s, M) M^2}{2} + \frac{c I_c (1 - \alpha) D(s, M) M^2}{2} - A(n)$$

**5. Some special cases**

**5.1. Huang's model**

If the potential value of the product does not change in infinite time then there is no need of preservation technology which we used here. So  $k \rightarrow 0, m(\epsilon) \rightarrow 0, s = c, \alpha \rightarrow 0$  (it means that the retailer also offers the full trade credit to his/her customer), demand does not depend on selling price and trade credit period, and production rate is infinite then this inventory model is identical to that of Huang (2003).

**5.2. Shah's model**

If the potential value of the product does not change due to preservation technology then there is no need of preservation technology which we used here. So  $m(\epsilon) \rightarrow 0, s = c, N \rightarrow 0$  (it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his/her customer) that is one level credit,  $\alpha \rightarrow 0$ , demand does not depend on selling price and trade credit and production rate is infinite then this inventory model is identical to that of Shah (1993).

**5.3. Goyal's model**

If the potential value of the product does not change in infinite time then there is no need of preservation technology which we used here. So  $k \rightarrow 0, m(\epsilon) \rightarrow 0, s = c, N \rightarrow 0$  (it means that the supplier would offer the retailer a delay period but the retailer would not offer the delay period to his/her

customer) that is one level credit,  $\alpha \rightarrow 0$ , demand does not depend on selling price and trade credit then this inventory model is identical to that of Goyal (1985).

### 6. Numerical analysis

The reduced deterioration rate,  $m(\epsilon)$ , is a function of the preservation technology cost “ $\epsilon$ ” such that  $m(\epsilon) = k(1 - e^{-k\epsilon})$ ,  $k' > 0$ , where  $k'$  is the simulation coefficient representing the percentage increase in  $m(\epsilon)$  per dollar increase in  $\epsilon$ , which means  $m(\epsilon)$  is an increasing function bounded above by  $k$ .

*Example 1* If  $a = 1,950$ ,  $b = 0.09$ ,  $c' = 0.08$ ,  $c = \$50$  per unit,  $h = \$15$  per unit,  $I_c = 0.15$ ,  $I_e = 0.10$ ,  $N = 0.05$ ,  $\alpha = 0.5$ ,  $k = 0.05$ ,  $k' = 0.01$ ,  $\epsilon = \$40$ , it gives  $m(\epsilon) = 0.016$ .

By using Theorem 4 we can easily obtain the optimal solution on three different sets of parameters as shown in Table 1.

*Example 2* If  $a = 1,950$ ,  $b = 0.09$ ,  $c' = 0.08$ ,  $c = \$50$  per unit,  $h = \$15$  per unit,  $I_c = 0.15$ ,  $I_e = 0.10$ ,  $s = 130$ ,  $\alpha = 0.5$ ,  $k = 0.05$ ,  $k' = 0.01$ ,  $\epsilon = \$40$ , it gives  $m(\epsilon) = 0.016$ .

By using Theorem 5 we can easily obtain the optimal solution on two different sets of parameters as shown in Table 2.

#### 6.1. Case study

A pharmaceutical company produces medicine. From the data it is calculated that the production cost per unit is \$50. Company observed that the demand pattern of the customer is  $as^{-be^{cM}}$  i.e. it depends on the selling price and the trade credit period offered, where  $a = 1,950$ ,  $b = 0.09$ ,  $c' = 0.08$ . The holding cost per unit is \$15 whereas ordering cost place by retailer is \$145. Here,  $I_c = 0.15$ ,  $I_e = 0.10$ . The trade credit offered by the company to his retailer is 0.1 years whereas retailer provides the trade credit period of 0.05 to his customers. Due to the strong position in the market, retailer can obtain full trade credit period whereas retailer provides partial trade credit to his customer where  $\alpha = 0.5$ . Preservation technology is used here to decrease the rate of deterioration with  $\epsilon = \$40$ .

In any decision-making situation, the change in the values of parameters may happen due to uncertainties. The decision-maker of the company observed that the deterioration rate, the selling price, and the production rate can changes as follows:

$$k = [0.03, 0.04, 0.05, 0.06, 0.07]$$

$$s = [65, 70, 75, 80, 85]$$

$$P = [2,300, 2,500, 2,700, 2,900, 3,100, 3,300, 3,500]$$

Now, in the front of the decision-maker it is a challenge to analyze how the optimal solution affected due to change in these parameters.

Table 1. Optimal policy in case of $M > N$									
Values of parameters/cases	P	M	A	s	$\Delta_1$	$\Delta_2$	$T_1$	$T^*$	TRC (\$)
1.1	2,900	0.1	145	75	-141.93	-76.59	0.10	0.13( $T_0$ )	1,998.78
1.2	3,900	0.1	95	75	-81.39	15.58	0.05	0.09( $T_0$ )	1,637.73
1.3	4,500	0.15	150	125	42.57	252.01	0.05	0.08 ( $T_0$ )	631.28

Table 2. Optimal policy in case of $M \leq N$								
Values of parameters/cases	P	M	A	N	$\Delta_3$	$T_1$	$T^*$	TRC (\$)
2.1	3,000	0.06	150	1.00	-136.06	0.10	0.13( $T_0$ )	2,949.84
2.2	4,500	0.09	90	1.10	10.86	0.04	0.08( $T_0$ )	20,468.00

In order to examine the implication of these changes, the sensitivity analysis will be of great help in decision-making.

### 6.1.1. Effect to deterioration rate

We now investigate the effects of varying rate of deteriorating in order to get more insight.

Figure 7 shows the deterioration rate at 0.03, 0.04, 0.05, 0.06, and 0.07 with other variables remain unchanged. It is shown that as the deterioration rate increases, the retailer's total inventory cost increases. A higher value of the deterioration rate results lower than the values for the optimal cycle time and the optimal order quantity, and higher value for the total inventory cost for the retailer.

### 6.1.2. Effect of selling price

To get the behavior of proposed model regarding selling, we investigate its effect on demand and the total inventory cost.

Figure 8 reflects the effect of selling price on demand and the total inventory cost of the retailer. It observes that as the selling price increases demand of the customer decreases and hence the total inventory cost of the retailer increases.

### 6.1.3. Effect of production rate

Sensitivity with respect to production rate has been performed here to get the effect on optimal policy of inventory management.

Figure 9 reflects the effect of production rate on production period, cycle time, and on the total inventory cost of the retailer. It is observed that as the production rate increases production period, cycle time decreases and the total inventory cost of the retailer decreases. So it is advisable to the decision-maker not to increase the production rate without the prior information about the customer's demand.

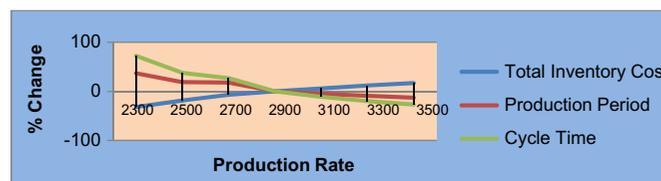
**Figure 7. Effect of deterioration of total inventory cost.**



**Figure 8. Effect of selling price on demand and total inventory cost.**



**Figure 9. Effect of production rate on total inventory cost.**



## 7. Discussion

Concept of preservation technology and the two-level trade credit has received a very little attention by the researchers while taking the decision regarding the manufacturing system. In this paper, we develop a production inventory model by assuming that the retailers hold the powerful position in the chain and can obtain the full trade credit offered by the supplier and retailer just offers partial trade credit to customers. We also assume that demand of the items not only depend upon the selling price but also credit period offered by the retailer. Here, retailer invests in preservation technology to control the deterioration rate of the product.

## 8. Conclusion

Our proposed production inventory model forms a general framework that includes many previous model as special cases such as Huang (2003), Shah (1993), and Goyal (1985). We also proposed theorems to find the optimal replenishment cycle time. Numerical examples are presented to illustrate the proposed theorems for finding the optimal cycle length. Sensitivity analysis with respect to deterioration rate, production rate, and selling price of items is also carried out to obtain a lot of managerial insights. The results of the paper not only provide a valuable reference for decision-makers in planning and controlling the inventory but also provide a useful model for many organizations that use the decision rule to improve their total operational cost.

## 9. Future work

There are several promising areas in which direction this model can be extended. This model can be extended by considering three different players such as supplier, manufacturer, and retailer to form integrated systems. One can also consider imprecise parameters in place of precise one.

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