Applying design of experiments to a compression refrigeration cycle

Nuno Ricardo Costa\textsuperscript{1,2}* and João Garcia\textsuperscript{1}

\textbf{Abstract:} Refrigeration cycles are used in a large diversity of industrial and domestic (residential and non-residential) equipment and their efficiency depend on several variables. To better understanding of how controllable variables impact on a compression refrigeration cycle efficiency, statistically designed experiments were conducted and data were analyzed. A quadratic polynomial model was fitted to Coefficient of Performance and variable settings to maximize cycle efficiency identified. Results give confidence to use the illustrated approach for refrigeration cycle design and operation improvement purposes.

\textbf{Subjects:} Heat Transfer; Thermodynamics; Heating Ventilation & Air Conditioning; Energy & Fuels; Engineering Productivity; Technology

\textbf{Keywords:} condensing pressure; COP; evaporating pressure; multiresponse; modeling; thermodynamic

\section{1. Introduction}

Without proper characterization of process and product, a considerable amount of guesswork about which input variables (control factors) have a significant effect on the quality characteristics (dependent variable or response) of interest will typically occur. Changing a single control factor...
while keeping the others fixed is an often-used practice, though it is strongly discouraged. The one-factor-at-a-time approach offers advantages only in exceptional conditions (Frey & Wang, 2006) so it is recommended to use an approach supported by statistical and mathematical techniques that has provided unequivocally evidence of its usefulness.

Experimental design and data analysis is an effective and commonly used approach in scientific investigations and technological applications in a wide variety of science fields, including in mechanical, chemical, and biotechnological engineering. Applications in product design and development comprise aircraft engines (Tappeta, Nagendra, & Renaud, 1999), bike-frames (Jeang, Liang, & Chung, 2008), copolymers (Ilbay & Çelik, 2009), electric motors (Gijo & Scaria, 2012), desonide creams (Lopes, Sarraguça, Prior, & Lopes, 2012), medical devices, and technological processes (Dixon, Eatock, Meenan, & Morgan, 2006; Steinberg & Bursztyn, 2010; Vlachogiannis, 2003), to cite only a few. In thermodynamic cycles, especially in refrigeration cycles, applications were not found, so this approach is illustrated here to investigate the called one stage refrigeration compression cycle (hereafter denoted as RC). This cycle has been used in a large diversity of domestic and industrial (residential and non-residential) equipment and explored from a thermodynamic point of view (Anand, Gupta, & Tyagi, 2013; Koelet, 1992; Rasmussen, 2012; Rasmussen & Shenoy, 2012; Tassou, Lewis, Ge, Hadawey, & Chaer, 2010), namely due to the recent refrigeration fluid restrictions related with environment protection as well as to the necessity of improvements in energy efficiency and energy savings (Bansal, Vineyard, & Abdelaziz, 2012; Palm, 2008). The objective here was to better understanding of how the temperature and the water mass flow rate in both evaporator and condenser impacts on RC and maximizing the cycle efficiency, because high-efficiency cycles are characterized by lower energy consumption and better refrigeration effects.

The remainder of the manuscript is organized as follows: Section 2 presents the experimental installation, refrigeration cycle principle, and Coefficient of Performance metric; experimental design, data analysis, and results are made in Sections 3 and 4, respectively; and conclusions are presented in Section 5.

2. RC—Experimental installation
The one stage RC (code: R632/25019) used in this study is a didactic unit produced by P.A. Hilton Ltd (see Figure 1). It includes a hermetic compressor (Aspera NEK6214Z), a condenser constructed from a thick-walled glass cylinder with machined brass end plates and a coil of copper tube inside (through which heating water flows), an evaporator constructed from a thick-walled glass cylinder with machined brass end plates and a coil of copper tube inside (through which cooling water flows), and an expansion valve (a float operated needle valve situated in the bottom of the condenser). The refrigeration fluid is R141b and integrated instrumentation enables to measure the condenser and evaporator pressures as well as temperatures in addition to water temperatures and flow rates.

Figure 1. Refrigeration cycle.
The description of the RC can be summarized as follows:

The hermetic compressor maintains a low pressure in the evaporator and this causes the refrigerant to evaporate at a low temperature, extracting (sensible) heat from the water and reducing water’s temperature. The low-pressure vapor formed in the evaporator is drawn into the compressor where its pressure is increased. The high-pressure fluid is then condensed and heat is transferred to the water that flows in the condenser. The high-pressure fluid collects in the bottom of the condenser and its level is controlled by a float operated expansion valve, which reaches an equilibrium position and the fluid discharged to the evaporator at the same rate as it is formed. When the warm fluid at high-pressure passes through the valve its pressure decreases to evaporator pressure and its temperature falls to the saturation temperature. On entering the evaporator, the low-pressure fluid and vapor separate themselves. The fluid is reevaporated, while the vapor mixes with the other vapor and passes to the compressor.

Two auxiliary apparatus were built for heating and cooling water in order to set the temperature in the inlet and outlet of both the evaporator and condenser at planned values. Hot water was produced in a gas burner, stored in a thermo-accumulator tank (SOLCAP-200 litres) to stabilize the temperature at specified values, and then pumped to the condenser (see Figure 2). Cold water was obtained by introducing ice water in a tank where current water was stored, and then pumped to the evaporator at desired temperature (see Figure 3).

Figure 2. Heating water system.

Figure 3. Cooling water system.
2.1. Refrigeration cycle: Coefficient of Performance

Refrigeration cycles are used in a wide variety of fields, for example, in food and pharmaceutical industries for product refrigeration and conservation, in health services to keep some medicines at low temperature, and in domestic and public rooms as air conditioner systems. To assess RC efficiency is required to know the refrigeration and electric powers. Electric power is the rate of energy consumption per time supplied to the compressor, which was measured with an analyzer Chauvin Arnoux (Qualistar plus CA 8335 - see Figure 1). Refrigeration power is a measure of the heat-extraction capacity of refrigeration equipments and is calculated by applying the first law of thermodynamics to open stationary systems, which states that the total energy of the system remains constant. Thus, in RC and under the assumption that heat losses in evaporator are negligible and the process is stationary, the energy received by refrigeration fluid from the water in the evaporator is equal to the energy transferred (released) by water to the refrigeration fluid.

Considering the schematic representation of inputs and outputs (energy and mass balance) in the evaporator (region delimited by the dash line) shown in Figure 4, refrigeration power ($Q_{\text{evap}}$) can be defined as

$$Q_{\text{evap}} = \dot{m}_{\text{water}} C_{\text{p,water}} (T_{\text{out, evap}} - T_{\text{evap}})$$

(1)

Notation and variable units are as follows:

$\dot{Q}_{\text{evap}}$—Refrigeration power (W); $\dot{m}_{\text{water}}$—Water mass flow rate in evaporator (kg/s); $C_{\text{p,water}}$—Specific heat of water at constant pressure (4.18 kJ/kgK); $T_{\text{evap}}$—Inlet water temperature in the evaporator (°C); $T_{\text{out, evap}}$—Outlet water temperature in the evaporator (°C).

The ratio between the Refrigeration ($Q_{\text{evap}}$) and Electric ($W_{\text{elect}}$) powers is a metric used currently to assess the refrigeration cycles efficiency (Dabas, Dodeja, Kumar, & Kasana, 2011; Dincer, 2004; Mackensen, Klein, & Reindl, 2002; Pfister, 2004). This metric is called Coefficient of Performance (COP) and is defined as

$$\text{COP} = \frac{Q_{\text{evap}}}{W_{\text{elect}}}$$

(2)

The higher the COP value is, the better cycle efficiency will be, which can be achieved by reducing energy consumption and increasing the refrigeration power (Dincer, 2004). COP values of compression refrigeration cycles used in domestic and industrial refrigeration vary from 1 up to 3 (Bjork, 2012). For small didactic cycles like this one used in the study reported here, the expected COP value will be equal to or slightly higher than 1. These units have demonstration and didactic purposes so its components do not have the best technical characteristics. For instance, the evaporator and condenser are made in glass, with very low heat transmission capacity, and the power of the compressor is not as high as that of compressors used in industrial or domestic equipment. As a result, refrigeration power and cycle performance values are not as high as desired.

Figure 4. Evaporator: control volume.
3. Design of experiments

Design and conduct experiments are not trivial tasks, though various authors have presented guidelines to help researchers and practitioners in planning, conducting, and analyzing data of experimental studies (Bisgaard, 1999; Coleman & Montgomery, 1993; Costa, Pires, & Ribeiro, 2006; Freeman, Ryan, Kensler, Dickinson, & Vining, 2013; Simpson, Listak, & Hutto, 2013; Tanco, Costa, & Viles, 2009). A careful management of statistical and non-statistical issues is crucial to successful case studies. For instance, experimental design selection is a critical activity, because using an inappropriate experimental design is sure to compromise study conclusions. To avoid spending time and effort running inappropriate experiments, Tanco et al. (2009) focused on experimental design selection, highlighting various key points and providing guidelines to help practitioners in selecting experimental designs that were validated based on examples from the literature.

To explore the relationship between dependent variable (COP) and four independent variables (or control factors) of the didactic unit that impact on RC efficiency, namely the inlet water temperature in condenser \((T_{\text{cond}})\), inlet water temperature in evaporator \((T_{\text{evap}})\), water mass flow in the evaporator \((m_{\text{evap}})\), and water mass flow in condenser \((m_{\text{cond}})\), a face-centered design (FCD) was selected. This experimental design consists of a two-level full factorial design \((2^4 = 16\text{ experiments})\), eight star points and four center points, which allow to estimate linear and non-linear terms that can be used for modeling the COP variable (response). The four center points are enough to produce the required design variance stability because the region delimited by factors range represents both the region of interest and the region of operability. Supported on authors’ expertise, preliminary experimental results (trial runs), and to simulate as much as possible real-life operating conditions of RC, factor levels (in coded and non-coded values) were set as displayed in Table 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>Coded value</th>
<th>(T_{\text{cond}}) (°C)</th>
<th>(m_{\text{cond}}) (g/s)</th>
<th>(T_{\text{evap}}) (°C)</th>
<th>(m_{\text{evap}}) (g/s)</th>
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</thead>
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<tr>
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<td>20</td>
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data analysis and results

The designed experiments were run in the thermodynamic laboratory of Setubal Polytechnic Institute—ESTSetubal, without any order (randomly), and the response results are displayed in Table 2. This data were analyzed using the software package STATISTICA® and a second-order model fitted to COP based on analysis of variance (ANOVA) results. The estimated regression coefficients are displayed in Table 3, and the model fitted to COP, after sent to the ANOVA error term some non-significant variables/interactions, is as follows:

\[
\hat{\mu} = 0.8767 + 0.2539 x_3 + 0.0791 x_4 + 0.0745 x_1^2 + 0.0776 x_2^2 - 0.1675 x_3^2 - 0.0305 x_1 x_4
\]

where \(x_i\) \((-1 \leq x_i \leq 1\) for \(i = 1, \ldots, 4\)) denotes the coded label of the ith independent variable.

This model shows good descriptive ability \((R^2 = 0.934; \text{Adjusted } R^2 = 0.915; \text{MS Residual} = 0.005)\), and graphical residual analysis presented in Figures 5–7 does not provide evidences of ANOVA assumptions (residuals Normality, Independence, and Homoscedasticity) violation. It includes statistically significant linear and quadratic terms, namely an interaction term, and one can see that \(x_3\) \((T_{\text{evap}})\) is the most important one to maximize COP, and has the biggest coefficient has the biggest coefficient (0.2539). In practice, the greater \(T_{\text{evap}}\) value is, the higher cycle efficiency (COP value) will
Table 2. Matrix of experiments and results

<table>
<thead>
<tr>
<th>Standard order of runs</th>
<th>T&lt;sub&gt;cond&lt;/sub&gt; (°C)</th>
<th>̇m&lt;sub&gt;cond&lt;/sub&gt; (g/s)</th>
<th>T&lt;sub&gt;evap&lt;/sub&gt; (°C)</th>
<th>̇m&lt;sub&gt;evap&lt;/sub&gt; (g/s)</th>
<th>COP</th>
<th>̇W&lt;sub&gt;elect&lt;/sub&gt; (W)</th>
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Figure 5. Normal probability plot.
be, though the quadratic effect of Tevap \((x_3^2)\) has a negative influence on COP value since its coefficient is lower than zero. The interaction term \(x_1 \times x_4 \times (T_{\text{cond}} \times m_{\text{evap}})\) is significant, but its coefficient is smaller, in absolute value, than all the other coefficients of the model fitted to COP. These experimental results are in accordance with the theoretical knowledge, since it is known that evaporator temperature has significant impact on COP (Kilicarslan & Mülle, 2004).

To achieve the highest COP value and respective optimal coded values \(x_i\), Solver optimization tool available in Excel® was used. The achieved COP value, slightly higher than 1 (COP = 1.23) for \(x_i = (-1, 1, 0.76, 1)\), is not surprising, taking into account the installation used here (Bjork, 2012). The performance of current refrigeration systems is, in fact, higher since they integrate components of higher quality (with better technical characteristics). The achieved COP value is low from a theoretical point of view. However, this does not mean that experimental methodology and study results are of no interest or unhelpful. One can’t ignore that small didactic units are not designed or developed with efficiency purposes. They are a valuable teaching aid for students, from craft and technician training at Polytechnics and Universities, and are used to help them in visualizing and understanding the events within the various components.
To validate the COP value obtained from the optimization process (COP = 1.23), two confirmatory experiments with variable settings at optimal values were run. Experimental runs and results are displayed in Table 4, and one can see that COP values are in agreement with those achieved from the optimization of model fitted to COP. Thus, one can argue that experimental methodology illustrated here was helpful to better understand the influence of selected control factors on refrigeration cycle performance.

5. Conclusions

Statistically designed experiments were performed and results analyzed with the objective of maximizing the efficiency of a compression refrigeration cycle, using a small didactic installation. A second-order model was fitted to Coefficient of Performance and considerable benefits result from it. Besides it expresses the functional relationship between design variables and the response, the model provides an estimate of the response at any point within the experimental region, which is useful for refrigeration cycle design and operation improvement purposes. Results show that, except for the inlet water temperature in the condenser, which must be set at low level, the remaining variables must be set at high level or close of it to maximize the cycle performance. Confirmatory experiments corroborated these results.

The design and analysis of experiments to investigate refrigeration cycles is a novel approach in thermodynamics and results give confidence to use this approach for refrigeration cycle design and operation improvement purposes. Therefore, as future research, we plan to apply this methodology in domestic and industrial equipment as well as in other thermodynamic cycles. Simultaneous optimization of models fitted to refrigeration and electric power responses is another alternative optimization strategy to maximize refrigeration cycle efficiency. To test other refrigeration fluids and compressor types can also be considered in future research studies.
Acknowledgments
The authors thank to IPS-ESTSetubal for allowing us to use the didactic refrigeration cycle and all the other necessary instrumentation as well as to the Mr Vitor Pinho for his relevant contribution in the experimental part of this study. We are also grateful to the two reviewers for their contribution to the improvement of this manuscript.

Funding
The authors received no direct funding for this research.

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Citation information
Cite this article as: Applying design of experiments to a compression refrigeration cycle, N. Ricardo Costa & J. Garcia, Cogent Engineering (2015), 2: 992216.

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