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APPLIED PSYCHOLOGY | RESEARCH ARTICLE

Optimal temporal placement of the call in beach volleyball

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Abstract: The call is a tactical component in beach volleyball attacks. Through the call, the setter indicates to his or her teammate an open spot in the opponent's court. In two experimental conditions, we investigated the interval between the call and the ball-hand contact ("call shot interval", CSI) of top-level athletes. We show that the probability that a given call is followed is dependent on the duration of the CSI and the number of call options. Longer CSIs result in an increased probability that the given call will be followed, whilst increasing the call options results in a decrease in probability. On average, there is a 50% probability that the call will be followed if the call precedes the shot by 460 ms and if a single call option ("line") is expected. If the attacker has to choose between three call options ("line", "cut", "no-one") a 50% probability that the call will be followed is observed at an CSI of 542 ms. It did not appear that gender influenced the ability to follow a call. We recommend that in practice and in competition, players and coaches should consider the proper duration of the CSI for effective calling.

Subjects: Sport and Exercise Science; Sports Performance Analysis; Sports Psychology

Keywords: choice reaction time; motor control; movement time; ball games; applied sports science

1. Introduction

In beach volleyball a typical rally consists of a serve by team A, followed by a serve receive, a set and an attack from team B. A typical defensive strategy for team A is to divide their court into two areas;



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Stefan Künzell is a professor for sport science at the Universität Augsburg. His research focuses on motor control and learning. In his applied research he investigates in skiing, climbing and volleyball how athletes can improve their skills. He develops practice programmes for top athletes as well as for beginners. In his basic research programme he investigates basic mechanisms of learning, attention and multitasking. The study reported in this paper was a result from a project that was funded by the Federal Government to help German athletes to win medals at the Olympics.

PUBLIC INTEREST STATEMENT

Every spectator of a beach volleyball tournament will notice that especially in women's teams the setter accompanies every attack with a loud hint for the attacker where to play the ball. This "call" is dependent on the location of the opponent's defender and the blocker. Sometimes, one gets the impression that the call comes too late, so that the attacker cannot follow the call. However, so far no one knows the minimal time interval between the call and the attacker's ball contact that is necessary to be able to follow the call. We conducted an experiment with elite German beach volleyball players. We manipulated the length of that interval and investigated whether or not attackers could follow the call. We found out that beach volleyball players need 542 ms from call to shot to have a 50% chance to follow the call if they have to decide between three call options.

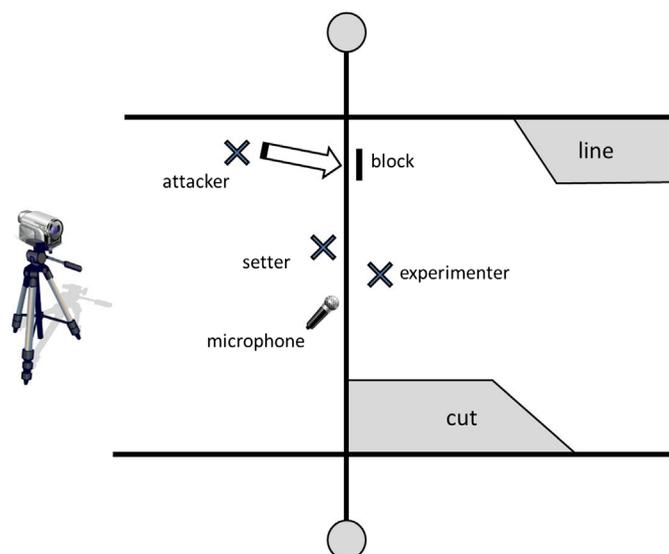
the defensive player covers one area, the blocker the other. The actual line-up to be applied is generally specified by the non-serving player through hand signals. The hand signals are concealed behind the player's back to shield the signal from view by the opposing players. As a rule of thumb a hard-hit ball into the defender's area or an off-speed shot into the blocker's area are the most promising attack options. Koch and Tilp (2009) analysed the distribution of the different attacking strategies. They found that in a world series tournament men clearly prefer hitting (59%) to off-speed shots (41%), whereas female beach volleyball players hit hard (49.5%) and place shots (50.5%) equally. In a recent study, Medeiros, Mesquita, Marcelino, and Palao (2014) analysed the efficacy of spikes and off-speed shots. They showed that younger male players (U 19 and U 21) were more efficient when hitting the ball with full speed, whereas senior players had an equal efficacy for hard-hit balls and off-speed shots.

In order not to reveal their defensive strategies, the defensive player takes up his or her assigned position as late as possible, so that the opposing attacker—who also has to keep an eye on the ball set by his teammate—is probably unable to recognize the defender's position. However, after setting the ball, the setter has the opportunity to analyse the opponent's defence constellation and support the attacker by calling an open spot in the opponent's defence. The typical calls are “line” and “cut”, which denote areas shown in Figure 1. Additionally, a “no-one” call denotes the situation in which the defending team does not block the attack. After any call, the attacker has the option either to neglect the given call and hit the ball down into the defender's area or to follow the given call. In the cases of “line” and “cut” calls, the attacker places an off-speed shot into the respective area, while in case of “no-one”, a hard driven ball along the sideline is the best option. Künzell, Schweikart, Köhn, and Schläppi-Lienhard (2014) analysed the effectiveness of a given call in the Swiss Championships of 2011. They found that attacks, which followed a good call, had a higher success rate compared to hitting with full speed. In male and female beach volleyball, this occurred equally.

However, the temporal placement of a call is eminently important for an efficient attack. In a case where the call is given too early, the defensive player might react to the call and therefore run to the area the opposing player had called. In case the call is given too late, the attacker might have not enough time to change his planned action and react to the call. Hence, the purpose of this investigation is to be able to define the interval between a call and an off-speed shot that is needed in order that a call is followed with a certain probability.

Figure 1. Experimental set-up.

Notes: Players attacked from either the right or the left side of the net, as they usually do in competition. So, the set-up could be arranged mirrored.



We define the call-shot-interval (CSI) as the time between the start of the setter's call and the moment the attacker touches the ball. Therefore, the CSI consists of the reaction time that the player needs to perceive and process the call and the time that the players need to execute the movement to follow the call.

Concerning reaction time there are several well-known laws that allow specific hypotheses. Hick's (1952) law states that choice reaction time (CRT) is dependent on the number of alternatives n according to the formula $CRT = b + a \cdot \log_2(n)$, where a and b are person-specific constants. Hick's law was observed in a variety of areas of application (for a review refer to Hawkins, Brown, Steyvers, & Wagenmakers, 2012). Although, after an applied call the situation is similar to a laboratory CRT-task there is also a difference: in a CRT-task, the participants have to wait until the imperative stimulus is applied. In beach volleyball, the attackers have to react to the ball that has been set, whether or not a call is given. They cannot wait for the call. Thus, it is not exactly the same situation as in CRT tasks, because the attackers cannot be sure of receiving a call. A similar situation is investigated in Go/No-go experiments. A well-known investigation is the "stop before eight" experiment developed by Slater-Hammel (1960). Participants had to lift the index finger at that moment when a rotating handle reaches a mark. However, they should not lift their finger if the handle stops before reaching the mark. Again, this does not exactly meet the situation of an attacker in beach volleyball. This is because he or she not only has to stop the initiated movement plan but also has to change the initiated movement plan to follow the given call. When estimating the reaction time another finding plays a role: Henry and Rogers (1960) discovered that reaction times are prolonged when the action, which has to be executed, becomes more complex. The findings were confirmed in speech production (Klapp, 2003). The high complexity of the volleyball attack leads to the assumption that reaction times are longer than in a laboratory experiment where the only action is to lift a finger.

In sports science, Roth (1989) investigated the time intervals that are needed to correct a movement. He conducted his experiments against the background of the General Motor Program Theory and the Schema Theory of Schmidt (1975). His aim was to confirm the hypothesis that a correction of the motor programme needed more time than a correction of the parameters (e.g. absolute force, direction, absolute timing) within the same motor programme. In one of his experiments indoor volleyball players had to execute a cross-court spike. In some trials a light flashed up indicating that the players had to execute a line spike (a correction of the parameter "direction") or an open hand tip (a correction of the programme). Roth (1989) identified the minimal correction times, which he defined as the time between the flash of the light and the ball-hand-contact, in which the probability of correction is exactly 50%. Mean minimal correction times were 490 ms for a correction of parameters and 745 ms for a correction of the programme. Roth (1989) conducted similar experiments in handball, table tennis and tennis. Overall, he found out that a programme change needed more time than a parameter change. Beyond this, he could find neither an effect of gender nor an effect of expertise. He concluded that movement correction times are independent of the performance level in the type of sport being executed. However, these results might be severely flawed by the fact that none of the players tested by Roth (1989) had ever experienced a movement change demanded by the flash of a light. We suggest that an expertise in changing movement plans might also depend upon the amount of practice in perception of the imperative stimulus and thus to be very specific (Eriksson & Smith, 1991). Hence, the actually required time intervals may significantly differ from the values reported by Roth (1989). Therefore, it seems to be of valuable interest that the required CSI duration based on real calls given by a teammate be reinvestigated.

In the following, the CSI is determined in two different conditions. In condition 1, a single call will be applied, whilst in condition 2, three different call options are used. Therefore, condition 2 is closely related to a situation in a typical competition. We hypothesize that the probability of following a call will increase with a prolongation of the CSI. Moreover, for a given CSI, in the three calls condition the probability should be lower than in the single call condition. In other words for a given probability of following a given call, the optimal CSI is shorter for the single call condition than in the three calls condition. It is an open empirical question if there is a difference between male and female athletes

as well as an expertise effect. However, the focus of research in these experiments is to give coaches and athletes recommendations about the relation between the temporal placement of a call and the resulting probability of it being followed by the attacker.

2. Methods

2.1. Participants

Participants were eight female and eight male beach volleyball players, all being members of the German national squad (mean age 21.3 years, SD 2.58 years). Due to injuries and time problems, only seven men and four women participated in both experimental conditions. The study was performed in accordance with the Declaration of Helsinki. All participants gave their informed consent.

2.2. Procedure

The experiment took place in a standard indoor beach volleyball court. Participants were tested in groups of four, where one was the setter, one the blocker and two were the attackers (see Figure 1). All attackers approached and attacked from their favourite side. The attackers alternated. They tossed a standard beach volleyball to the setter who set the ball, as it is usual in practice situations. The attackers then had to hit the ball into the opposing court. We instructed them to hit cross-court in case no call was applied. Calls were always given in the attacker's familiar code. (Some elite players use individual code names to denote the different call options.) After 10 trials for each attacker, there was a small break to shag the balls and to enable recovery. In the opposing court, a player stood on an elevated platform and blocked the attacks at the net. We tested the players under two different experimental conditions, single call and three calls (see below). Participants played 5 times 10 attacks in single call condition on one day and 5 × 10 attacks in three calls condition on the other day.

The experimenter manipulated the CSI. Because the exact time of the ball-hand contact was unknown, the CSI could be varied only by subjective estimation. The experimenter had been trained in previous pilot studies in manipulating the time intervals.

2.2.1. Experimental condition 1: single call

In condition 1, the attackers had two options. In 50% of the attacks, in a randomly fixed sequence, the experimenter applied a "line" call. In this case, participants should play a long-line off-speed shot over the blocker. We marked the target zone with coloured skipping ropes (see Figure 1). In the other 50%, no call was applied and a cross-court hard driven ball was to be hit. In 10 trials, due to errors in the setting of the ball, we could not evaluate the attack.

2.2.2. Experimental condition 2: three call options

The goal for condition 2 was to detect the CSI in a situation that is more similar to competition. The setting was the same as in condition 1, but with four options. In a randomly fixed sequence, the experimenter applied 11 "line"-calls, 11 "cut"-calls, and 11 "no-one"-calls as well as 17 attacks without a call. Again, the standard attack was hitting cross-court, which was supposed to be executed when no call was given. Participants successfully followed the "line" call if they placed an off-speed shot into the marked zone shown in Figure 1 and for the "cut" call into the cut zone, respectively. According to the coach's directive, in case of the "no-one" call the attacker had to hit into the line zone. The blocker had to block attacks according to the called direction and in case of the "no-one" call, he or she had to leave their hands down. Again, due to errors of the setting and technical errors in recording 34 cases could not be evaluated.

2.3. Measurement

We videotaped all attacks and recorded the calls by a wireless microphone that was interlaced into the net. We measured the CSI by analysing the audio track of the camcorders recordings, which had a sampling rate of 48 kHz. The gap between the first peak caused by the call and the peak caused by the ball-hand-contact was defined as the CSI. The successful following of the call was evaluated by

analysing the video track. A “line” or “cut” call was followed if the ball landed in the respective target area and was played as an off-speed shot, not as a hard-hit ball. A “no-one” call was followed if the ball was hit into the line zone.

2.4. Statistical procedure

The aim of the experiment is to model the probability of the ability to follow a call for a given CSI. We suppose that there is a negligible influence of the setter because all our participants are elite players and are able to pass a tossed ball perfectly. We discarded the very few ill-set passes from our analysis. Additionally, we assume that there is no influence of the side of attack, as it was constantly the favoured side for all attackers. Effects due to learning or fatigue were disregarded in our experiment because when highly experienced players are attacking this is a strongly automatized movement pattern and is practised repeatedly in standard exercises. We computed a standard logistic regression and applied the Wald test to test the significance of the regression parameters. Additionally, we computed the confidence intervals. The methodology is documented in Appendix A. The logistic regressions were executed separately for each player and each condition. We also computed the logistic regression for the pooled data of all players per condition to enhance reliability. We analysed several different interactions between conditions, gender and players. The relevant combinations are discussed in the following section.

3. Results

In condition 1, we analysed 345 attacks with a call and 345 attacks without a call. In condition 2, we analysed 124 “cut” calls, 128 “line” calls, 127 “no-one” calls, and 237 attacks without a call.

Temporal distribution of the CSI, shown in Figure 2, reveals that calls cover an interval between 200 and 1,000 ms. Descriptive statistics are displayed in Table 1. On average, the experimenter called 96 ms later in condition 1 than in condition 2, which results in shorter CSIs within condition 1. This reflects the caller’s task to make calls in a CSI range that is close to the critical CSI where the participant could or could not follow the call.

In condition 1, participants followed a given call 210 times, that is 60.9% of all valid trials where a call was applied. In nine trials, they played an off-speed line shot although no call was given, which is 2.6% of all attacks without a call and represents the rate of speculation. In condition 2, a given call was followed 225 times, that is 59.4% of all valid trials where a call was applied. The rate of followed calls differed only slightly for different call types (“cut” 63.7%; “line” 60.6%; “no-one” 54.3%). In 17 trials, an off-speed shot was played without a call, which is 7.2% of all attacks without a call and again represents the rate of speculation.

Figure 2. Temporal distribution of the CSI of condition 1 (left, grey bars) and condition 2 (right, hatched bars).

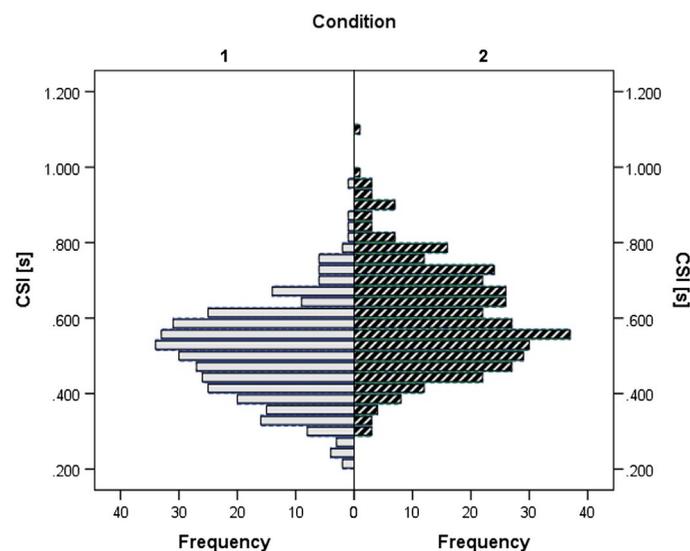


Table 1. Descriptive statistics of the CSI in conditions 1 and 2

	Mean [s]	Median [s]	SD [s]	Min [s]	Max [s]
Condition 1	0.508	0.511	0.122	0.209	0.950
Condition 2	0.602	0.585	0.137	0.292	1.095

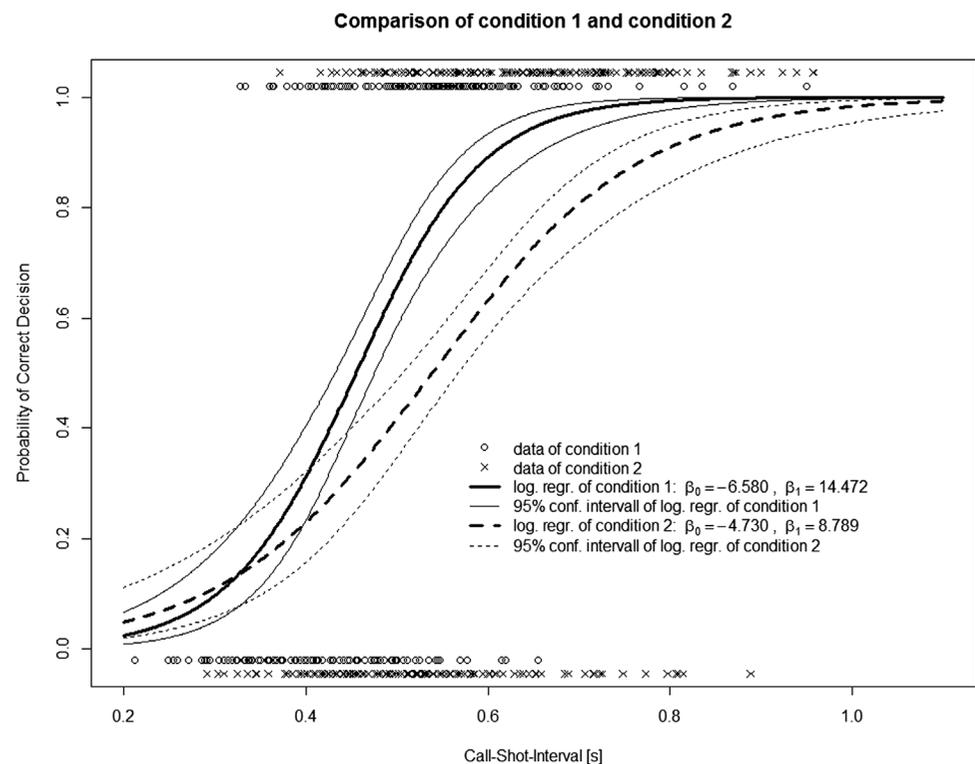
Notes: SD = standard deviation, Min = minimum, Max = maximum.

To compare both conditions we only regarded attacks of athletes who participated in both conditions. The probability of following a call given for both conditions with dependence on the length of the CSI is shown in Figure 3.

The CSIs of the individual athletes that result in a 50% probability to follow a call vary between 0.378 s and 0.556 s for condition 1 and 0.430 s and 0.709 s for condition 2. The mean CSI for condition 1 is 0.475 s and 0.541 s for condition 2, respectively. However, the inspection of the confidence intervals reveals that these individual results are not reliable. This is due to the too small number of attacks per person. However, the number of attacks could not be increased further because of the tight schedule of the top-level athletes and the potentially increasing influence of fatigue. Consequently, we keep our focus on an inter-individual analysis to increase the significance of our results, knowing that these results only present averages. This leads to viable results, since the averages are taken over a relatively homogenous group of German national squad players. The CSI of the pooled data that results in a 50% probability of following a call is 0.460 s for condition 1 and 0.542 s for condition 2. This is close to the means of the individual data reported above. The β_0 -coefficient of condition 1 is lower than in condition 2, which results in a shift of the regression curve of condition 2 to the right towards a longer CSI. This difference is not significant ($p = .10$). The β_1 -coefficient of condition 1 is higher than in condition 2, which results in a steeper slope in condition 1. This difference is highly significant ($p = .01$). Thus, the steepness of the slope can be traced back to the difference in the experimental conditions. Moreover, for values of CSI > 447 ms the ranges of the 95% confidence intervals do not overlap for condition 1 and condition 2.

Figure 3. Probability of following the call over the Call-Shot-Interval (in seconds).

Notes: Solid lines indicate condition 1; dashed lines condition 2; the bold lines represent the probabilities; while the thin lines denote the 95% confidence interval; the dots (condition 1) and the crosses (condition 2) represent single data points, which denote trials where the attacker either followed or did not follow the call.



Comparing the performance of male and female athletes, we analyse all 16 athletes that participated in one or both of the conditions for each condition separately. We included the main effects for the gender and the CSI as well as the interaction. For condition 1, both coefficients, the gender main effect as well as the interaction between the gender and the CSI differ significantly from zero. Since the gender is a binary variable, the gender main effect is an additive shift applied on the β_0 coefficient and the interaction is an additive shift on the β_1 coefficient in a simple logistic regression with only the CSI as an explanatory variable. Thus, we conclude that β_0 is significantly lower for male athletes compared to females ($p = .03$) and shifts the graph to the right, while β_1 is significantly higher for males compared to females ($p = .04$) and leads to a steeper ascent. However, the 95% confidence intervals overlap for all CSIs. In condition 2, we do not observe any significant differences between male and female athletes. To analyse the effect of expertise we performed a median split by the German beach volleyball ranking list. The logistic regression done in the same way as for the gender analysis revealed no significant differences between the upper half and the lower half of the ranking list do not show significant differences.

4. Discussion

The results show clearly that the ability to follow a call is dependent on the length of the CSI. Moreover, comparing condition 1 and condition 2 there is a clear difference concerning the relation between CSI and the probability of a call to be followed. Whilst within short CSIs < 450 ms the probability of following a call is very low in both experimental conditions, differences emerge with longer CSIs. When having to follow a single call (“line”) the probability to follow the given call rises faster than having to follow one out of three possible calls. This is perfectly in line with the theoretical prediction given by Hick (1952) and Hawkins et al. (2012).

Condition 1 is similar to the experiment by Roth (1989) in which he investigated the change of the motor programme. The differences to our experiment are that Roth investigated indoor volleyball players and did not use a call but used a light flashing as the imperative stimulus. Furthermore, participants in his experiment had to play an open hand tip instead of a line roll shot. Roth observed a CSI of 745 ms for a 50% probability to follow a stimulus, which is notably longer than 460 ms that we found. We suggest that the reason for this considerable difference is that we used a familiar stimulus that athletes typically use in practice and in competitions. Moreover, detecting a flashing light might distract attention from the approaching ball more than listening to a call.

What do these findings add to tactics in beach volleyball? Recently, Kredel et al. (2011) investigated the movement initiation time and decision time of elite and near elite beach volleyball players in a defence situation whilst receiving a shot. In a laboratory situation without calls, elite female defensive players initiated their movement on average 120 ms after ball-hand-contact, with a rate of correct reactions of 82%. Near elite female players initiated their movement earlier (30 ms before ball-hand-contact) with a lower rate of correctness (70%). Male athletes initiated the movement later than females (245 ms after ball-hand-contact for elite, 110 ms for near elite) and had a higher rate of correctness (95% elite, 73% near elite). On the contrary, Künzell et al. (2014) analysed the rate of success of attacks in the Swiss national championship. They found that 68.8% of the attacks where the elite female attacker followed the call were kill shots, i.e. a direct rally point for the attacker (elite men: 62.1%). Comparing the high decision correctness and the low success in the competition of the defensive players it can be concluded that they initiate their movement too late. An obvious recommendation for the defensive player could be to listen to the opponent’s call. In a series of interviews the top-level beach volleyball defensive athletes, Schläppi-Lienhard and Hossner (2015) analysed the decision strategies. Only one athlete reported that he would listen to the opponent’s call, no other athlete mentioned this possible cue. Therefore, listening to the opponent’s call could be a major tactical improvement of the defence in beach volleyball. On the contrary, the advice for attackers is not to give standard English calls (“line”, “cut”) but to define code words, eventually in the non-English mother tongue.

In our experiment, we presupposed that the default action after a set is the hard-hit attack. This may not always be the case. There could be situations, in which the attacker already had decided to play an off-speed shot instead of a hard-hit ball, perhaps due to a suboptimal pass. Here, the call could be helpful in indicating the direction of the shot. Changing only the direction instead of the whole movement pattern—the “motor program” in Schmidt’s (1975) terminology—might be possible within shorter CSIs. This situation needs further investigation.

Last but not least a hint for coaches in beach volleyball. In a usual attacking situation, as a rule of thumb, the optimal moment to give a call (i.e. a CSI of 538 ms) is in the attack approach immediately after the hitter is beginning to move his or her centre of mass from the lowest point in an upward direction.

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Appendix A

If a call i ($i = 1, \dots, n$) was successfully followed by an attacker, the dependent variable Y_i was coded 1, else 0. Let csi_i be the explanatory variable. We use the standard logistic regression model.

$$\ln\left(\frac{P(Y_i = 1)}{1 - P(Y_i = 1)}\right) = \beta_0 + \beta_1 \cdot csi_i.$$

Thus, the joint density of (Y_1, \dots, Y_n) is given by

$$\prod_{i=1}^n \left(\frac{\exp(\beta_0 + \beta_1 \cdot cs_i)}{1 + \exp(\beta_0 + \beta_1 \cdot cs_i)} \right)^{y_i} \cdot \left(\frac{1}{1 + \exp(\beta_0 + \beta_1 \cdot cs_i)} \right)^{1-y_i}$$

The estimated parameters $\hat{\beta}_0$ and $\hat{\beta}_1$ maximize the above density, i.e. they are the maximum likelihood estimates. They can be found by the Newton–Raphson algorithm (see Christensen, 1997). Furthermore, for each fitted value f_i , $f_i = \hat{\beta}_0 + \hat{\beta}_1 \cdot cs_i$ we calculate an asymptotic standard error s_i . According to Christensen (1997, p. 367), it holds

$$s_i = \left(X \cdot (X^T \cdot D \cdot X)^{-1} \cdot X^T \right)_{ii}$$

Thereby, X is the model matrix

$$X = \begin{pmatrix} 1 & cs_1 \\ \vdots & \vdots \\ 1 & cs_n \end{pmatrix}$$

and D is a diagonal matrix with the i th entry equal to $F(f_i) \cdot (1 - F(f_i))$, where F is the logistic cumulative distribution. For this calculus, we used the free statistic software R (R Core Team, 2012). The 95% confidence interval depicted in Figure 3 is given by

$$[F(f_i - 1.96 \cdot s_i); F(f_i + 1.96 \cdot s_i)].$$

To test whether the individual parameters are significantly different from zero, the Wald test is applied. With the notation from above, the test statistic given by

$$T := \frac{\hat{\beta}_i}{\hat{\sigma}_i}$$

with

$$\hat{\sigma}_i := (X^T \cdot D \cdot X)^{-1}$$

The null hypothesis $\beta_i = 0$, this test statistic T is standard normal distributed. Hence, the cited p -values within the text are given by

$$2(1 - N(|T|))$$

This is the probability for a standard normal random variable of not being in the interval $(-|T|; |T|)$.



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