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TEACHER EDUCATION & DEVELOPMENT | RESEARCH ARTICLE

The identification of teaching interactions used in one-to-one teaching of number in the early years of schooling

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Abstract: This research paper reports on phase one of an investigation of video recorded intensive one-to-one teaching interactions with 6–7-year-old students who were in their second year of schooling in Australia and identified by their teacher as low attaining in early number. The two-phased study from which this paper emerges was originally conducted in 1998 as part of my Bachelor of Teaching Honours (Research) program at Southern Cross University Lismore, New South Wales. That study identified teaching interactions particularly suited to one-to-one teaching in the *Maths Recovery Program*, a program designed for these students who were at risk of failure in early number. Since that time a great deal has not changed with limited literature available that comprehensively reports on teaching interactions in intensive one-to-one settings. Revisiting the original study is considered timely given the increasing number of withdrawal and intensive programs now funded and adopted by schools and yet, rarely reported on in terms of the effectiveness of the teaching interactions that occur in such settings. This paper then presents a discussion of a preliminary series of teaching interactions that either positively and or negatively influence an intensive one-to-one teaching and learning setting.

ABOUT THE AUTHOR



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Bronwyn Ewing is a senior lecturer in the School of Curriculum at QUT. In her research role, Bronwyn works with industry partners, government and private sectors and community groups in a range of collaborative research and commercial projects with a specific focus on mathematics education, multisensory/modal pedagogy and social theory of learning to enhance the mathematical learning of a range of equity groups, including students who are Indigenous, special needs and ESL. This paper contributes to further understandings of working with such groups to improve their mathematics achievement. It also works to inform undergraduate and postgraduate primary mathematics students that I teach in my academic role.

PUBLIC INTEREST STATEMENT

Low attainment in mathematics has been identified as a strong predictor of later achievement in numeracy and literacy. It is also predictive of the need for intervention through intensive one-to-one mathematics teaching programs in the early years to prevent low levels of academic performance in mathematics in the later years. This research article provides insights into phase one of a two-phased study that focused on teaching interactions in one-to-one intensive mathematics teaching settings. The purpose of the study was to identify teaching interactions used in one-to-one teaching settings with a view to understanding a teacher's practice when interacting intensively with students who are low attainers in mathematics. The article presents a series of teaching interactions identified from the literature and video-observed data to establish an initial series deemed suitable for working with young students in intensive one-to-one settings. It is particularly useful for Principals and teachers who are in positions of leadership and who delegate specialised roles for teachers, such as those intensive one-to-one positions.

Subjects: Childhood; Curriculum Studies; Early Years; Education; Mathematics; Teaching & Learning

Keywords: one-to-one intensive teaching in early number; early childhood mathematics; mathematics intervention; low attainers in early number; scaffolding; teacher education in mathematics; double bind; illusion of competence; post-question

1. Introduction

Education determines a child's economic future and is significant to a child's social and emotional development, their sense of identity and sense of place in the world. Unfortunately, failure in mathematics in the early years has been shown to be a strong predictor of later achievement in numeracy and literacy (Duncan et al., 2007; Desoete & Stock, 2013; Smith, Cobb, Farran, Cordray, & Munter, 2013). It is also predictive of the need for intervention through intensive one-to-one early number teaching programs in the early years to prevent low levels of academic performance in number in the later years.

As far as can be ascertained, little if any empirical evidence exists which allows me to determine which teaching interactions used in intensive one-to-one teaching settings connected with positive student achievement. Currently, there is minimal literature available which specifically focuses on the interactions of one-to-one teaching in a mathematics setting (Supekar et al., 2013; Wright, Martland, & Stafford, 2006). Other studies demonstrate that small group tutoring and peer tutoring impacts on student mathematics achievement but do not focus specifically on the interactions of teaching to explain this impact (McCourt, 2006; Namkung & Fuchs, 2012; Powell, Fuchs, & Fuchs, 2013). In light of these findings, understanding the interactions of one-to-one teaching might serve as a guide to understanding a teacher's role when interacting with students who are low attainers in number and which enable positive student achievement.

Low attainment refers to students who are 6–7 years of age and in their second year of schooling. In the Maths Recovery Program, students who are at risk of failure in early number learning and identified by their teacher and the Maths Recovery Teacher using an interview-based assessment *Schedule for Early Number Assessment (SENA)* (Wright et al., 2006) are placed in individualised teaching cycles of length 12 to 15 weeks (Ewing, 2005). The purpose of the cycles is to advance children's early number learning to a level at which they are likely to learn successfully in a regular classroom.

The low attainment of 6–7-year-olds in number provided the purpose for the study: to identify teaching interactions used in one-to-one teaching settings with a view to understanding a teacher's practice when interacting intensively with students. The following questions guided the process:

- (1) What research understandings of intensive one-to-one teaching provide the basis for the development of theoretical ideas? and
- (2) How do these ideas compare with the empirical data observed from video recorded intensive one-to-one teaching sessions?

In what follows, a brief discussion of the research literature is provided to identify one-to-one teaching interactions which are then followed by a discussion of the methodology used to investigate the research questions. Following this, a pragmatic approach is adopted, to puzzle out and problem solve to identify interactions observed in empirical data and the dialectics between data and generalisations. To do this necessitates a brief summary of the Maths Recovery program.

1.1. Maths Recovery Program

The Mathematics Recovery (MR) Program is an early intervention programme for students who are 6 to 7 years of age and in their second year of schooling. Developed by Robert Wright over a three-year period (1992–1995) (Wright, 1989, 1991a, 1991b, 1993) in Australia and more recently with Jim

Martland and Ann Stafford (Wright, Martland, & Stafford, 2006), the program is used in classrooms in the United Kingdom, Ireland and the USA. It was used as the foundation for the Count Me in Too Programme in New South Wales, Australia (Gould, 2001). The MR programme was funded by the Australian Research Council (1992–1995) (AM9180064). The programme draws extensively on the constructivist teaching experiment work of Steffe (1990, 1991), von Glasersfeld (1990, 1991) and Cobb, Wood, and Yackel (1991). This work was designed to investigate children’s mathematical knowledge and learning in instructional classrooms and individualised settings. The teaching experiments of Steffe and others involved selecting students who were considered to be low attainers in their class and withdrawing them several times per week for individual teaching sessions over extended periods.

2. Literature review

Drawing on the research literature, this review summarises important theoretical ideas identified as underpinning mathematics teaching in the early years of schooling. These ideas include: (1) constructivism: social constructivism, (2) sociocultural theory. Each one will now be addressed.

2.1. Constructivism

The understanding that students can construct their mathematical knowledge has been at the centre of debates for over thirty years (cf. Cobb & McClain, 2001; Kyriacou & Goulding, 2006; Kyriacou & Issitt, 2007; Roth, 2014). This is so because two cognate approaches, constructivism and social constructivism began to influence mathematics education over that time. Constructivism has its origins in cognitive theory and the work of Piaget (1936) whilst social constructivism is influenced by the work of Vygotsky (1930, 1934). Both are claimed to have significantly influenced the way mathematics has been taught and learned in classrooms, and continues to do so (Cobb & Yackel, 1998; Cobb & Steffe, 2011; Ernest, 1996; von Glasersfeld, 1995; Waschescio, 1998).

There are two major premises to constructivism. First, the child’s knowledge, knowledge, attitudes and experiences brought to a learning context provide the starting point for learning. Second, as the interplay of that knowledge, and the attitudes and experiences, students begin to construct new knowledge based on previous learning. This knowledge is grounded in and develops further from previous experiences. Thus, when something is said to make sense and or is meaningful, it is this association of knowledge that is addressed.

In the 1990s, some researchers (see e.g. von Glasersfeld, 1991, 1995) explored the application of Piaget’s (1977) theory of assimilation and accommodation to the mathematics classrooms to further understand how students construct their mathematical understanding. What was found was that learners constantly strove for equilibrium, that is, the cognitive stability that occurs through the process of assimilation and accommodation. Assimilation occurs when new information meshes with existing understanding. Through this meshing, a learner is said to be accommodating this new information to fit with their cognitive schemata and schemes. Equilibrium occurs because of these two processes and is crucial to a student’s cognitive development. Knowledge then is not a commodity that rests outside the knower, where it is simply passed from the teacher to the child, but rather it is an individual’s constructive activity.

2.1.1. Social constructivism

The early work of Cobb et al. (1991) with American teachers in the 1990s was instrumental in supporting them in renegotiating classroom social norms so that they and their students together constituted a community of active and engaged learners—a forecast perhaps of the rise of social constructivism. This meant classroom learning involved small-group collaborative activities and whole-class discussions of students’ interpretations and solutions (Cobb & Yackel, 1998). They found that the interplay between students’ thinking and mathematical concepts was increasingly important and therefore required the teacher to make instructional decisions and changes to their teaching practices in order to accommodate this interplay (Cobb & Yackel, 1998; Fennema, Sowder, & Carpenter, 1999).

Social constructivism builds on the constructivist position and rests on the premise that what children can do with assistance is more indicative of their cognitive development than what they can do alone (Brown, Metz, & Campione, 1996; Marti, 1996). Moreover, the focus is on the interplay between language and thought (Sierpinska, 1998) and cognitive development and culture (Lave & Wenger, 1991; Saxe, 1991). Researchers who claim that priority should be given to social and cultural processes (Engestrom, 1996; Forman & McPhail, 1993; Levine, 1996; Minick, 1996; Voigt, 1994) draw mainly from Vygotsky's (1930) contention that social interaction and culture are constitutive of an individual's cognitive development.

2.1.2. *Socio cultural theory*

According to Vygotsky's sociocultural theory (Renshaw, 1992, p. 5), learning is a communal activity. That is: Culture is not an overlay on a basic substrata of individual development, but is a constitutive element of individual development. That is, learning for any individual is a process of appropriating "tools for thinking" that are made available by social agents who initially act as interpreters and guides in the individual's cultural apprenticeship.

Using qualitative methodologies such as ethnographic research, researchers applied Vygotsky's sociocultural theory to investigate the significance of culture and social interaction with students in mathematics classrooms (Engestrom, 1996; Forman & McPhail, 1993; Levine, 1996; Minick, 1996; Voigt, 1994). Minick (1996), e.g. suggests that there is much to learn from exploring the connections between social practice and cognition through the face-to-face encounters of teachers and pupils in the classroom. One way of doing this is to explore the influences of curriculum and teaching materials on teachers and learners. Similarly, Voigt (1994) found that negotiation of meanings is a necessary condition for mathematics learning. He pointed out that this was the case when "students' understandings differed from the understanding the teacher wants the students to gain" (p. 215). Such differences are seen to be crucial to negotiations of meanings in the classroom. Hence communication between students and teacher and individual expertise should be supported in classrooms cultures, including one-to-one teaching contexts.

To summarise, the theoretical ideas addressed in this section highlight several important points. First, children bring to a learning context their knowledge, attitudes and experiences which provide the entry point into building on from what they bring to a context. Second, it was proposed that children are capable of constructing mathematical ideas with the support from a teacher who is able to guide and build on a student's cognitive constructions. Here, the interplay between the student's mathematical development and a teacher's practice has a critical role because both are active participants in the teaching and learning context. The task of the teacher is to alter or change their practice to accommodate a child's active learning and to negotiate mathematical meaning as part of this process. The next section provides the methodological approach used in the study.

3. Methodology

In this study, three methodological principles were used. They formed the basis of Vygotsky's (1930) approach to the analysis of higher psychological functions. The first involved studying the processes, i.e. "process analysis as opposed to object analysis" (p. 65). The second was explanation vs. description, i.e. "analysis that reveals real, causal or dynamic relations as opposed to enumeration of a process's outer features, that is, explanatory, not descriptive" (p. 65). And the third was "developmental analysis that returns to the source and reconstructs all the points in the development of a given structure" (p. 65).

This process of abductive reasoning (Vygotsky, 1930; Walton, 2004) allowed for the development of an initial series of teaching interactions from video observations, progressing to more likely possible explanations which provided the basis for constructing the case studies in phase two. These interactions were compared and contrasted with the research literature until a fit was obtained and possible explanations of the observations could be made. Tasks and settings observed during the study are described as they arise.

3.1. Research context

The current study focuses on the teaching interactions of one-to-one teaching sessions. The study was conducted in New South Wales, Australia. The data for the study consisted of videotaped recordings of one-to-one Maths Recovery teaching sessions conducted by four teachers. The video-recorded sessions were part of a larger trial study of the implementation of Maths Recovery into primary schools (Wright, 1989; 1991a, 1991b, 1993). Analysis of these recordings focussed on teaching and specifically, on the interactions between teachers and students. Phase One of the study was undertaken in order to develop a series of teaching Interactions and involved a total of 18 h of recordings.

3.1.1. Ethics

As the study focused on a small aspect of the much larger trial study in the 1990s and discussed previously, it was encapsulated in the ethics for that study. Permission to use the data was granted by the project Chief Investigator Robert Wright in 1995. All observable data was de-identified by the project team prior to the commencement of this study.

3.1.2. Sampling: Selection of video recordings of teachers

Purposeful sampling was suited to the investigation because it provided representative samples of teaching interactions from four teachers in a range of intensive teaching settings which could then be compared with the research literature (Patton, 1990; Silverman, 2007). This sampling allowed for “information-rich cases” that could be study in depth to provide opportunities for learning about the interactions in intensive one-to-one teaching settings (Patton, 1990, p. 169).

3.2. Puzzling out and problem solving: Steps to analysis

In this study, I adopted a method of analysis that was informed by Cobb and Whitenack’s (1996) approach to longitudinal analysis of data on teaching in the form of video recordings and transcripts. In that work, they emphasised “that the development of theoretical constructs should occur simultaneously with data collection and analysis” (p. 224). They identified that this approach was found to be consistent with Glaser and Strauss (1967) constant comparative method. That is, theory development occurs simultaneously with data collection and analysis.

Drawing on Cobb and Whitenack’s (1996) approach, I used three critical steps to the analysis to develop theory about teaching interactions in intensive one-to-one settings. Step one involved the analyses of videotaped records of the teaching sessions. The researcher viewed the videotapes of each teaching session and wrote notes of three kinds—descriptions, explanations and protocols. The descriptions focused on the teaching setting and interactions. The protocols illustrated the interactions and the explanations provide an elaboration of the interactions identified in the protocol. Each of the descriptions, explanations and protocols was coded by teacher, interaction, child, date and setting.

4. Interactions in one-to-one teaching: Analysis

Viewing one-to-one teaching in early number as interactive was central to this study. In what follows, discussion and analysis of eight interactions are provided and include scaffolding, double bind, illusion of competence, questions, post-question wait-time and questioning and prompting. A necessary feature of the interactions is that they can be applicable across settings and tasks. The settings and tasks are described as they arise.

4.1. Scaffolding

Bruner’s (1996) notion of scaffolding referred to the gradual release of teacher control and support as a consequence of children’s increasing mastery. Effective scaffolders focus children’s attention on the task and keep them motivated and working throughout the session (Wood, 1990, p. 140). Wood (1990) divides the learning task into accessible components and directs the child’s attention to the essential and relevant features. According to Díaz, Neal, and Amaya-Williams (1990) assert that “the teacher who scaffolds, demonstrates and models successful performance while keeping the task at

a proper level of difficulty, avoiding unnecessary frustration and encouraging children's independent learning" (p. 40).

Clay and Cazden (1990) assert that within the zone of proximal development the child is not a passive recipient of the adult's teaching, nor is the adult simply a model of expert, successful behaviour. Rather the adult and child engage in joint problem-solving activities, where both share knowledge and responsibility for the task (p. 218). The Maths Recovery teacher performs the crucial function of scaffolding the task to make it possible for the child to reflect on the strategies and thinking involved and gain confidence in their own solutions, reducing the need to continually refer to the teacher for approval.

Scaffolding can be seen in the following protocol. The child placed the numeral cards from "12" to "16" in order on the table. The teacher asked the child to identify the numerals on the table.

- T: Right, count these ones for me.
C: (Uses finger to point at numeral cards) twelve, eleven.
T: Twelve. What comes after twelve?
C: (Pauses for four seconds) eleven.
T: Twelve. You count up to twenty for me.
C: One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, sixteen, sixteen.
T: Start again and I will point to the numbers when you get to them.
C: One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve (stops after "12") twelve.
T: (Pauses for seven seconds points to numeral cards "12" and "13") ten, eleven, twelve (indicates with an upward inflection).
C: Thirteen, fourteen, fifteen, sixteen.
T: Let's try that again. We'll start with number ten, eleven (points to numeral card "12").
C: Twelve (pauses for six seconds) twelve, eleven.
T: Ten.
C: (Immediately responds) ten.
T: Eleven, twelve.
C: (Counts with teacher) eleven, twelve, thirteen, fourteen, fifteen, sixteen (as teacher points to numeral cards).
T: Right (with an upward inflection in the voice to indicate the child's success).

To the observer, it appeared that the teacher broke the task into simple components. The teacher paused and allowed time for the child to think about the task and instructionally supported him when necessary.

The teacher was successful with keeping the child focused and working through the task. The above protocol seems consistent with Bruner's (1985) description of scaffolding:

If the child, is enabled to advance by being under the tutelage of an adult or a more competent peer, then the tutor or the aiding peer serves the learner as a vicarious form of consciousness until such a time as the learner is able to master his own action through his own consciousness and control. When a child achieves that conscious control over a new function, it is then that he is able to use it as a tool (p. 24).

Determining the optimal amount of scaffolding requires a high degree of craftsmanship and regulation on the part of the teacher. Scaffolds are adjustable and temporarily used to help extend the range of work and accomplish tasks not otherwise possible.

In summary, the Maths Recovery teacher is constantly adjusting the amount of scaffold, to take account of new learnings of the child and anticipating and supporting the child's next step. In Wright et al. (2006) underlying principles of Maths Recovery teaching, the teacher is involved in ongoing observation and continual micro-adjusting of teaching based on his or her observations. Micro-adjusting is informed by the student's performance on a task. When a student is not succeeding, the teacher adjusts the task to one closely related to the original task. A child's programme must be adjusted from lesson to lesson and from moment to moment within lessons. That support keeps a 17 child at the cutting edge of his or her competencies, in his or her continually changing zone of proximal development. In contrast to scaffolding is a double bind.

4.2. Double bind

According to Mellin-Olsen (1991, p. 52) "scaffolding learning can be considered as complementary to double binding". Mellin-Olsen states that scaffolding promotes independence and supports the pupil's responses to his or her learning. Contrary to scaffolding, a double bind produces dependence and "promotes mock cooperation and purposeless reproduction of knowledge" (p. 52). Mellin-Olsen (1991) states that Bateson (1972) introduced the notion of a double bind in the context of mother-child relations in various cultures. He used the term "double bind" to refer to a double communication from mother to child in which the two modes of communication contradict each other. According to Mellin-Olsen (1991) a double bind structure "can be considered as an explanation of the socialisation of the individual to her society" (p. 40). He states that as a tool for socialisation, the double bind is hidden in messages to the student. For example, "you should take control, but not control too much; you can do it your own way as long as you are not breaking the rules" (p. 40). Mellin-Olsen outlines Bateson's (1972) explanation of important features of the double bind. Four of these interactions seen as particularly relevant to Maths Recovery teaching include,

- (1) A double bind is a state within the individual as a result of messages sent to her in a communication system.
- (2) The messages communicated to the individual contain a contradiction.
- (3) The individual has no opportunity to escape the communication system by, for instance, turning her back on it.
- (4) The communication situation is repeated.

The following protocol provides an example of how a double bind can occur in a Maths Recovery teaching session. The teacher placed a hundreds chart on the table then placed a plastic counter to cover each of the numerals "35", "38", "39", "40" and "46".

T: (Points to the counter covering the numeral "46") right can you count this for me?

C: Forty-one.

T: (Taps with her finger on the numeral "46" to indicate the child has not counted correctly) forty-six (with an upward inflection to indicate that she wants the child to continue while pointing at "45" which is uncovered).

C: (Commences before teacher finishes speaking) forty-seven.

T: Counting backwards, forty-five (taps several times in quick succession as before to indicate an error).

C: (Speaks at the same time as the teacher speaks) forty-five, forty-four, forty-three, forty-two, forty-one, fifty (in coordination with the teacher pointing to the numerals "45" to "40").

T: (Teacher continues to point to the numeral "40").

- C: (Slaps hands on table and sighs) nine.
- T: (Points to each numeral in turn) forty-six, forty-five, forty-four, forty-three, forty-two, forty-one (with an upward inflection as before while pointing to the numeral “40”).
- C: (After thinking for four seconds) fourteen (indistinct) fifteen (rests head on hands which are flat on the table).
- T: Forty. You’re guessing, think!
- C: Ninety-two. Has it got a nine in it?
- T: Certainly has, forty-five, forty-four, forty-three, forty-two, forty-one (points to numeral “40”).
- C: Fif.
- T: (At the same time that the child says ‘fif) forty.
- C: Forty.
- T: (Points to the numeral “39”) something nine. What is it?
- C: Thirty.
- T: Thirty-nine.
- C: (In coordination with the teacher pointing to the numerals from “38” to “35”) thirty-eight, thirty-seven, thirty-six, thirty-five.
- T: Good, okay (to indicate that the task has been completed).

In this protocol, the teacher asked the student to count “46” with the intention of assisting the student to count backwards from that number, however, the child did not appear to understand this intention, instead, counting “47”. There could be several explanations for why the child did not count “46”. First, the child’s difficulty could have been with how the teacher’s intention was communicated. The child was not clear about the teacher’s intentions which resulted in the child “41” and then later “47”. Second, there may have been a mismatch between the level the child was assessed at on the SENA interview assessment and the level of the task the child was expected to work through. Whilst the task of the teacher is to design instructional tasks which are just beyond the cutting edge of the child’s current number knowledge, the evidence in this protocol indicates that the child is struggling to achieve with this task and thus trying to work well beyond her/his cutting edge (Wright et al., 2006). Finally, the child’s responses and gestures indicate she/he is experiencing difficulties but cannot escape from the situation, instead is required to repeat the process counting different numbers in “mock” cooperation with the teacher.

In the next protocol, the teacher changed the instructional setting from the hundreds chart to a row task, that is, a row of dots on a strip of cardboard. Her intention was to assist the child to learn the numeral “39”. The child experienced difficulty counting backwards fluently from “46”.

- T: This number is forty-two (places plastic counter on dot on row task). What is this number? (Places counter on five dots before first counter).
- C: Forty-three.
- T: Forty-two (taps with finger on the counter to indicate the child has not counted correctly).C: Forty-three, forty-two (in a questioning tone).
- T: Forty-two (points with finger on the dot to indicate the child has not counted correctly).
- C: Forty-one, forty, forty-nine.
- T: (Waits for thirteen seconds then with an upward inflection) we just counted that, forty-two, forty-one, forty.
- C: Fifteen (pauses).
- T: No! We just counted that. What did we count?
- C: Twenty (speaks in questioning tone).

- T: (Points to counter) thirty-nine.
- C: Thirty-eight, thirty-seven (with teacher tapping with finger and pointing to dots).
- T: Right, this number here is forty-five. What's this number here? (Taps and points to dot with plastic counter on it) forty-five.
- C: Six that is.
- T: What's this one? Forty-five (taps with finger on dot).
- C: Forty-four , forty-two.
- T: Forty-three (points to dot and taps with finger).
- C: Forty-three, forty-two, forty-one, forty (pauses and softly says) twenty-nine. Is that the same number as before?
- T: Forty-five, forty-four .
- C: Thirty-nine? (Says this as teacher is counting) fifteen.
- T: Forty-three, forty-two, forty-one, forty.
- C: Fifteen.
- T: No. You just said it! You're counting backwards from forty-five, you get back to forty it's...
- C: Fifteen.
- T: Thirty-nine (points to dot which represents "39") thirty-eight, thirty-seven.
- C: Thirty-eight, thirty-seven.
- T: We've just done that three times. You have to concentrate ...so that you can learn how to do it.

In this task, counting down from "46" to "39" was repeated causing more confusion for the child. In this situation, the child was unable to escape the communication between himself and the teacher. Instead, the child seemed to be dependent on the teacher to give him the right response. In this instructional setting, it appeared that the teacher created a double bind by repeating the learning task when the child had difficulty with counting backwards and did not understand what to do. This is contrary to Wright et al. (2006) first underlying principle of Maths Recovery teaching, viz., teachers exercise their professional judgement in selecting from a bank of instructional settings and tasks, and varying this selection on the basis of on-going observations. Further, it is in contrast to the idea that the teacher is tasked with making instructional decisions and changes to their practice to accommodate the interplay between thought and language and meaning construction (Cobb & Yackel, 1998). In the next section, an illusion of competence is presented.

4.3. Illusion of competence

According to Yackel (1990, p. 155), "when a teacher and a student develop a solution together, with the student following teacher directives, an illusion of understanding can be created". This, according to Yackel, Gregg (1992) termed an illusion of competence. Yackel, in viewing the illusion of competence, states that by studying the situation it is possible to determine whether or not one of the participants is attempting to communicate aspects of their mathematical thinking that they consider is not readily apparent to others. As the Maths Recovery teacher supports the student with finding solutions to arithmetical problems the teacher must be aware of their own attempts with helping students develop viable solutions. Yackel explains that the teacher could unintentionally inhibit a student's attempts to solving a problem. The teacher relieves the student of the responsibility of finding a solution to a problem or prevents the student from finding which aspects of the problem need further clarification.

In the following protocol an illusion of competence is identified when the teacher relieved the student of her obligation to find the numeral "6". The teacher had numeral cards, each with a numeral in the range of "11" to "20" written on them. She placed the cards in a pile on the table and selected the numeral card "16".

T: OK could you make me number sixteen (places numeral card “16” for the child to see clearly).

C: (Looks in tray which has plastic digits in it. Touches plastic digits “1” and “2” which are together on the table to show twelve).

T: That’s number twelve, we’ll put the two away (pushes plastic digit “2” to the side). See if we can make, we’ll leave the one because you’ve got a lovely pattern over there.

C: (Reaches for plastic digit from pattern).

T: Just find a number out of here (picks up plastic digit “6” from the tray and places it on the table).

C: (Picks up plastic digit “6” and places it beside the plastic digit “1” to show “16”).

T: Good.

The teacher clearly solved this problem for the child by identifying the plastic numerals needed. By saying “good”, the teacher indicated successful solution on the part of the child when in fact the child did not solve the problem at all.

The following protocol is another example of an illusion of competence. The teacher set up the problem and then solved it for the child. The child was asked to identify the numeral card “12” and then “13”.

T: Do you know which one is twelve there? Maybe if I spread them out that might be a bit easier (numeral cards from “11” to “20” are spread on the table). That one is twelve (points to numeral card “12” and places the numeral cards “13” and “14” close to one another). What have we got next? Twelve, eleven, twelve (prolongs pronunciation).

C: Fourteen (picks up numeral card for “14”).

T: Thirteen, thirteen yes. See how this one’s got one, two, three? (Points to row of numeral cards “1” to “10” placed on table). Well, this one has got to have one, two, three (pointing to “11” “12”) with a three in it (slides numeral card for “13” beside numeral card for “12”). There we go thirteen, thirteen. What’s the next one? Thirteen.

To summarise, both these examples reveal how the teacher relieved the student of her obligation to solve the problem and that finding a solution became the teacher’s responsibility not the student’s. The student was not given enough time to reflect and think about a personally meaningful solution to the problem. Instead, the teacher intervened and solved the problem for the child. This is at odds with one of the underlying principles of Maths Recovery teaching, viz., the teacher should provide the child with sufficient time to solve a given problem. When this principle is observed the student is likely to be engaged in episodes which involve sustained thinking, reflection on her thinking and reflecting on the results of her thinking. It is also at odds with von Glasersfeld’s work (1990, 1991, 1995) which argues that putting children on centre stage where they are supported by their teacher, allows students to become active participants, not the other way around where the teacher becomes the active participant and the child the passive learner. In the next section post-question wait-time is discussed.

4.4. Post-question wait-time

Post-question wait-time is the length of time that a student has to respond to a question (Brophy & Good, 2009). “The length of pause following questions should vary directly with their difficulty level” (p. 362). In Reading Recovery teaching, Clay and Cazden (1990) assert that students, when they read texts at an instructional level, “use a set of mental operations, strategies in their heads that are just adequate for the more difficult bits of the text” (p. 207). During this process, the student engages in deliberate attempts to solve new problems with familiar information. Depending on the difficulty of the problem, wait-time would be crucial for a student in Reading Recovery as they build up understanding of their learning.

In the following protocol, the child was asked to identify the numeral card “13” which was placed on the table. To the observer, it appeared that the teacher could have given the student more time to understand the problem that was to be solved.

T: One more before we go. What’s that number? (Places card with “13” written on it on table waits five seconds).

C: Ohh

T: (Slides plastic digits, “1” and “3” and places them above the card numbered “13”). It’s a hard one (in a soft voice). We make it here like this (waits for two seconds). Is it threeteen or is it thirteen? (Points to plastic digit “3” and “1” and waits five seconds). Thirteen (points to “3” then “1”).

The child may have benefited from having more time to respond to this question. When the teacher intervened, the child appeared to be thinking about the question. If she had more time she may have succeeded in giving an appropriate response. Instead, the teacher solved the problem for her.

In the next protocol a child is given enough time to solve the problem with minimal talk from the teacher. The child was asked to count backwards from “10” and then to arrange in descending order, numeral cards from “1” to “10”.

T: Count backwards for me this time. Start at number ten. Count backwards to number one.

C: (Immediately) ten, nine, eight, seven, six, five, four, three, two, one.

T: Can you put those numbers now starting at number ten and going backwards? (Randomly places numeral cards on table). We’ll start at number one up here (points to left hand side of table).

C: (Places cards in order from “1” to “10”. The child takes forty-five seconds to arrange cards during which time the teacher observes and does not speak).

In summary, in a one-to-one teaching setting, a student gradually builds a repertoire of strategies for solving arithmetical problems. Depending on the level of difficulty of the problem a student needs time to think about the problem and to reflect on their thinking. In accordance with Brophy and Good’s (2009) explanation of post-question wait-time is Vygotsky’s sociocultural theory and the underlying principles of Maths Recovery teaching discussed previously. A natural concomitant of post-question wait-time is questioning and prompting.

4.5. Questioning and prompting

According to Lyons, Pinnell, and DeFord (1993, p. 157) questioning and prompting takes much practice and experience. They state that Reading Recovery teachers develop the skill to observe closely what the student is doing, decide what kind of information the student needs to attend to, and then select the prompt or question that will help the student become a more independent problem solver. Lyons et al. assert that a Reading Recovery teacher “learns when, why, how, and under what conditions questions can and should be asked and how to tailor questions to fit the demands of the text and specific student needs” (p. 159).

The following protocol provides an example of questioning and prompting by a Maths Recovery teacher. The teacher arranged counters by twos on the table and asked the child to count by twos to “20”.

T: Let’s see if you can count by twos for me again today.

C: (immediately) two, four, six, eight, ten (pauses after “10”) twelve, fourteen, sixteen, eighteen.

T: (Sequentially places out pairs of blue counters to make a 5×2 array) and two more?

C: Twenty.

T: That's right. You really had to think about those last ones didn't you? What were you doing while you were thinking so hard? (Arranges red counters as before to make a second 5×2 array).

C: Um, I don't know.

T: What were you doing in your mind to help you get the answer?

C: (immediately) I was counting by twos and I went all the way back to two then I counted on from two.

T: Did you? (Touches two counters and moves them slightly to indicate counting by "2"). Let's do it a little bit quicker with our counters.

C: (immediately) two.

T: (Touches counters as before to indicate counting by twos) two.

C: Four, six, eight, ten (begins to count slowly from "12") twelve, fourteen, sixteen, eighteen, twenty.

T: Good boy. Why is it faster like that?

C: Because you can see two, you can see a two pattern and then you can see sev ... a four pattern, then you can see eight, and then you can see all the other patterns.

T: So when you are doing it, you can see some patterns can you? (Places a card over "18" counters and leaves "2" counters exposed). What can you see there?

C: Two.

T: (Moves card along counters as child begins counting by twos).

C: Four, six, ... twelve (begins to count slowly) fourteen, sixteen, pauses) eighteen, twenty.

T: How come you were so quick on that one? (indicates fourteen with card).

C: Because I saw a four pattern there (points to four blue counters).

T: So you just went four (points to four blue counters) teen.

C: (At the same time as the teacher) teen.

T: (Moves card as before to indicate counting by twos).

C: Sixteen, eighteen, twenty.

As can be seen in the above protocol, the teacher made the necessary micro-adjustments to her questioning and prompting based on the observed behaviours of the student. It appears that she was triggering the experiences that the student may have had with counting by twos.

In the following protocol, the teacher provided scaffolding for the child as she attempted to solve the addition problem "42" plus "23". As can be seen in the protocol the teacher supported the child always "stretching" and questioning her thinking.

T: (Places plastic numerals "42" and then "23" beneath).

C: Forty-two plus twenty-three.

T: How would you do that?

C: (Fiddles with numerals and thinks for sixteen seconds).

T: Think of what you did with your tens and ones bundles. How did you work it out? (Pauses for ten seconds). What did we do with the tens and ones bundles?

C: We added.

T: Which one were the ones? Which were the ones row?

C: (Points to the ones row).

T: Okay, three ones?

C: Make three.

- T: And what are we going to add that up to? Three.
- C: (Thinks for fifteen seconds).
- T: Not sure? What if we made...
- C: (Immediately) Five!
- T: What are you adding together?
- C: Six.
- T: These are the tens aren't they? (Points to numerals in "10s" column) two tens and four tens make six tens (places plastic numeral "6" below "4" and "2"). What else?
- C: Five, sixty-five!
- T: (Places plastic numeral "5" below "2" and "3").

In summary, Vygotsky's (1934) theory of a zone of proximal development discussed previously can be useful to understanding the importance of questioning and prompting. A teacher of Reading Recovery or Maths Recovery becomes more aware of a student's learning and previous experience and micro-adjusts their teaching according to this. There are similarities between the questioning and prompts of the Reading Recovery teacher and those of the Maths Recovery teacher. In each case, teachers become sensitive to a student's learning and make crucial decisions based on their observations of students.

5. Discussion and conclusion

This paper has presented phase one of an investigation that focused on teaching interactions in intensive one-to-one settings in the Maths Recovery Program. In doing so, it presented a preliminary series of interactions identified as either positively or negatively influencing intensive teaching and learning settings. There were commonalities among the teaching interactions seen to be positive, that is, scaffolding, post-question wait-time, questioning and prompting, the overarching intention of the Maths Recover Program (Wright et al., 2006) and sociocultural theory (Vygotsky, 1934). Of particular significance was the communication between the teacher and student and the manner in which the teacher worked to keep the child at the cutting edge of her/his learning—with the lowest threshold at which instruction can begin identified from the SENA assessment, the upper threshold, the cutting edge must also be determined (Vygo is to advance children's tsky, 1934, p. 104). Once the thresholds have been identified the role of scaffolding comes to the fore. Evidenced in the excerpts was the teacher working through a process of gradual release of teacher control and support as the student's mastery increased. Critical to scaffolded interaction is the wait time after questioning and the prompting that occurs through the skilful observations of the teacher. These important aspects were identified in the excerpts provided. Knowing when to ask and tailor the questions to fit the demands of the task and the student's specific needs takes practice and experience. When this practice and experience is less evident, dependence and mock cooperation is likely to be the outcome. In the excerpt referring to a double bind, the student purposelessly repeated what the teacher was stating. Consequently, the interaction became a double bind with no escape for the child and with the teacher repeating the same task (Mellin-Olsen, 1991). Coupled with this outcome is the likelihood of an illusion of competence (Yackel, 1990). That is, as the student demonstrated an increasing dependency on the teacher and repeated what the teacher was counting, the teacher could be unintentionally inhibiting the student's attempts at finding a solution or seeking clarification of aspects of what had been asked. The dilemma here is that the teacher may be reinforcing to the student what she/he may already know about her/himself, that is, a failure in mathematics; this reinforcement is in contradiction to the core principles of the Maths Recovery Program which is to advance children's early number learning to a level where they are likely to succeed in a regular classroom.

In a new era where so much more is known about how teachers teach mathematics and how students learn, this paper can argue based on observations in phase one that a teacher who consistently exhibits teaching interactions which enhance learning will be a more effective teacher. Their observational skills and reflective practice enables them to make important decisions at crucial

moments in a student's learning. They create a learning environment which supports good teacher and student interactions. The students are not passive recipients but active, motivated participants who are supported throughout their learning. This paper has provided the basis for the development of case studies in phase two of the study where the progression to more likely possible explanations is presented.

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