Duroisin & Demeuse, *Cogent Education* (2015), 2: 1049846
**http://dx.doi.org/10.1080/2331186X.2015.1049846**

cogent education

CURRICULUM & TEACHING STUDIES | RESEARCH ARTICLE

# What role for developmental theories in mathematics study programmes in French-speaking Belgium? An analysis of the geometry curriculum's aspects, framed by Van Hiele's model

Natacha Duroisin[1]* and Marc Demeuse[1]

Received: 06 March 2015
Accepted: 29 April 2015
Published: 14 June 2015

*Corresponding author: Natacha Duroisin, Faculty of Psychology and Educational Sciences, Methodology and Training Unit, University of Mons, Place du Parc, 18, Mons 7000, Belgium
E-mail: natacha.duroisin@umons.ac.be

Reviewing editor:
Kris Gritter, Seattle Pacific University, USA

Additional information is available at the end of the article

**Abstract:** One possible way of evaluating set curricula is to examine the consistency of study programmes with students' psycho-cognitive development. Three theories were used to evaluate matching between developmental theories and content proposed in the mathematics programmes (geometry section) for primary and the beginning of secondary education. These were considered in the light of more recent work. Qualitative analysis was performed on the basis of the geometrical thinking model proposed by Van Hiele and this paper focuses on this model. The results obtained can be used to identify gaps where the programmes fail to take adequate account of child development. These results highlight the lack of precision in the wording of programme items, which makes them hard to analyse on the basis of scientific knowledge. The classifications performed revealed instances of lack of coherence that raise doubts about the supposedly progressive nature of the set content.

## ABOUT THE AUTHORS

Natacha Duroisin is a research fellow under a grant from the F.R.S. – FNRS (National Fund for Scientific Research) at the Methodology and Training Unit, Faculty of Psychology and Educational Sciences, University of Mons, Belgium. She has a doctorate in psychological sciences and educational sciences. Her research focuses on the application of cognitive processes to spatiality in connection with education and study programmes.

Marc Demeuse is a professor and the head of the Methodology and Training Unit, Faculty of Psychology and Educational Sciences, University of Mons, Belgium. He has a doctorate in psychological sciences and is a statistician. His work mainly relates to the analysis of educational policy and the evaluation of school learning. He is editor of the journal *Evaluer. Journal international de recherche en éducation et formation.*

## PUBLIC INTEREST STATEMENT

In the context of research work that aims to understand how children-adolescents experience space and how school sets about formalising spatial learning, this article compares the available knowledge about students' psycho-cognitive development with the treatment of knowledge relating to space in school study programmes. In order to investigate students' understanding of space (and identify their difficulties in understanding formalised space), the authors have chosen to focus on geometry, as one aspect of formalising the understanding and description of space. This paper presents two major interests for the international community. On the one hand, it describes an educational system which presents a particular situation as regards curriculum. On the other hand, it proposes an qualitative approach that can be replicated in other fields of study and it allows to raise awareness the community of researchers, teachers and political class on the importance of consider developmental theories when evaluating and designing study programmes.

**Subjects: Child Development; Mathematics; Mathematics & Numeracy; Mathematics Education; Science Education; Teaching & Learning**

**Keywords: comparative analysis; study programme; developmental psychology; geometry; skills; educational psychology**

## 1. Introduction

Study programmes may be evaluated from a variety of viewpoints, such as form, content coverage, coherence of structure and progression, terminology used, pedagogical orientation, didactic approach or consistency with learners' psycho-cognitive development (Duroisin, Soetewey, & Demeuse, 2013). In French-speaking Belgium, the use of large numbers of study programmes due to the complicated structure of the education system has led the research team to study curriculum coherence in previous work (Demeuse, Duroisin, & Soetewey, 2012), among other means through comparative analysis (Soetewey, Duroisin, & Demeuse, 2011). In the present study, a different approach has been used, involving verifying the internal consistency of programmes and the educational continuum proposed in the light of a recognised developmental model.

In the context of research work that aims to understand how children and adolescents experience space and how school sets about formalising spatial learning, this article compares the available knowledge about students' psycho-cognitive development with the treatment of knowledge relating to space in school study programmes. In order to investigate students' understanding of space (and identify their difficulties in understanding formalised space on the basis of sensory space), the authors have chosen here to focus on geometry, as one aspect of formalising the understanding and description of space.[1]

Focusing on a specific education network (the public network organised by the Federation Wallonia-Brussels, Desoete, Roeyers, and De Clercq, 2004), a stream of education (transitional education) and a given subject (mathematics), we analysed the study programmes for primary education (grades 1–6) and the first three years of secondary education (grades 7–9). The Van Hiele's model of geometrical thinking, which was used as an interpretative key, enabled us to evaluate the integration and coherence of developmental concepts across this set of programmes, which one would expect to be coherent, which reflect the mathematics curriculum across the different education networks.

## 2. Curriculum and study programmes: particularities of French-speaking Belgium

The situation in Belgium with regard to the curriculum is an interesting one. Firstly, the country has three highly autonomous systems (there is no common curriculum authority for the three systems, or even a permanent forum for consultation between them), and secondly, within these systems, there are numerous subsidised public and private structures with considerable room for manoeuvre, including in the definition of study programmes.

This is because, since 1831, Article 24 of the Belgian Constitution has guaranteed freedom of education. This applies to parents (in terms of choice of school), but also to schools, which enjoy considerable autonomy with regard to their educational approach. The so-called "School Pact Law" (1959) enshrines three fundamental principles of the Belgian educational system: freedom of choice of school for parents, an end to tensions between the networks and the provision of education free of charge. This law was introduced before education was devolved to the Communities in 1989, with powers in this area being handed to the parliaments of the country's three linguistic communities (the Flemish Community, the German-speaking Community and the French Community, now called the "Federation Wallonia-Brussels"). Education is therefore no longer a competence of the federal State; strictly speaking, there is no national curriculum, but rather three curricula, one per community. Under the "School Pact Law", two major categories were identified: the public networks and the subsidised independent networks. Each of these networks has different school authorities, which have genuine responsibility for organising teaching in one or more schools. Thus, in the case of the public networks, the school authority is always a public law entity: "public" education is organised by the network of the

Federation Wallonia-Brussels (FWB) or by the network of cities and provinces. For the subsidised independent networks, the school authority is a person or entity under private law, and "independent" education is organised within a faith-based network (mainly Catholic) and a non-denominational network, consisting of schools that define their educational and teaching projects on non-religious bases (mainly active learning approaches based on thinkers such as Decroly or Freinet). In addition to formal education provided in schools belonging to a network and subsidised by the government (the Federation Wallonia-Brussels for the purposes of this article), parents also have the right to educate their children at home, under the supervision of the school inspectorate.

In northern Europe, the term "'curriculum' (…) is by tradition associated with formal documents describing purposes, aims and content for what a particular group of students should be taught and learn throughout their study course" (Westbury, 2007). These are "published by national authorities" (Sivesind, 2013). Although the Belgian education system does in fact have a "curriculum" which "offers a planned, structured and coherent overview of educational guidelines on organising and managing learning in the light of the expected outcomes" (Demeuse & Strauven, 2006, p. 11), the compilation of study programmes is left to the different educational networks. To compile their programmes, they must take account of framework documents (such as the Missions decree) and ensure the attainment of the requirements set out in the core skills at the end of the first stage of secondary education (grade 8), and in the terminal attainment levels, at the end of the second and third stages of secondary education (grades 9–12). As far as the authors are concerned, these framework documents constitute the French-speaking Belgian curriculum. However, they are essentially a list of skills to be attained, whereas a curriculum generally goes further than this (2006, p. 9). It is in fact the study programmes, specific to each network, that specify, among other things, objectives, teaching methods, materials, evaluation processes for measuring the achievement of the objectives and so on (e.g. D'Hainaut, 1985; De Landsheere, 1979; Nadeau, 1988; Roegiers, 1997); these programmes are derived from the educational and pedagogical projects which are also specific to each educational network, or even to a school authority. According to Article 5, 15° of the Missions decree of 24 July 1997, which establishes a framework for all compulsory education in French-speaking Belgium, a study programme is "a repository of mandatory or optional learning situations and learning content, and of methodological guidelines that a school authority defines in order to attain the skills set by the government for a grade, stage or cycle".[2] Thus, the network of the Federation Wallonia-Brussels has programmes that it sets, the network of cities and provinces refers to the curricula of the Provinces and Municipalities, and the independent networks have their own programmes. Specifically, in a given grade (in the same sector, form and option), the course content is determined by programmes which differ from one another, being drawn up independently by each network in accordance with the common attainment levels. The principle of educational freedom thus inevitably leads to plurality in the approach to the prescribed themes, and hence results in a wide variety of study programmes. As each network compiles its own study programmes for each level of education (nursery education, primary education, transitional secondary education, qualifying secondary education and vocational secondary education) and for the different subjects (mathematics, French, science, geography, physical education, etc.), the number of programmes available and in use for compulsory education is high. The multitude of programmes is therefore an aspect that can be considered in research by focusing on the evaluation of their consistency. As we have already shown in other articles (e.g. Soetewey et al., 2011), some instances of failure at school (including in international surveys such as PISA and external non-certificative evaluations conducted in French-speaking Belgium) may be due to a series of inconsistencies in the implementation of the curriculum. This article emphasises a different approach to the evaluation of the consistency of the curriculum, involving verifying the internal consistency of study programmes and the educational continuum which is set out in the light of a given developmental model within a single network education, that organised by the Federation Wallonia-Brussels, under the direct authority of the minister responsible for compulsory education. The authors postulate that even within an educational network's programmes, inconsistencies with child attainment levels can be identified, and these may be a source of failure. This analysis was conducted in several subject areas, but this paper covers the work in the field of geometry, through the successive programmes of the network of the Federation Wallonia-Brussels covering the period of primary school and the beginning of the secondary education and representing 9 of the 12 years of compulsory education.

## 3. The development model of geometrical thinking according to Van Hiele

In order to verify the consistency of study programmes with learners' psycho-cognitive development, it was necessary to select appropriate development models. In the literature, two types of models can be identified. First, there are general development models which relate in particular to learners' psycho-cognitive development, and second, there are specific models that focus on the development of a particular psycho-cognitive field. In this study, the chosen general development models provided pointers for understanding how the transition is made from an intuitive knowledge of space to the formalism taught in school. Thus, consideration was given to Piaget's concepts relating to concrete thinking and formal thinking (Piaget, 1947)—or, to use the terms of Chevallard and Julien (1991), sensory space (space made accessible by the senses) and geometrical space (the theoretisation of space)—and those of Vygotsky (1986) which present the model of conceptual thinking in three phases.

Although, as Houdé and Leroux indicate (2013, p. 155) "Piaget's theory is the only one to describe, if not explain, the genesis of the normative structures of human intelligence from a constructivist perspective that links ontogenetic construction with the scientific genesis of logical and mathematical knowledge", Piaget's work has been and still is today the subject of frequent criticism (Montangero, 2001). Among the objections raised are the fact that, through his research, Piaget attributed excessive power to action, that he focused exclusively on the logical and mathematical structures of the "'epistemic' subject—an excessively abstract and general concept [...] sometimes forgetting the 'real psychological' subject" (Houdé & Leroux, 2013, p. 3), that he confined children to one particular stage at any given time (the staircase model), and that these theories fail to take account of differential psychology by neglecting to explain the significant intra- and interindividual variability in subjects' performance. The Swiss researcher's work should therefore be used with a little caution, taking account of the results of the "new child psychology" (Houdé, 2011). However, recent studies (Barth, 2001; Duval, 2005; Emprin, Douaire, & Rajain, 2009) have emphasised that the transition from concrete thinking to abstract thinking is difficult for many students, and that practice in moving from one kind of thinking to the other needs to start in primary education. Thus Mathé (2008) argues that it is necessary to begin the work of abstraction in the third cycle of primary education, around the age of 10 or 11 years, so that the process of conceptualisation can be implemented gradually, avoiding an abrupt move from one stage to another with the change from primary to secondary.

The model of conceptual thinking originally developed by Vygotsky (1986, 2012) lists three main stages of development. The first is "thinking based on unorganised groupings". During this period, children group objects together on the basis of "chance associations formed from what they perceive (grouping by trial and error, organisation by visual field, reassembled congeries or 'heaps')" (Chaoued, 2006, p. 64). At this stage, children can give a name to the grouping they have formed, but fail to collect similar objects together. The second stage is that of "thinking based on grouping into complex sets". At this point, children manage to break away from their egocentric thinking to establish links between isolated and concrete objects. As Chaoued mentions (2006, p. 64), "[...] the links between the various components are 'concrete' and 'factual' rather than abstract and logical. The final phase of this stage is pseudo-conceptual thinking, which 'is a transitional passage between thinking in sets and thinking based on genuine concepts'". To reach this stage, two developmental paths in thinking must converge: synthesising and separation. "The primary function involved in complex thinking is the allocation into sets or synthesis of phenomena with common features. The second path leading to conceptual thinking follows the process of separation or analysis of phenomena by dissociating them or abstracting certain of their characteristics" (Chaoued, 2006, p. 64).

The main model used in this article (Van Hiele, 1959) specifically considers the field of geometry. For the development of learners' geometrical thinking, this model focuses on language and the formation of simple axioms for primary and lower secondary education. It is constructed hierarchically and reflects five levels of understanding of geometrical concepts (Fuys, 1985). A description of Van Hiele's model is given in Table 1. The first level, "identification", is achieved when students are able to recognise shapes from their overall appearance. At this level, students do not need to list the properties of the given shape.

The second level, "analysis", is achieved by students when they succeed in distinguishing and abstracting some of the properties of a geometrical shape, though without establishing logical relationships between them. The third level, "informal deduction", is reached when learners are able to establish the logical relationships between multiple properties of one or more shapes. When students are able to understand what a theorem is or, for example, to construct a proof, this shows that they have reached the fourth level, "formal deduction". The final level relates to university education, and refers to different axiomatic systems. These different levels are typically reflected in the school system (Belkhodja, 2007). In Van Hiele's view, education must reflect the levels of the model.

As Wirszup (1976) indicates, the different levels described by Van Hiele are inherent to the development of thinking processes. The same author mentions that the transition from one level to another is not a spontaneous process concurrent with students' biological development and dependent purely on age. This development is achieved on the basis of what has been learned, and hence of taught content and recommended teaching methods. In this context, the differentiated instruction can be an important element (Forsten, Grant, & Hollas, 2002). Tomlinson (2001) identifies different elements of the curriculum that can be differentiated (contents, process and products). The transition from one level to another is thus not spontaneous and does not depend on students' maturity (or age) only; it may be accelerated (Fuys, Geddes, & Tischler, 1988), according to Van Hiele, by education based on five successive phases (inquiry/information; directed orientation; explanation; free orientation; integration) (Figure 1).

**Table 1. Description of Van Hiele's model (1959)**

| Level | Level of acquisition (Belkhodja, 2007) | Name of level | Description | |
|---|---|---|---|---|
| 1 | Before school | Identification - Visualisation - Global perception | Recognising shapes without stating properties | Visual level |
| 2 | During primary education | Analysis | Distinguishing and abstracting some properties of a geometrical shape without relating them to one another | Descriptive level |
| 3 | During lower secondary education | Informal deduction | Relating properties of one or more shapes to one another | Logical level |
| 4 | During upper secondary education | Formal deduction | Constructing deductions and simple proofsUnderstanding a theorem | |
| 5 | Higher education - university | Rigour | Comparing axiomatic systems-Producing theorems in different axiomatic systems | |

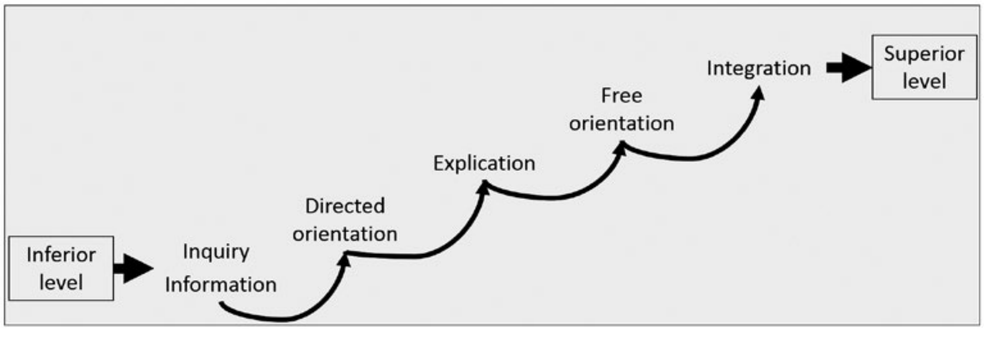



**Figure 1. Illustration of the five successive phases in Van Hiele's model (free representation based on Van Hiele, 1959).**

According to Gutierrez (1992) and Usiskin (1982), the levels described by Van Hiele have several characteristics, three of which are presented below:

- They are sequential and ordered (a higher level can only be attained if the lower level has been acquired).
- They are continuous (the transition from one level to the next is performed continuously, as "acquisition of a thinking level by a student is gradual and it can be observed along the time" (Gutiérrez, 1992, p. 32).
- They have their own language (a term may have a different meaning depending on the level).

This last characteristic is a source of many problems in teaching and learning. Given the difference in the level of educational attainment between students and teachers, they do not use the same language or the same axioms and therefore do not approach the material in the same way. It is therefore necessary for teachers to adapt their language to their students. Similarly, from the curriculum viewpoint, we will see that an item may be attached to one or more levels in the model depending on how it is interpreted (see Section 6.3).

Although the model proposed by Van Hiele builds on Piaget's work (Colignatus, 2014), it is also distinct from it. While Van Hiele (1986, p. 5) states that "an important part of the roots of my work can be found in the theories of Piaget," his theory departs from Piaget in two main ways. First, in their theses, the Van Hiele test the idea, empirically defining and developing levels of abstraction in the understanding of mathematics and defending the notion of a link that is independent of students' chronological age (Colignatus, 2014): they do not think that the levels of understanding are related to a particular chronological age. Second, they think that the development theory proposed by Piaget fails to take account of learning, and fear that the developmental stages (the pre-operational and concrete operational stages) are not enough to enable geometrical concepts to be understood. Moreover, Van Hiele recognises the important role played by language, and in this sense also draws inspiration from Vygotsky's theory (Knight, 2006).

Besides the fact that the Van Hieles' model was designed in the light of general development theories, other reasons guided the choice of this model. First, we wanted to select a model that had already been tested and/or validated by several authors (Crowley, 1987; Lunkenbein 1982; Marchand, 2009; Usiskin, 1982). Second, the chosen model needed to reflect the content covered in current study programmes (Yildiz, Aydin, & Kogce, 2009). Third, it needed to determine the progress of teaching/learning with some precision and illustrate the main phases through which students must pass in order to progress in geometry (Marchand, 2009).

## 4. Research and methodology questions

As several authors mention (Hemmi, Lepik, & Viholainen, 2013; Saint-Pierre, Dalpé, Lefebvre, & Giroux, 2010), development models are useful for devising study programmes which are adapted to students' educational level. However, the programmes that are the subject of our study provide no information about the model on which they are based. We therefore decided to use a model that can be used to analyse the official documents. The aim of our study is therefore to evaluate the integration and coherence of this development model. The choice was made of the model of geometrical thinking proposed by Van Hiele, for the reasons set out above; this was compared with the geometry sections of mathematics study programmes in primary education and the first three grades of secondary education (education organised by the Federation Wallonia-Brussels). This analysis, focusing on the learning of geometry, is of interest both with regard to the understanding of this particular field, but also as an approach that can be replicated in other fields of study. Two issues are examined in particular. They can be formulated as follows:

In the study programme sections devoted to geometry, can we identify levels of development of geometrical thinking such as those proposed by Van Hiele?

Are skills covered according to each level of development of geometrical thinking described by Van Hiele?

To answer these questions, our research was conducted in two stages. In the first stage, a literature review was conducted in order to identify the theoretical model (and underlying theories) that could be used in order to compare the development of spatial understanding in students and the learning prescribed during the period from the beginning of primary education to the end of the third year of secondary education. In the second stage, comparative qualitative analysis was conducted between the development model of geometrical thought proposed by Van Hiele and the learning planned in the mathematics study programmes. Theoretical concepts from the work of Piaget and Vygotsky were only used to add precision to certain critical points in the programmes (concrete and formal thinking, the space dedicated to perception, observation… in primary and secondary education).

Specifically, each study programme includes a series of items corresponding to an entry point defined in terms of knowledge, know-how or skill. The analysis was performed on the basis of the structure of the programmes. An item is a phrase or sentence corresponding to a coding unit. Each of these units was associated with one of the levels of the model proposed by Van Hiele (Table 2 shows the theoretical elements used to perform the analysis of the programmes). Once assigned to one of the levels of the model, the items were grouped according to the four cross-cutting skills described in the mathematics section of the core skills. These four cross-cutting skills are (1) analysing and understanding a message; (2) solving, reasoning and arguing; (3) applying; and (4) structuring and synthesising.

The coding was carried out in two stages. To start with, the work was carried out on a small number of items by two researchers working independently who had a good knowledge of the study programmes concerned and of Van Hiele's model. Next, they compared their results. After agreeing on a common approach, they then performed the coding of all the items in the programmes. According to the conventional formula for the calculation of intercoder reliability (Clermont, Desbiens, Malo, Martineau, & Simard, 1997), the concordance rate was 94%. Although this percentage does not include the small number of

**Table 2. Theoretical elements that were used to analyse the curricula**

| **Theoretical elements from Van Hiele** |
|---|
| Model of geometrical thinking in five levels<br>• Identification/Visualisation Global perception<br>• Analysis<br>• Informal deduction<br>• Formal deduction<br>• Rigour |

**Table 3. Examples of item classification performed with reference to Van Hiele's model**

| **Items from the curricula (*educational level expressed as a grade and/or cycle*)** | **Level in Van Hiele's model** |
|---|---|
| Recognising regular polygons among other plane figures. *(primary cycles 2 and 3)* | Level 1 |
| Comparing the rectangle and the square (in terms of sides and angles). *(primary cycle 3)* | Level 2 |
| Recognising right angles, acute angles, obtuse angles, complementary angles, *supplementary angles. (1st grade of secondary—cycle 1 S)* | Level 1 |
| Determining the relative positions of vertices, edges and faces. *(1st grade of secondary—cycle 1 S)* | Level 2 |
| Comparing radius and diameter. *(2nd grade of secondary—cycle 1 S)* | Level 3 |

items that were used to define the coding principles, it does include a category called "unclassifiable items", which contains the items that could not be associated with a level of the model (we discuss this at Section 6.4).

To perform this classification, the researchers confined themselves to the item as it appears in the original text, without trying to interpret it. An example of item classification is given in Table 3.

## 5. Premises of the analysis, based on certain aspects of Piaget's and Vygotsky's theories

When comparison is made between part of the set curriculum and the theoretical model proposed by Piaget concerning concrete and formal thinking, it is noticeable that, overall, the study programmes for primary (6–11 years) and secondary (12–13 years) education do take account of learners' cognitive development. Thus, reliance on concrete thinking is mainly observed at primary level, while abstract thinking becomes more prominent in the second stage of secondary education (14–15 years). It is also noticeable that the activities set during primary education relate almost exclusively to the perception, observation and recognition of familiar objects, solids, plane figures, movements of objects, associations, comparisons and classifications of objects, plane figures and so on.

Study programme content for primary education is thus consistent with Piagetian theories, since it encourages the development of concrete thinking. As mentioned earlier, recent studies have stressed that the transition from concrete thinking to abstract thinking is a difficult one for many students, and that practice in making this transition should be provided from primary school onwards. Mathé (2008) recommends beginning the work of abstraction gradually in primary education, avoiding an abrupt transition from one stage to the next coinciding with a change of school level. Yet, compared with the model of conceptual thinking developed by Vygotsky, it is precisely in terms of progression that the set study programme is problematic.

To illustrate this, the following example is taken from the study programme for the third grade of secondary education, and concerns the trigonometry of the right-angled triangle (Figure 2).

Contrary to what is advocated by Vygotsky, no reference is made to progression. Only the key elements to be taught are identified. The study programme makes no mention of the links between the theoretical concept presented and the right-angled triangle, which is only mentioned in the title, or even of the link with the orthonormal coordinate system. Moreover, no illustration is provided to enhance understanding of the subject matter to be taught, although, according to Article 5, Point 15 of the "Missions" decree of 24 July 1997, which governs all compulsory education in French-speaking Belgium, a study programme is "a repository of learning situations, compulsory or optional learning content and methodological guidelines defined by a school authority with a view to achieving the competencies set by the government for a grade, stage or cycle".

## 6. Results of the comparative analysis of Van Hiele's development model of geometrical thinking and the study programmes

Van Hiele's model can be used to examine in more detail the development of geometrical thinking from the viewpoint of the study programmes.

### *6.1. Do the educational cycles correspond to the levels of development of geometrical thinking?*

To answer this question, a graphical representation of the results in terms of cycles is given in Figure 3. The first three cycles refer to primary education, while the last represents the first stage of secondary education.

**Figure 2. Excerpt from the study programme for the third grade of secondary education.**

*Trigonometry of the right-angled rectangle*
*Definition of cosine, sine and tangent of an acute angle. Use of the calculator.*
*Basic formulae* $\sin^2 alpha + \cos^2 alpha = 1$, *tan alpha = sin alpha/cos alpha*

**Figure 3. Illustration of the relationship of the curricula with Van Hiele's levels.**

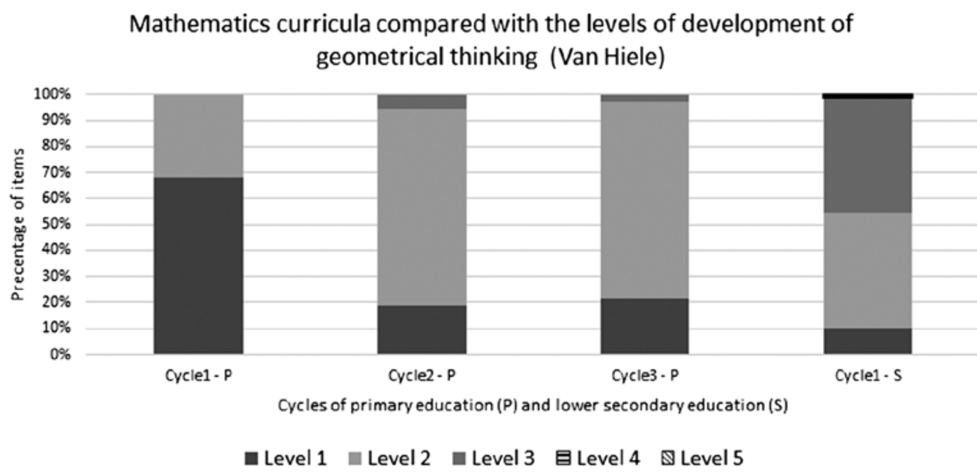



Figure 3 highlights the ample space (67.9%) devoted to the first level of the model (Identification) in the first cycle of primary education (Cycle 1-P). Less work is done on this level in the other cycles (Cycle 2-P: 18.9%; Cycle 3-P: 21.6%; Cycle 1-S: 10.3%). Subsequently, there is more coverage of level 2 (Analysis): 75.7% of the items listed for primary cycles 2 and 3 refer to the model's second level. For the Cycle 2-P and Cycle 3-P, a start is made on level 3 (more in Cycle 2-P than Cycle 3-P). In the first cycle of secondary education (Cycle 1-S), the main focus of work is on levels 2 and 3 (Informal Deduction) (level 2: 44.1% and level 3: 44.3%). A start is made on level 4 in this cycle (1.3%). Overall, the skill levels covered in the education cycles are consistent with the levels of development of geometrical thinking described by Van Hiele.

### *6.2. Are skills properly covered according to each level of development of geometrical thinking?*

On the basis of the reflections of Belkhodja (2007, p. 140) who states that "when the focus is on developing skills in order to achieve basic learning at school, it becomes necessary to define the associated levels of development", we now need to consider how well the skills taught match the developmental levels of geometrical thinking. To this end, two types of visual representation have been used.

In the section devoted to mathematical education, the core skills, it will be recalled, contain four cross-disciplinary skills that need to be developed during primary education and the first stage of secondary education. These four skills are: (1) analysing and understanding a message; (2) solving, reasoning and arguing; (3) applying; and (4) structuring and synthesising. As mentioned earlier, the items that appear in the study programmes were first assigned to one of the levels of the model; they were then sorted into these four cross-disciplinary skills.

Figure 4 shows how the skills are distributed (across all study programmes) between the five levels proposed by Van Hiele.

Examination of the results obtained reveals an uneven distribution of skills between the five levels of the presented model: skill 1 only appears in the first two levels of the model, and very little work is done on skills 2, 3 and 4 in the first level, although one might have expected the use of more visual formats (tables or graphs). Moreover, skills 3 and 4 are largely worked in the second level but few worked in the third level (<12% of items).

When one considers the differences in the distribution of skills as a function of educational cycle, further conclusions can be drawn. Figure 5 describes the situation for each cross-disciplinary skill. The educational cycles (cycles 1P, 2P and 3P for primary education and cycle 1S for secondary education) are detailed in each figure for each skill.

**Figure 4. Distribution of skills between the five levels proposed by Van Hiele.**

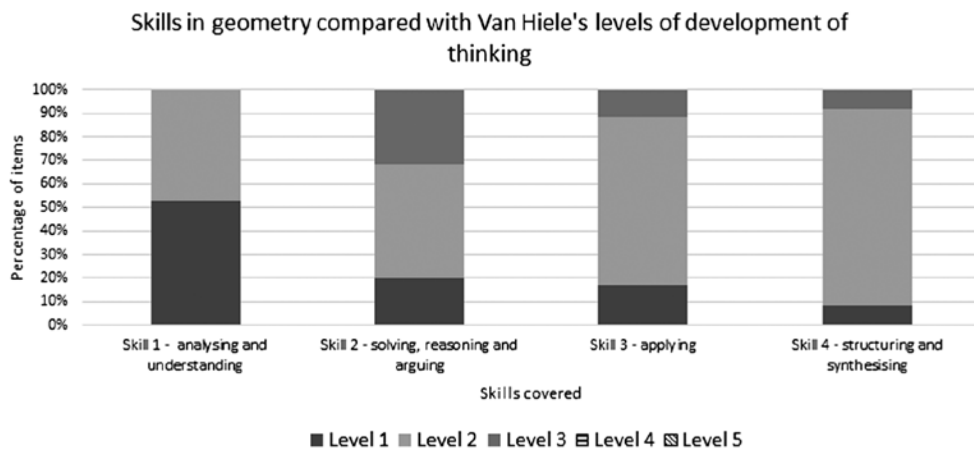
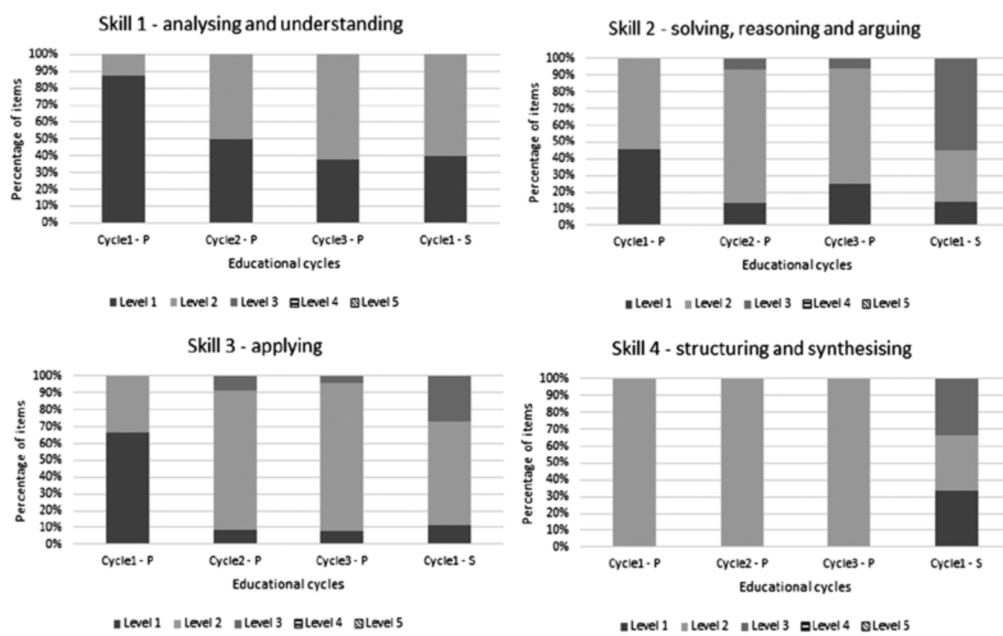



**Figure 5. Distribution of skills between the five levels proposed by Van Hiele as a function of educational cycle.**




The skills are not all worked on in each level of the model in the different educational cycles. Thus, for skill 1 (analysing and understanding), in the first cycle of primary education (Cycle 1-P), virtually all items (87.5%) relate to the first level of the model. For skill 2 (solving, reasoning and arguing), in the second (Cycle 2-P) and third (Cycle 3-P) cycles of primary education, there is a marked emphasis on the second level of the model (Cycle 2-P: 80%; Cycle 3-P: 68.7%), but little work is done on the first level (Cycle 2-P: 13.3%; Cycle 3-P: 25%). For skill 3 (applying), there is considerable coverage of level 1 of the model during the first cycle of primary education (Cycle 1-P: 66.7%) and relatively little during subsequent cycles (Cycle 2-P: 8.7%; Cycle 3-P: 7.7%; Cycle 1-S: 10.7%). In these last three cycles, the level on which the most work is done is level 2 (Cycle 2-P: 82.6%; Cycle 3-P: 88.5%; Cycle 1-S: 57.1%). For skill 4, the first and third levels of the model are not worked on during primary education. In the first cycle of secondary education (Cycle 1-S), this skill is covered in the same proportions (33.33%) as the first three levels of the model.

### *6.3. Limitations of the approach given the vagueness of the study programme items*

One limitation of the approach used is the lack of precision in some items (see the example given in Figure 6). This means that some cannot be assigned to one level rather than another. Depending on the choices made by teachers during lessons—choices subject to very little guidance from vague headings—the use of skills at very different levels can no doubt be observed.

**Figure 6. Excerpt from the study programme for the third grade of secondary education.**

*Thales' triangles - Ratios and proportions*
*Construction and calculation problems, investigation and demonstration of properties.*
*The following problems will be covered at the minimum:*
*- (...),*
***- Section of a prism or a pyramid by a plane parallel to a face.***

The excerpt from the study programme of the third grade of secondary education illustrates the difficulties faced by researchers when assigning items to one of the levels of Van Hiele's model. Reading the wording shown in bold, one wonders how teachers are supposed to understand it, and hence how the researcher should classify it. Is the idea to encourage abstraction, or rather to rely on concrete facts that lead to a better understanding of the concept? Should this exercise be done with 3D solid or a solid represented on a sheet of paper? In other words, should teachers get students to visualise the section of a prism, or to construct the section of a prism? Should students calculate the area of the section, as indicated at the start of the wording? Should Thales' theorem be worked on from a geometrical or an algebraic viewpoint?

### 6.4. Limitations of Van Hiele's model as revealed by the study programmes analysed

Although Van Hiele's model made it possible to analyse the study programmes and identify important points for an in-depth examination of how effectively the development of students' geometrical thinking is taken into account, the model's limitations were also revealed. The first of these is that it is sometimes difficult to assign items relating to the activities that students must complete to one of the given levels. This is the case for activities such as constructing and measuring. It may therefore be useful when classifying items to refer to the work of Duval (2005). This author stresses the important place of student's activity, particularly the spatial visualization. He notes that "the way of seeing a geometrical figure depends on the activity for what it is used" (p. 5) and he distinguishes four common entries in geometry (botanist/land surveyor/constructor/inventor-handyman). In the same way that it was done using the model of Van Hiele in this paper, it might be interesting to use the Duval's model to classify items of programmes. The second limitation is that the model fails to consider geometrical content that is close to the algebraic paradigm; for, as Duroisin (2013) and Duroisin, Soetewey, and Canzittu (2013) emphasise, the sections dedicated to "geometry" in the mathematics study programmes (formal education) sometimes fall within an algebraic paradigm.

## 7. Using developmental theories to assess the educational continuum set forth in the study programmes

Chapter III of the decree defining the priority missions of elementary (primary) education and secondary education and organising the structures for accomplishing them (Ministère de la Communauté française de Belgique, 1997) is entitled "Specific objectives common to elementary education and the first stage of secondary education". In section 1, Article 13, it is noted that "In mainstream education, nursery school and the first eight grades of compulsory education represent an educational continuum structured into three phases, aimed at giving all students the Core Skills necessary for their social integration and the pursuit of their studies". The reference to an "educational continuum" should be taken to mean that continuity of learning is intended and encouraged from nursery school to the end of the first stage of secondary education (and even beyond that point, until the end of compulsory education). Although the documents that define legal frameworks (decrees, the policy declaration known as the *Contrat pour l'Ecole*, etc.) advocate such an educational continuum, it has to be concluded that in the study programmes, sufficient thought has not necessarily been given to continuity of learning between primary and secondary education. Our use of development models has brought to light a lack of coherence in the progression between the content supposed to be taught in primary and in secondary education. Some examples of this incoherence in the study programme sections on "solids and shapes" (for primary education), and "geometry" (for secondary education) are listed in Tables 4 and 5.

**Table 4. Examples of incoherence taken from the curricula (primary and first grade of secondary)**

| Primary ed., 8–10 years | Primary ed., 10–12 years | Secondary ed., 1st grade |
|---|---|---|
| **Tracing** foldable templates of solids (cubes and cuboids) on paper (squared or plain) | **Tracing** foldable templates of solids (cubes and cuboids) on paper (squared or plain) | **Recognising** foldable templates of cubes, cuboids and right prisms |

**Table 5. Examples of incoherence taken from the curricula (primary and second grade of secondary)**

| Primary ed., 8–10 years | Primary ed., 10–12 years | Secondary ed., 2nd grade |
|---|---|---|
| **Using** translation, rotation and symmetry in concrete expressive activities: physical education, painting, etc. | **Moving** plane figures and distinguishing translation, rotation, orthogonal symmetry and central symmetry | **Discovering** an axis of symmetry in a figure |
| | **Comparing** and **classifying** plane figures using the following criteria: the number of sides and angles; relationships between the sides; relationships between the angles; the presence of axes of symmetry | **Discovering** symmetries and rotations in regular polygons |

It is clear from these tables that there are significant inconsistencies with regard to the primary/secondary transition. The content of secondary education study programmes appears to ignore learning that has taken place in earlier grades.

## 8. Discussions and conclusions: towards the complementarity of development models

Development models are an important reference tool for both designing and evaluating study programmes. This study relates to the second of these approaches.

Overall, the analysis conducted here highlights the fact that the programmes have little basis in specific developmental theories in the field of geometry. These programmes focus during primary education almost exclusively on work on concrete objects, taking little account of recent research highlighting the value of starting the process of abstraction in the third cycle of primary education. Similarly, they neglect multi-modal and multi-sensory learning (hands-on learning) in secondary education. By creating representations with hands–body–mind and multisensory experiences, appropriate material, language and symbols adapted, the learning can allow for the process of abstracting the ideas from reality and can help to represent it. For future study programmes, it would seem useful to give a priori consideration at the design stage to integrating the findings of this new research (Duval, 2005; Mathé, 2008; Perrin-Glorian, Mathé, & Leclercq, 2013).

Comparison of the current programmes with the model developed by Van Hiele shows that the educational cycles do coincide with the levels of development of geometrical thinking. However, there is insufficient emphasis on the intermediate stages that would allow students to master more easily the skills targeted later on in the study programme.

Comparison with developmental theories also raised questions about the progression of learning during the transition from primary to secondary. Although the attempt is made in numerous framework documents (decrees, the *Contrat pour l'Ecole*, etc.) and much research to solve the problems encountered during this transition, it would also seem necessary to rewrite the study programmes, paying particular attention to learning previously acquired, and thus contributing to the development of a coherent continuum. If the aim is to construct a true curriculum, this requirement is especially important given that in French-speaking Belgium there is no official manual, common to all teachers, nor even a common certification system besides those used at the end of primary education and after the first cycle of secondary education. Incidentally, the difference in results between these two tests was the main factor that raised questions about the existence of a genuine, coherent curriculum in geometry (Houdement, 2007; Usiskin, Andersen, & Zotto, 2010).

Moreover, this raises significant difficulties involved in allocating to a particular level a number of items which are very vague and open to a wide range of interpretations. Although this difficulty of classification raises doubts about the reproducibility of our study and about the overall results, more importantly it leaves one wondering about the difficulties teachers must experience when they are preparing their lessons. The lack of precision in the drafting of the items and/or the lack of illustrations is likely to lead to significant departures from the study programme, depending on the level of students, and significant disparities of the kind observed between schools. This difficulty is even greater for secondary school teachers, given that they are not necessarily trained to teach the subjects for which they are responsible (which also raises the issue of teacher training). Furthermore, the lack of precision in the wording of programme items can be also source of many problems for teachers who are not robust and uncomfortable with mathematics. The lack of understanding about geometry knowledge makes more difficult the interpretation of these curriculum statements. Given the constraints that teachers face, one solution is to overhaul the study programmes, giving due consideration to the primary/secondary transition, avoiding repetition, supporting the progression of learning and building on recent research in developmental psychology.

Finally, it should be noted that this study only considered the study programmes produced by the network organised by the Wallonia-Brussels Federation. Similar work could be carried out on the documents issued by the other educational networks to verify the programmes' internal consistency.

**Funding**
Natacha Duroisin gratefully acknowledge the financial support of National Fund for Scientific Research in Belgium? (Fonds De La Recherche Scientifique - FNRS) [grant number 5046941].

**Author details**
Natacha Duroisin[1]
E-mail: natacha.duroisin@umons.ac.be
Marc Demeuse[1]
E-mail: marc.demeuse@umons.ac.be
[1] Faculty of Psychology and Educational Sciences, Methodology and Training Unit, University of Mons, Place du Parc, 18, Mons 7000, Belgium.

**Citation information**
Cite this article as: What role for developmental theories in mathematics study programmes in French-speaking Belgium? An analysis of the geometry curriculum's aspects, framed by Van Hiele's model, Natacha Duroisin & Marc Demeuse, *Cogent Education* (2015), 2: 1049846.

**Notes**

1. Although other spatial components are also treated in the research (such as geography and physical education), this article relates purely to geometrical aspects.
2. The terms "cycle" and "level" are used interchangeably here. In French-speaking Belgium, primary education is made up of three cycles (Cycle 1-P: 6–7 years; Cycle 2-P: 8–9 years; Cycle 3-P: 10–11 years). Secondary education is also composed of three cycles (Cycle 1-S: 12–13 years; Cycle 2-S: 14–15 years; Cycle 3-S: 16–17 years). In this article, the first four cycles of compulsory education are considered (from the start of Cycle 1-P to the end of Cycle 1-S).

**References**

Barth, B. M. (2001). *L'apprentissage de l'abstraction* [Learning of abstraction]. Paris: Retz.
Belkhodja, M. (2007). *La visualisation en géométrie dans trois et deux dimensions en tant que compétence à développer à l'école* [The visualization on geometry in three and two dimensions as a skill to develop at school] (Vol. 1, Published thesis). Université de Laval, Quebec.
Chaoued, A. (2006). *L'enseignement scientifique à l'école de base. Approches didactique, anthropo-culturelle et épistémologique des curricula scientifiques de l'enseignement de base en Tunisie* [Scientific education at the basic school Didactic, anthropo-cultural and epistemological approach of the scientific curricula of basic teaching in Tunisia] (Doctoral thesis). Université de Rennes II – Haute Bretagne, France.
Chevallard, Y., & Julien, M. (1991). Autour de l'enseignement de la géométrie au collège [Around the teaching of geometry in college]. *Petit x, 27*, 41–76.
Clermont, G., Desbiens, J.-F., Malo, A., Martineau, S., & Simard, D. (1997). *Pour une théorie de la pédagogie. Recherches contemporaines sur le savoir des enseignants* [For a theory of pedagogy. Contemporary researches on the teachers' knowledge]. Quebec: les Presses de l'Université de Laval.
Colignatus, T. (2014). *Pierre Van Hiele and David Tall: Getting the facts right*. Paper presented at ARXIV, USA, Cornell University Library.
Crowley, M. (1987). The Van Hiele model of the development of geometric thought. In M. Lindquist & A. Shulte (Eds.), *Learning and teaching geometry, K-12* (pp. 1–16). Reston, VA: National Council of Teachers of Mathematics.
D'Hainaut, L. (1985). *Des fins aux objectifs de l'éducation* [Purposes to the objectives of education] (4è éd.). Bruxelles: Labor-Nathan.
De Landsheere, G. (1979). *Dictionnaire de l'évaluation de la recherche en éducation* [Dictionary about evaluation of

educational research]. Paris: Presses universitaires de France.

Demeuse, M., Duroisin, N., & Soetewey, S. (2012). Implications du choix des référentiels dans les évaluations nationales et internationales. Le cas de l'enseignement des sciences dans l'enseignement belge francophone [Implications of the choice of framework in national and international assessments. The case of science education in French-speaking Belgian]. *Education comparée. Revue de recherche internationale et comparative en éducation, 7*, 123–154.

Demeuse, M., & Strauven, C. (2006). *Développer un curriculum d'enseignement ou de formation* [To develop an educational or training curriculum]. Bruxelles: De Boeck. http://dx.doi.org/10.3917/dbu.demeu.2006.01

Desoete, A., Roeyers, H., & De Clercq, A. (2004). Children with mathematics learning disabilities in Belgium. *Journal of Learning Disabilities, 37*, 50–61. http://dx.doi.org/10.1177/00222194040370010601

Duroisin, N. (2013, October). *La place de la géométrie dans les programmes d'études de mathématiques* [Which place for geometry in the mathematics curriculum]. Paper presented at Journées scientifiques, Cclépodi, Changements dans les curricula et reconfigurations des disciplines scolaires, ESPE Toulouse, France.

Duroisin, N., Soetewey, S., & Canzittu, D. (2013). *On the importance to consider developmental psychology in the process of writing a curriculum*. Paper presented at European Conference on Educational Research, Istanbul, Turkey.

Duroisin, N., Soetewey, S., & Demeuse, M. (2013). Concevoir un programme d'études et ancrer ce travail de conception sur des propositions théoriques et méthodologiques, une tâche difficile? [Develop a study programmes and anchor this design work on theoretical and methodological proposals, a difficult task?]. *Mesure et évaluation en éducation, 36*, 109–137. http://dx.doi.org/10.7202/1025742ar

Duval, R. (2005). Les conditions cognitives de l'apprentissage de la géométrie: Développement de la visualisation, différenciation des raisonnements et coordination de leurs fonctionnements [Cognitive conditions of the geometric learning: Developing visualisation, distinguishing various kinds of reasoning and co-ordinating their running]. *Annales de didactique et de sciences cognitives, 10*, 5–53.

Emprin, F., Douaire, J., & Rajain, C. (2009). L'apprentissage du 3D à l'école, des situations d'apprentissage à la formation des enseignants [Learning 3D to school, learning situations to teacher training]. *Repères, 77*, 23–52.

Forsten, C., Grant, J., & Hollas, B. (2002). *Differentiated instruction. Different strategies for different learners*. Peterborough: Crystal Springs Books.

Fuys, D. (1985). Van Hiele levels of thinking in geometry. *Education and Urban Society, 17*, 447–462. http://dx.doi.org/10.1177/0013124585017004008

Fuys, D., Geddes, D., & Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education, 3*, 1–7.

Gutierrez, A. (1992). Exploring the links between Van Hiele and 3-dimensional geometry. *Topologie Structurale, 18*, 31–47.

Hemmi, K., Lepik, M., & Viholainen, A. (2013). Analysing proof-related competences in Estonian, Finnish and Swedish mathematics curricula—towards a framework of developmental proof. *Journal of Curriculum Studies, 45*, 354–378. doi:10.1080/00220272.2012.754055

Houdé, O. (2011). *La psychologie de l'enfant* [The child psychology] (5th éd.). Paris: Presses universitaires de France.

Houdé, O., & Leroux, G. (2013). *Psychologie du développement cognitif* [Cognitive psychology]. Paris: Presses universitaires de France.

Houdement, C. (2007). A la recherche d'une cohérence entre géométrie de l'école et géométrie du collège [Looking for consistency between geometry at school and at college]. *Repères, 67*, 69–84 .

Knight, K. (2006). *An investigation into the change in the Van Hiele levels of understanding geometry of pre-service elementary and secondary mathematics teachers* (Published thesis). University of Maine, Orono.

Lunkenbein, D. (1982). Géométrie dans l'enseignement au primaire [Teaching of geometry at primary]. *Instantanés mathématiques, 5*, 5–15.

Marchand, P. (2009). Le développement du sens spatial au primaire [The development of spatial sense at primary]. *Bulletin AMQ, XLIX*, 63–79.

Mathé, A. C. (2008). Confrontation aux objets et processus de conceptualisation en géométrie à la fin de l'école primaire, rôle des interactions langagières [Confrontation objects and design process in geometry at the end of primary school, role of linguistic interactions] (contribution 3). *Internationale Conference "Efficacité et équité en education"*. Université de Rennes*, 2*, 1–14.

Ministère de la Communauté française de Belgique. (1997). *Décret définissant les missions prioritaires de l'enseignement fondamental et de l'enseignement secondaire et organisant les structures propres à les atteindre* [Decree defining the priority missions of primary education and secondary education and organizing the structures to achieve it]. Retrieved 29 October, 2014, from http://www.gallilex.cfwb.be/document/pdf/21557_018.pdf

Montangero, J. (2001). Pourquoi tant de critiques à l'œuvre de Piaget? [Why much of critics to the work of Piaget?]. *Intellectica, 33*, 245–273.

Nadeau, M. (1988). *L'évaluation de programme: Théorie et pratique* [Programmes evaluation: Theory and practice]. Québec: Presses de l'Université Laval.

Perrin-Glorian, M.-J., Mathé, A.-C, & Leclercq, R. (2013). Comment peut-on enseigner la continuité de l'enseignement de la géométrie de 6 à 15 ans? Le jeu sur les supports et les instruments [How can we teach the continuity of teaching geometry from 6 to 15 years?]. *Repères, 90*, 5–41.

Piaget, J. (1947). *La psychologie de l'intelligence [The psychology of intelligence]*. Paris: Armand Colin.

Roegiers, X. (1997). *Analyser une action d'éducation ou de formation [To analyze an education or training action]*. Bruxelles: De Boeck.

Saint-Pierre, M.-C., Dalpé, V., Lefebvre, P., & Giroux, C. (2010). *Difficultés de lecture et d'écriture. Prévention et évaluation orthophonique auprès des jeunes* [Difficulties in reading and writing. Prevention and orthophonic therapy evaluation with youth]. Québec: Presses de l'Université du Québec.

Sivesind, K. (2013). Mixed images and merging semantics in European curricula. *Journal of Curriculum Studies, 45*, 52–66. http://dx.doi.org/10.1080/00220272.2012.757807

Soetewey, S., Duroisin, N., & Demeuse, M. (2011). Le curriculum oublié: Analyse comparée des programmes de sciences en Belgique francophone [Forgotten curriculum. A comparative analysis of science programmes in French-speaking Belgium]. *Revue Internationale d'Education de Sèvres, 56*, 123–134. http://dx.doi.org/10.4000/ries

Tomlinson, C. (2001). *How to differentiate instruction in mixed-ability classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry* (Doctoral thesis). University of Chicago, Chicago.

Usiskin, Z., Andersen, K., & Zotto, N. (2010). Future curricular trends in school algebra and geometry. In *Proceedings of a Conference. Research in Mathematics in Education*. Iowa State University, Iowa.

Van Hiele, P. M. (1959). The child's thought and geometry. In T. P. Carpenter, J. A. Dossey, & J. L. Koehler (Eds.), *Classics in mathematics education research* (pp. 60–67). Reston, VA: NCTM.

Van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Orlando, FL: Academic Press.

Vygotsky, L. (1986). *Thought and langage* (Revised ed.). Cambridge, MA: MIT Press.

Vygotsky, L. (2012). *Thought and langage* (Revised and expanded edition). Cambridge, MA: MIT Press.

Westbury, I. (2007). Making curricula: Why states make curricula, and how. In F. M. Connelly (Ed.), *The SAGE handbook of curriculum and instruction* (pp. 45–65). Toronto: Sage.

Wirszup, I. (1976). Breakthroughs in the psychology of learning and teaching geometry. In J. I. Martin & D. A. Bradbard (Eds.), *Space and geometry: Papers from a research workshop* (pp. 75–97). Columbus, OH: Mathematics and Environment Education ERIC Center for Science.

Yildiz, C., Aydin, M., & Kogce, D. (2009). Comparing the old and new 6th–8th grade mathematics on curricula in terms of Van Hiele understanding levels for geometry. *Procedia - Social and Behavioral Sciences, 1*, 731–736. http://dx.doi.org/10.1016/j.sbspro.2009.01.128