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Student constructs of mathematical problems: Problem types, achievement and gender

Mei-Shiu Chiu[1*], Huei-Ming Yeh[2] and David Whitebread[3]

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*Corresponding author: Mei-Shiu Chiu, Department of Education, National Chengchi University, 64, Sec. 2, Zhinan Rd., Taipei 11605, Taiwan, ROC
E-mail: chium@nccu.edu.tw; meishiuchiu@gmail.com

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Abstract: This study aims to understand students' constructs regarding mathematical problems. Fifty-one Taiwanese primary students' constructs are elicited using interviews with the repertory grid technique based on their responses to creative and non-creative problems. The results of qualitative data analysis show that students' initial constructs can be categorized into three cognitive constructs (perception, strategy, and goal, each with deep and surface orientation constructs) and one affective construct. The results of correlation analysis reveal that deep strategy for both creative and non-creative problems and affect for creative problems are related to mathematics achievement. The results of multivariate analysis of variance (MANOVA) reveal that students have fewer surface strategies and more surface goals for creative problems than those for non-creative problems. No significant gender differences or interaction effects between problem types and genders occur for either construct. The results reveal implications for teachers in the development of effective pedagogy for teaching different types of mathematical problems.

Subjects: Child Development, Education, Educational Psychology, Social Sciences

Keywords: mathematics learning, personal constructs, problem types, repertory grid technique

ABOUT THE AUTHORS

Mei-Shiu Chiu is an educational psychologist and a professor in the Department of Education, National Chengchi University, Taiwan. Her research interests focus on interactions between emotion/affect, cognition, and culture for diverse knowledge domains (e.g. mathematics, science, and medicine) in relation to teaching, assessment, and large-scale surveys.

Huei-Ming Yeh is an assistant professor of the National Taiwan University and a visiting staff in the Department of Anesthesiology, National Taiwan University Hospital. Her research expertise includes anesthesiology, malignant hyperthermia, cancer epidemiology, public health, and clinical medical education.

David Whitebread is a senior lecturer in Psychology & Education in the Faculty of Education, University of Cambridge, UK. He is a developmental psychologist and early years education specialist (having previously taught children in the 4–8 age range for 12 years). His research interests are concerned with children's development and implications for early years and primary school children's learning and education.

PUBLIC INTEREST STATEMENT

Students' approaches to mathematical problems are influenced by cognitive (perceptions, strategies, and goals) and affective factors/constructs (e.g. confidence, control, and alerts). Students cognitively approach mathematical problems in deep ways (understanding the innate structure of the problem) and/or surface ways (repeating the wording in the problem). Students' mathematics achievements relate to deep strategies for both creative and non-creative problems and to affective constructs for creative problems. The results imply that teachers need to develop students' deep strategies in solving any problems and to increase student positive affect in solving creative problems for promoting achievements. Students have fewer surface strategies and more surface goals for creative problems than those for non-creative problems. The results suggest that students need creative problems to develop better learning strategies (i.e. fewer surface strategies). Students, however, may need to identify surface goals in solving creative problems, and teachers should develop suitable pedagogies to support this student need.

1. Introduction

Problem-solving has long been a focused activity in mathematics learning and teaching (O'Shea & Leavy, 2013; Schoenfeld, 1985). International student achievement studies also recognize this and include diverse problems into their assessment programs to reflect the need to cultivate high-quality citizens for human and country development (Organization for Economic Co-operation and Development, 2007). Relatively, few studies, however, focus on teaching and learning approaches to different types of mathematics problems (Chiu, 2009). Educators need to have a thorough knowledge of how different learners interact with different types of mathematical problems, in order to provide students with adaptive and effective teaching. A basic step for advancing knowledge in this area may be to understand student constructs of the diverse problems that they solve in their real mathematics classrooms.

This study aims to identify student personal constructs of mathematical problems, elicited by the repertory grid technique, and to investigate issues related to the constructs. The constructs can be seen as being situated in real education contexts and can be used to understand learning practices when the mathematical problems are selected from students' real learning context (Lave & Wenger, 1991; Veenman, van Hout-Wolters, & Afflerbach, 2006). The identification of formative constructs of mathematical problems may help develop effective pedagogies to enhance student mathematical problem-solving (Fuchs et al., 2003).

1.1. Components of mathematical problem-solving

A number of researchers have proposed an overview of the knowledge, behavior, and processes relevant to mathematical problem-solving. For example, Polya (1945, 1962) proposes four steps (understand, plan, carry out, and review) for solving mathematical problems. Mason, Burton, and Stacey (1996) suggest key moments (getting started, getting involved, mulling, keep going, insight, being skeptical, and contemplating) in the process of mathematical problem-solving. De Corte (2004) posits that competent mathematics learners need five mathematical dispositions: domain knowledge, heuristics methods, meta-knowledge, self-regulatory skills, and beliefs about self and mathematical learning. Researchers have also emphasized the importance of affect in mathematics learning (Hannula, 2002; McLeod, 1988, 1992; Turner, Thorpe, & Meyer, 1998).

The above propositions suggest that competent mathematics learners exercise both cognitive and affective constructs in solving problems (Furinghetti & Morselli, 2009; Mevarech & Fridkin, 2006; Roebers, Cimeli, Röthlisberger, & Neuenschwander, 2012). Well-developed mathematical problem solvers/learners are expected to learn actively, think independently, adjust to diverse tasks and environments automatically, generate effective strategies, and direct their behavior towards their goals intentionally (Boekaerts, 1999).

1.2. Constructs of diverse mathematical problem types

Diverse mathematical problems have been introduced in mathematical classrooms in line with the trend of interest in constructivism (Ministry of Education, Taiwan, 1993; National Council of Teachers of Mathematics, US, 1995). Accordingly, many different mathematical problem types are used in schools and examinations in an effort to develop student multiple abilities. The dichotomy of open and closed (Boaler, 1998), ill- and well-structured (Jonassen, 1997; Nitko, 1996), and routine and non-routine problems (McLeod, 1994) is often used by researchers to compare new and traditional mathematics. This study will use creative problems to represent the open-ended, ill-structured problems (e.g. problem-posing tasks) and non-creative problems to represent closed-ended, well-structured problems (e.g. single-answer calculation problems). All problems are actually learned by students in their classrooms, so that student constructs of mathematical problems can be viewed as a reflection of student real-life educational practices.

A wide range of cognitive and affective constructs are likely to emerge from diverse mathematical problems. Cognitive constructs of problem-posing tasks include decoding, representing, processing, implementing, and imagining, which are thought to be relevant to deep learning (Kotsopoulos & Cordy, 2009; Singer & Voica, 2013). Cognitive constructs related to problem-solving tasks may include the representation of the situational model of the problem (Voyer, 2011). Affective constructs are related to complex, realistic, or non-routine problems (Middleton & Spanias, 1999). Relatively, few studies focus on affect in relation to problems that students really work on in mathematical classrooms (Seegers, Putten, & Brabander, 2002; Vermeer, Boekaerts, & Seegers, 2000). Identifying student constructs in relation to their diverse problem-solving tasks performed in the real world can, therefore, further enable us to delve into issues at the intersection of cognition and affect for mathematics learning (Schoenfeld, 1989).

1.3. Relationships between constructs of mathematical problems and achievements and gender

Research indicates that different problems have different interactions with different learners. Students tend to have higher achievement in solving simplified word problems (with solution-relevant information only) than complicated ones (with solution-relevant, explanatory and situational information; Voyer, 2011). Researchers also suggest relationships between student achievement and affective concerns (Cooper & McIntyre, 1993; McLeod, 1994; Middleton & Spanias, 1999) and validate relationships between achievements and desirable affects (e.g. learning/mastery orientations/goals) in empirical studies (Pintrich, 2000; Shih, 2005). The detailed role of affect in mathematical problem-solving for different problem types may need to be further clarified using numerical data.

Males tend to have slightly more desirable achievements and attitudes in mathematics than females (Grootenboer & Hemmings, 2007; Seegers & Boekaerts, 1996). Gender differences in mathematics achievements, however, may diminish in solving real-life problems (Boaler, 1998). Interactions between genders and constructs of problems merit further investigation.

1.4. Methodology concerns

With the aim of identifying student constructs of mathematical problems, this study drew on a qualitative research procedure, the repertory grid technique (Kelly, 1955) along with interviews, to collect data. The repertory grid technique has been widely used by educational researchers as a useful tool for eliciting people's personal constructs (Lehrer & Franke, 1992; Middleton, 1995; Pope & Denicolo, 2001) and "precedent assumptions," "predicative categories," or extended meanings of personal worlds (Williams, 2001, p. 346). Constructs are viewed as serving as the building blocks (i.e. components) of human thinking. The repertory grid technique may also be used as a foundation for facilitating dialogs between interviewers and interviewees. Based on the above review of literature, this study aims to address three research questions.

(1) What are students' constructs in relation to diverse mathematics problems, as situated in their daily mathematics-learning context?

(2) Are there relationships between the constructs and achievements?

(3) Are there differences in the constructs between problem types and genders?

2. Method

2.1. Participants

Data were taken from a larger project on learning and teaching mathematics (Chiu, 2009; Chiu & Whitebread, 2011; Whitebread & Chiu, 2004). The participants were 51 Grade 5 students (aged 10–11) from four classes of a primary school in Taipei, Taiwan. The students were selected by balancing class, gender, and previous semester mathematics achievement (24 girls and 27 boys, 15 low achievers, 16 middle achievers, and 18 high achievers), recommended by their mathematics and class teachers based on the students' prior mathematics achievements (cf. Method: Measure 2: Achievements).

The four classes of the participants were normal classes and taught by different mathematics teachers. They were mostly assessed by non-creative mathematical problems in school tests and taught in a formalistic way with slight differences among the four mathematics teachers (Chiu, 2009; Chiu & Whitebread, 2011). All students were taught the same topics at around the same time, using the same mathematics textbook. The results of MANOVA showed that there were no significant differences among the four classes and between the boys and girls in their prior mathematics achievements and solutions to the focused problems.

2.2. Focused problems and problem types

The participants were interviewed about four problems on a fractions' topic (Problems 1–4) presented in the same mathematics textbook used in the participants' school. All problems had been attempted by the students during the teaching of the topics. Problems 1–2 are creative problems, with an infinite number of correct answers, while Problems 3–4 are non-creative problems, each with one single correct answer. The problems are shown below.

- Creative problems

  Problem 1: Please use the calculation procedure, 7÷5 = 1 2/5, to make a mathematical (word) problem.

  Problem 2: Mother made several pizzas and Betty got 3/4 of the pizza. What are the ways by which the pizzas could be divided? (Please list out all of the ways by which the pizza could be divided.) ("Betty got 3/4 of the pizza", in which "of the pizza" may be explained as either "of the total number of pizzas" or "of one pizza.")

- Non-creative problems

  Problem 3: Thirty-six scenery postcards are packed in one box. Divide the 10 boxes of postcards equally among 9 people. How much of a box of scenery postcards will one person get?

  Problem 4: Two ribbons (of equal length) are equally divided among six people. How much ribbon will one person get?

2.3. Measure 1: student interview with the repertory grid technique

After the teaching of the fractions' topic, the students were interviewed individually with the repertory grid technique. The interviewer (also the corresponding author) began the interview by asking a student to teach the interviewer how to solve certain problems (i.e. Problems 1–4), a technique referred to by Laurillard (1997, p. 134) as "teachback." The interviewer then requested additional problem-solving methods after the student provided the first. The teachback technique enabled the students to experience their authentic problem-solving again and is likely to facilitate the elicitation of the personal construct of situated meanings.

In the second phase of the interview, students' personal constructs in relation to the four problems were elicited with the repertory grid technique. The interviewer asked the student to randomly choose three problems from the four focused problems and to separate the three problems into "two similar problems" and "one different problem." The interviewer then asked the student to elicit his or her construct of the "similarities" and "differences" between the problems. The interviewer repeated the procedure until it was considered that no further constructs were being expressed.

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In the third phase, the students were asked to give scores of 5 (strongly agree) to 1 (strongly disagree) according to the contents of the very left column of their grids (cf. Appendix A) for each construct. The students' perceptions and rationales for the rating were explored by follow-up questions, such as "Why?" and "What do you mean by 'good marks'?" Each interview lasted 16–60 min. To elucidate the repertory grid technique, the Appendix A shows sample interviews (four students' completed repertory grids and solutions to the four problems), one student's teaching back of how to solve some problems, and coding methods.

2.4. Measure 2: Achievements
Two kinds of student achievement data were collected from the school. These included (1) prior achievements (i.e. previous semester achievements, mostly based on results from three school tests in the previous semester, supplemented with the teachers' assessments) and (2) later achievements (i.e. results from a school test, after the teaching of four topics in the school textbook, one of which was the fractions' topic).

2.5. Data analysis
To answer Research Question 1, the interviews were recorded and transcribed. The responses in each interview were used to create a set of repertory grids for each student (cf. Appendix A). Student personal constructs elicited by the repertory grid technique were analyzed following a generic qualitative methodology with elements of grounded theory (Bowen, 2006; Kahlke, 2014; Miles & Huberman, 1994; Strauss & Corbin, 2007). The major analysis procedure included open coding, theme finding, and meaning interpreting. Rate of agreement on the final code, as described in the Results section, between two raters was 94%. Disagreements were resolved by discussion.

To answer Research Questions 2–3, the frequencies of the categories of the constructs identified by answering Research Question 1 were recorded for each student. Quantitative data were analyzed by statistical analysis using R software, Version 3.1.0 (10 April 2014) (R Core Team, http://www.R-project.org/). Research Question 2 was answered by correlation analysis. Research Question 3 was answered by MANOVA with problem types as the within-subject independent variable and genders as the between-subject one. All statistics were judged based on Bonferroni correction using the significant level of $\alpha = .05/n$, where n is the number of the whole family of tests (Abdi, 2007). The largest test family in this study included 28 tests and, thus, $\alpha = .001786$ (.05/28) was used throughout this paper to facilitate consistent interpretation. The results obtained by quantitative data analysis were interpreted with help from the results of qualitative data analysis for Research Question 1.

3. Results and discussion

3.1. Categories of students' personal constructs (Research Question 1)
Based on the experiences of three researchers and on the discussions between them, the students' personal initial constructs (over the four problems) were categorized into four major constructs: three cognitive (perception, strategy, and goal) and one affective. Each major construct included two or three minor constructs. There were deep and surface orientations for each of the minor constructs of perception, strategy, and goal. Surface orientations are students' constructs concerning the surface statements of problems or simple knowledge, while deep orientations are indicated where the students go beyond surface statements of problems, or produce comparatively sophisticated comments with regard to knowledge and meaning. In a sense, affects are deep orientations, as these are personal reflections and go beyond surface statements of problems. Meanings and examples of each minor construct are shown below.

3.1.1. Perception

- *Conditional knowledge*: Indicating conditions of a problem with surface statements (e.g. "tell you the item" and "with answers") or deep orientations (e.g. "give a well laid out problem" and "application problem").
- *Content knowledge*: Indicating relevant mathematical subject-matter knowledge with surface statements (e.g. "fractions" and "whole numbers") or deep orientations (e.g. "mixed numbers" and "proper fractions").
- *Outcome prediction*: Indicating the status of the answer or predicting what the answers would be like with surface statements (e.g. "answer is a figure" and "without answers") or deep orientations (e.g. "no fixed answers" and "various answers").

3.1.2. Strategy

- *Mathematical procedure*: Indicating mathematical procedures used to solve the problem with surface statements (e.g. "calculation" and "division") or deep orientations (e.g. "big (number) divided by small (number)" and "equally divided").
- *Thinking process*: Indicating thinking processes when solving a problem, with surface statements (e.g. "think") or deep orientations (e.g. "reasoning back").
- *Representation*: Indicating the tools or methods used to represent their knowledge with surface statements (e.g. "word" and "write out") or deep orientations (e.g. "write an answer" and "write procedures").

3.1.3. Goal

- *Major answers*: Indicating the single, major answer aimed to be obtained for a given problem with surface statements (e.g. "make a problem" and "how to divide") or deep orientations (e.g. "where the calculation procedures come from" and "how to create figures").
- *Minor answers*: Indicating relevant alternatives to or sub-answers of a main answer with surface statements (e.g. "calculating how many boxes" and "asking how many pizzas mother made") or deep orientations (e.g. "asking you how many persons and how many pieces of pizzas are there").

3.1.4. Affect

- *Confidence*: Indicating how difficult a problem is, implying a reflection in regard to personal ability in solving a problem successfully (e.g. "easy," "hard," and "hard to say").
- *Control*: Indicating a feeling of making personal choices or exercising personal control over the problem (e.g. "express freely," "by yourself/on your own," and "use your methods").
- *Alerts*: Indicating a unique characteristic of a problem, something worth paying attention to, or an alert, warning, or reminder (e.g. "being cheated" and "a trap").

Out of all 460 student constructs for the four focused problems, 26% were perceptions, 42% were strategies, 27% were goals, and only 5% were affects. The percentages, the above categories, and the descriptive statistics of the seven constructs (deep perception, surface perception, deep strategy, surface strategy, deep goal, surface goal, and affect) in Table 1 imply that mathematical problem-solving is mostly perceived by students as a cognitive affair. In solving the problems, 95% of their effort was invested in cognitive issues.

The constructs can be seen as the references made by students to successfully solve a problem and can be used to design teaching programs to enhance student mathematical problem-solving. Pape, Bell, and Yetkin's (2003) and Fuchs et al.'s (2003) initiatives are examples of multiple-construct self-regulated teaching programs for mathematics, but they based their teaching designs on models for general learning. The constructs obtained by this study are domain-specific and context-based (Op't Eynde, De Corte, & Verschaffel, 2006) and may be used to design mathematics-specific student-centered teaching programs.

3.2. Relationships between the constructs of different problem types and achievements (Research Question 2)

The three cognitive constructs (each with both deep and surface orientations) and one affective construct form the seven constructs (deep perception, surface perception, deep strategy, surface strategy, deep goal, surface goal, and affect) for statistical analysis to answer Research Questions 2 and 3. The reason for this is that an initial analysis shows that the major constructs with two orientations reveal more meaningful results than the minor constructs (e.g. alerts, content knowledge, and thinking process).

The results of correlation analysis reveal that the construct of deep strategy for creative problems is related to prior achievement (r = .213, p = .001) and that for non-creative problems is related to both prior achievement (r = .312, p < .0005) and later achievement (r = .272, p < .0005) (Table 1). The construct of affect for creative problems is related to both prior achievement (r = .263, p < .0005) and later achievement (r = .205, p = .00162). The other pairs of correlations between the constructs and achievements are not significant.

The results show that deep strategy for both creative and non-creative problems tends to be the most effective construct in relation to mathematics achievement. In other words, higher achievers in mathematics tend to solve problems using deep strategies, such as reasoning back, writing procedures, and big (number) divided by small (number) as indicated in examples taken from the previous section on constructs for answering Research Question 1. The result is consistent with past findings that students with deep approaches to learning are normally high achievers (Whitebread & Chiu, 2004). Some studies, however, doubt the relationships between deep approaches and achievements (Leung, Ginns, & Kember, 2008; Trigwell, Ashwin, & Millan, 2013). This study uses student constructs elicited by the repertory grid technique as the measure of deep approaches, and the previous studies normally use questionnaires. Future research can clarify the role of deep approaches in achievements by comparison between different measures.

Table 1. Means (Ms) and standard deviations (SDs) of the frequency of constructs by problem types and genders and correlations (*r*) between the constructs of different problem types and achievements

Problem type	Construct	All students		Boy		Girl		Prior achievement		Later achievement	
		M	SD	*M*	SD	*M*	SD	*r*	*p*	*r*	*p*
Creative problems	Deep perception	.510	.920	.41	.930	.620	.920	.119	.385	.087	1.000
	Surface perception	.550	1.150	.74	1.430	.330	.700	−.089	1.000	.055	1.000
	Deep strategy	.270	.630	.26	.590	.290	.690	.213	.001	.100	.873
	Surface strategy	.820	1.310	.59	.840	1.080	1.670	−.043	1.000	−.138	.152
	Deep goal	.310	.650	.37	.690	.250	.610	−.024	1.000	.112	.530
	Surface goal	1.550	1.420	1.52	1.250	1.580	1.610	−.059	1.000	.026	1.000
	Affect	.160	.540	.22	.640	.080	.410	.263	.000	.205	.00162
Non-creative problems	Deep perception	.570	.880	.44	.800	.710	.950	.092	1.000	.099	.873
	Surface perception	.690	1.090	.85	1.290	.500	.780	−.048	1.000	.050	1.000
	Deep strategy	.750	1.250	.63	1.080	.880	1.420	.312	.000	.272	.000
	Surface strategy	1.960	1.720	1.67	1.640	2.290	1.780	.027	1.000	−.085	1.000
	Deep goal	.040	.280	.07	.380	.000	.000	−.128	.258	−.014	1.000
	Surface goal	.570	1.100	.56	1.120	.580	1.100	−.141	.138	−.073	1.000
	Affect	.270	.780	.30	.670	.250	.900	.159	.047	.150	.082

Affective constructs for creative problems are related to mathematics achievements, while those for non-creative problems are not. The previous results for answering Research Question 1 have indicated that affect is much less perceived by certain students as a construct than is cognition. Affects or emotions may be traditionally ignored or regarded negatively in professional practices (Rhodes-Kropf et al., 2005) but may be utilized to regulate teaching and learning as suggested by research on emotional intelligence (Arora et al., 2010; Austin, Saklofske, Huang, & McKenney, 2004) and mathematics education (Hannula, 2006).

The examples from the previous section on constructs show that the feeling of control (e.g. "express freely," "by yourself/on your own," and "use your methods") is related to the creative problems, while that of alerts (e.g. "being cheated" and "a trap") is related to the non-creative problems. Helen's sample repertory grid in the Appendix A has the affective construct of "easy" for the two creative problems (Problems 1–2) and that of "being cheated" for one of the non-creative problems (Problem 3). Both Helen's constructs and the sample constructs presented in the previous section suggest that affective responses tend to be more positive toward creative problems than non-creative problems. This may be part of the reason for the significant correlation between achievements and affective responses to creative problems. The results also imply that higher achievers tend to enjoy creative problems more than lower achievers. Research has indicated that constructivist pedagogy that requires students to construct their own knowledge may favor high achievers (Kroesbergen, Luit, & Maas, 2004). Educators may need to pay attention to help low achievers in mathematics to gain enjoyment in solving creative problems.

3.3. Differences between problem types and genders in the constructs (Research Question 3)

The results of MANOVA reveal that there are significant differences between creative and non-creative problems in the frequencies of surface strategy ($p < .0005$) and surface goals ($p = .001$) (Table 2). Students tend to have fewer constructs of surface strategy for creative problems ($M = .820$, SD = 1.310) than for non-creative problems ($M = 1.960$, SD = 1.720). However, they have more constructs of surface goals for creative problems ($M = 1.550$, SD = 1.420) than for non-creative problems ($M = .570$, SD = 1.100) (Table 1). No significant differences between creative and non-creative problems occurred for the other five constructs. No significant gender differences and interactive effects between problem types and genders occurred for either construct (Table 2).

Students have more surface strategies for non-creative problems (e.g. calculation and division) than those for creative problems (e.g. word and writing). The results imply that non-creative problems invite students to invest in surface strategies than creative ones. Researchers have consistently advocated real-life problems that invite deep approaches to learning as deep approaches are related to high-quality learning outcomes, such as high achievements (Biggs, 2001), as this study also

Table 2. Results of MANOVA for problem types by genders

	Problem type		Gender		Problem type × Gender	
	F	*p*	*F*	*p*	*F*	*p*
Deep perception	.108	.744	1.011	.320	.079	.779
Surface perception	.227	.636	2.089	.155	.027	.871
Deep strategy	3.353	.073	.343	.561	.522	.474
Surface strategy	14.425	.000	2.248	.140	.106	.746
Deep goal	6.407	.015	.755	.389	.074	.787
Surface goal	11.915	.001	.024	.877	.008	.928
Affect	.288	.594	.333	.566	.212	.647

shows. More creative problems may need to be included in mathematics classrooms to encourage students to use less surface strategies.

Students invest in identifying surface goals for creative problems (e.g. make a problem, how to divide, and asking how many pizzas mother made) more than for non-creative problems (e.g. calculating how many boxes). The results imply that creative problems invite open-ended solutions and need students to pay extra attention to finding diverse solutions for one single creative problem, while one non-creative problem normally asks for one single solution. Pape et al. (2003) indicated the importance of the use of rich mathematical tasks in enhancing self-regulated learners. Rich, complicated problems, however, may decrease achievement (Voyer, 2011). Educators need to acknowledge student needs and pay attention to identifying surface goals for creative problems. Teachers need to develop effective strategies with additional patience to guide students to identify surface goals of creative problems.

4. Conclusion

4.1. Summary of the findings

Student constructs of creative and non-creative problems were identified using interviews with the repertory grid technique based on mathematical problems taken from the school textbook. Student constructs were categorized into four major constructs: three cognitive (perceptions, strategies, and goals) and one affective. Each of the three cognitive constructs included minor constructs of deep and surface orientations. The student constructs of mathematical problems identified offer a unique perspective of interaction between students and problems and can be used to model effective pedagogies.

The finding of relationships between the construct and related issues has implications for teachers in developing effective pedagogy to increase student achievement and to teach different types of mathematical problems. The finding of a significant correlation between achievements and frequent use of deep strategies for both creative and non-creative problems and affect for creative problems suggests that effective pedagogies for enhancing student achievements should take this into account. Teachers need to guide students to use deep strategies in solving problems and pay attention to increasing positive student affect in solving creative problems. The finding that students have fewer surface strategies and more surface goals for creative problems than for non-creative problems suggests that students need creative problems for less surface strategy use and need to pay attention to identify surface goals in solving creative problems.

4.2. Limitations of this study and implications for future research

This study has successfully identified student constructs regarding mathematical problems and found interactions between the constructs, problem types, and achievements. This study, however, is based on qualitative and relationship studies for a set of focused problems in a specific educational context. Future research can validate these findings with experimental methodology (e.g. teaching experiments) and diverse problem types (e.g. multiple kinds of creative and non-creative tasks such as problem posing, solving, and reasoning tasks) in other cultural contexts. This study is likely to go beyond a case study and create a thorough model with a relatively large sample size, using both qualitative and quantitative methodologies.

The repertory grid technique may be treated like a game in order to start interviews with students. The technique assists the students in focusing on the focused problems and generates an enormous amount of qualitative and quantitative research data. However, the use of the repertory grid technique via individual interview is time-consuming, and that via group or computer testing may be developed and investigated by future research to facilitate data collection. Further research can also explore diverse uses of data analysis methods related to the repertory grid technique. For example, the technique could be used as a student assessment tool to gain a deep understanding of students' constructs, learning approaches, and problem-solving behaviors.

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Author details
Mei-Shiu Chiu[1]
E-mail: chium@nccu.edu.tw; meishiuchiu@gmail.com
Huei-Ming Yeh[2]
E-mail: y.y.hhmm@hotmail.com
David Whitebread[3]
E-mail: dgw1004@cam.ac.uk
[1] Department of Education, National Chengchi University, No. 64, Sec. 2, Zhinan Rd., Taipei 11605, Taiwan, ROC.
[2] Department of Anesthesiology, National Taiwan University Hospital, No. 7, Chung-Shan S Rd., Taipei 100, Taiwan, ROC.
[3] Faculty of Education, University of Cambridge, 184 Hills Rd., Cambridge CB2 8PQ, UK.

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Appendix A. Sample interview: four students' repertory grids and solutions to the problems and the coding

Four students' repertory grids and solutions to the four problems are presented below. They were chosen by balancing gender, deep/surface constructs, and achievements. (Helen and Harry were among the highest 33% achievers, and Louis and Lisa were among the lowest 33% achievers based on prior achievements).

Harry, boy, high achiever, with surface components only

Similarity	P1	P2	P3	P4	Difference
P3P4, How many one person can get (*SG*)	1	1	5	5	P1, How to divide (*SG*)

P1: Seven boxes of chocolates are equally divided among five people.

P2: Three pizzas are equally divided among four people. Each person got 3/4 pizza.

P3: 10÷9 = 1 1/9. Ans: 1 1/9 box

P4: 2÷6 = 2/6. Ans: 2/6 ribbon

(Result: All correct)

Part of Harry's teachback of how to solve the problems is presented below.

(*Problem 1*)

I (Interviewer): Could you teach me how to solve this problem?

H (Harry): Seven cakes are divided among five people …

I: Any other methods?

H: This method is quick.

I: Why "quick?"

H: I don't know …

(*Problem 2*)

H: … The line in the middle means "division," so put "3" here, and put "4" below (the line). This is "3 pizzas divided amongst four people, and how much Betty gets …"

(*Problem 3*)
H: … "Numbers of pieces" is no use at all. The point is "numbers of boxes." So, just divide 10 boxes among 9 people, and it is "10 divided by 9."

Helen, girl, high achiever, with deep components

Similarity	P1[a]	P2	P3	P4	Difference
P1P2, Easy, calculating using division (*A, SS*)[b]	5[c]	5	1	5	P3, Being cheated (reversing the numbers of a fraction), to think (*A, DS, SS*)
P2P4, Not mixed numbers (*DP*)	1	5	1	5	P1, Mixed numbers (*DP*)
P3P4, Calculating out answers (*SS*)	1	1	5	5	P2, Thinking about how to divide (*SS, SG*)

P1: Thirty chocolates are packed into seven boxes. Divide them equally among five people. How many boxes of chocolates will one person get?

P2: Mother made three pizzas. Divided them equally among four people. Betty got one of them.

P3: 10÷9 = 10/9 = 1 1/9. Ans: 1 1/9 box

P4: 2÷6 = 2/6. Ans: 2/6 ribbon

(Result: All correct)

Louis, boy, low achiever, with deep components

Similarity	P1	P2	P3	P4	Difference
P1P2, Finding the items of the problem (*DG*)	5	5	1	1	P4, Finding answers from the items of the problem (*DG*)
P3P4, How many can one get from the division (*SG, SS*)	1	1	5	5	P2, How can it be divided (*SG*)
P3P4 Asking how many (*SG)*	1	1	5	5	P1 Making a problem (*SG*)

P1: Seven basketballs are equally divided among five persons. How many basketballs will one person get?

P2: Three pizzas are equally divided among four persons. Betty got one of these.

P3: 10÷9 = 10/9 = 1 1/9. Ans: 1 1/9 box

P4: 2÷6 = 2/6. Ans: 2/6 ribbon

(Result: All correct)

Lisa, girl, low achiever, with surface components only

Similarity	P1	P2	P3	P4	Difference
P3P4, Division (*SS*)	No	1	5	5	P2, How to divide (*SG*)
P3P4, Division (*SS*)	1	No	5	5	P1, Making a problem (*SG*)

P1: Seven bottles of a drink are equally divided among five people. How many bottles of the drink will one person get?

P2: Three pizzas were equally divided among four people. Each person got 3/4 pizza.

P3: 9÷10 = 9/10. Ans: 9/10 box

P4: 6÷2 = 3. Ans: 3 ribbons

(Result: P3 and P4 are not correct.)

[a]P1 = Problem 1, P2 = Problem 2, P3 = Problem 3, P4 = Problem 4

[b]Coding for students' personal constructs, in italics: DP = deep perception (i.e. perception in deep orientations), SP = surface perception, DS = deep strategy, SS = surface strategy, DG = deep goal, SG = surface goal, A = affect.

[c]Students were asked how much they agree that each problem had the characteristic of each personal construct on the first column (5 = strongly agree to 1 = strongly disagree).