



Received: 22 March 2017  
Accepted: 18 November 2017  
First Published: 27 November 2017

\*Corresponding author: S. Deva Arul,  
Department of Statistics, Government  
Arts College, Coimbatore, Tamil Nadu,  
India  
E-mail: [devaarul\\_s@rediffmail.com](mailto:devaarul_s@rediffmail.com)

Reviewing editor:  
Guohua Zou, Chinese Academy of  
Sciences, China

Additional information is available at  
the end of the article

## STATISTICS | RESEARCH ARTICLE

# Design and development of relational chain sampling plans [RChSP(0, $i$ )] for attribute quality characteristics

S. Deva Arul<sup>1\*</sup> and Vijila Moses<sup>1,2</sup>

**Abstract:** The purpose of this research article is to provide a novel algorithm for the Relational Chain Sampling Inspection in order to emphasize quality products from the production sources. It is evident that whenever the items are costly or destructive, the customers expect good items. Hence, the qualities control practitioners' emphasis on zero defective in such production process. But the Operating Characteristics curve of zero plans poorly discriminate the bad lots from a good one. Hence Chain Sampling Plans were recommended by Prof Dodge in the year 1955. The consumer expects that if the past results of inspection are to be used to sentence the current lot then the results of current sample should be used appropriately with previous results. Hence on the improvement of Chain Sampling Plans, the new sampling plans known to be Relational Chain Sampling Plans is being developed and is termed as RChSP(0,  $i$ ). The measure of sampling plans such as operating characteristics function for Relational Chain Sampling Plan is derived and provided. The Designing Procedure for RChSP(0,  $i$ ) is given and indexed through standard quality levels. Tables are constructed to facilitate Quality control practitioners to select and implement the RChSP(0,  $i$ ).



S. Deva Arul

### ABOUT THE AUTHORS

S. Deva Arul is an Assistant Professor of Statistics in Government Arts College, Coimbatore India. He has served in the School of Computer Science at Karunya Institute of Technology and Sciences; a prestigious deemed University in India. He has nearly 20 years of teaching experience in India and abroad in the field of Statistics, Computer Science and Mathematics sciences with extensive research and academic experience. He is one of the global leading instructor and contributor to the student community. He is the author and coauthor of more than 25 research papers in the field of Statistics, Probability and Image processing. He has produced several PhD scholars. He is a member of Indian Society for Technical Education.

Vijila Moses is an Assistant Professor in Karunya University, Coimbatore, India. She has rich teaching experience in the field of Mathematics and Statistics. She is a part-time research scholar in the Department of Statistics, Government Arts College, Coimbatore, India.

### PUBLIC INTEREST STATEMENT

Acceptance Sampling Plans control and improve the quality of the finished or partly finished products at strategic points of production process. The purpose of this research article is to provide a novel algorithm for a modern production process in order to control quality of the products before shipping from the production sources. It is evident that whenever the items are costly or destructive, the customers expect good quality items. The new sampling plan algorithm emphasizes on zero defective with chaining rule in the production process and it gives better shouldering effect on the Operating Characteristics curve. On the improvement of Chain Sampling Plans, the new sampling plans known to be Relational Chain Sampling Plans is being developed and is termed as RChSP(0,  $i$ ). The efficiency measures of relational sampling plans are derived and the parameters are determined using designing procedure. Tables are constructed to facilitate quality control practitioners for practical application in industries.

**Subjects: Science; Mathematics & Statistics; Statistics & Probability**

**Keywords: sampling plans; relational chain sampling plans; attribute quality characteristics; consumer risk; producer risk; AQL; LQL**

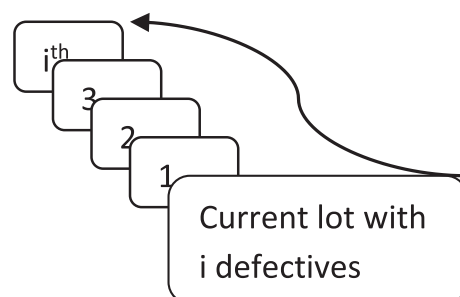
### 1. Introduction

In Industries, to control quality of the finished or partly finished product, several types of acceptance sampling plans are being developed by researchers. Dodge (1955) has introduced the ChSP-1 plans for small sampling and costly situations. The Chain Sampling Plans developed by many authors are only of partial chaining or one sided chaining. That is, it will chain only the past lots to decide about the current lot or it may defer the decision until few sample results are obtained. Hence to off-set the disadvantages, Devaarul and Rebecca Jebaseeli Edna (2012) have developed a new two-sided complete chain sampling plans. But many quality control practitioners argue that if defectives occur then chaining of the lots should done on the basis of matching the number of defectives with that of number of lots. Hence in this paper, a novel relational chain sampling plans is being developed by relating the results of current lot with that of number of previous lots. Suppose one defective occurs in the current sample, then preceding one lot result is considered for chaining. If two defectives are found, then the results of preceding two lots are considered for chaining which itself gives pressure on producer. In general, if  $i$  defectives are found, then the preceding  $i$  lots are considered for making a decision on the current lot. Hence previous inspection results are used in relation with the number of defectives of current lot for making a decision. The Relational Chain Sampling plan gives more protection to the consumer while retaining pressure on the producer. Comparison between ordinary chain and relational chain sampling plans shows better discrimination of bad lots whenever there is increase in non-conformities. And at the same time there is evident of high probability of acceptance when the quality of the lot is maintained. In an industry if the production is in steady state process, then correlation between the lots with reference to number of non-conformities is negligible and hence independence exists between the lots (Schilling, 1982). These plans are more efficient with respect to small sample size and cost of inspection (Figure 1).

Many quality control practitioners insist that compromise in quality is not advisable since in many industries defectives may lead to severe loss. Thus, non-conformity is to be maintained in the production process so as to sustain in the competitive market. But zero defective in any production industries is like a mirage in quality control sections. However acceptance criteria with zero acceptance number plans are more emphasized which gives pressure on the producer to maintain the quality of the lot. Dodge (1955) found that sampling plans does not discriminate between good and bad lots whenever the acceptance constant is zero. To overcome this drawback, he recommended Chain sampling plans instead of zero acceptance single sampling plans. But the consumers are not satisfied even if the producer maintains the specified quality level due to non-zero acceptance number. Hence to protect the producer and at the same time to satisfy the consumer, Relational Chain Sampling Plans are being developed by the authors.

Dodge and Stephens (1966) have given new type of chain sampling inspection plans. Rebecca and Joyce (2013) have studied chain sampling plan for variable fraction non-conformities. Balamurali and Usha (2013) have designed optimal Variables Chain Sampling Plan by Minimizing the Average

Figure 1. A schematic structure of relational chain sampling.



Sample Number. Vijayaraghavan and Sakthivel (2011) have used Bayesian methodology in studying Chain sampling inspection plans. Latha and Jeyabharathi (2014) have contributed towards Bayesian Chain Sampling Plans. Clark (1955) has developed OC curves for chain sampling plans. Frishman (1960) have developed an extended chain sampling plans. Dodge and Stephens (1974) have contributed towards evaluation of OC of Chain sampling plans through Markov Chain approach. Sampling procedures for inspection by attributes and designing through AQL can be found in ISO 2859-1 documents (ISO 2859-1, 1989).

Sundarrajan (1978) has given procedure and tables for construction and selection of Chain sampling plans. Kuralmani and Govindaraju (1993) have contributed towards the selection of Conditional sampling plans for the known AQL and LQL. Raju (1992) has developed OC functions for certain Conditional sampling plans. Suresh and Devaarul (2002) have developed Mixed Sampling Plans with Chain Sampling as attribute plan. Even though several authors have extended their research work using Dodge's chain sampling, literature is scarce in case of Relational Chain Sampling Plans. Hence an attempt has been made to design and develop RChsp(0, i) plans. Section 2 consists of operating procedure of Relational Chain Sampling Plans. The related sampling measures are derived and given in Section 3. Designing procedure is given in Section 4. Constructions of tables and illustrations are given in the remaining sections with necessary theorems and proof.

## 2. The operating procedure of relational chain sampling plans. RChSP (0, i)

The algorithm for sentencing a lot or batch is as follows:

- (1) Draw a random sample of size  $n$  units and count the number of non-conformities. Let it be  $d$ .
- (2) If  $d = 0$  accept the current lot.
- (3) If  $d = 1$ , accept the current lot provided preceding 1 lot is accepted. Otherwise reject it.  
If  $d = 2$ , accept the current lot provided preceding 2 lots are accepted. Otherwise reject it.  
If  $d = 3$ , accept the current lot provided preceding 3 lots are accepted. Otherwise reject it.
- (4) In general, if  $d = i$ , accept the current lot provided preceding  $i$  lots are free from defectives. Otherwise, reject it.

## 3. Measures of RChsp(0, i) sampling plans

One of the best measures of Sampling Plan is the Operating Characteristics Curve which reveals the power of discrimination between good and bad lots.

### 3.1. Operating characteristics curve (OC curve)

The OC curve gives the practical performance of acceptance sampling plan. The Locus of this curve is  $(p, P_a(p))$ . The axis represent  $p$ -proportion defective of the lot presented for inspection and the ordinate  $P_a(p)$  is the probability of acceptance of the lot or a process.

**THEOREM** The Probability of acceptance of RChsp(0, i) is

$$P_a(p) = P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots + P_{i,n}P_{0,n}^i$$

*Proof* Let  $n$  be the sample size.

$P_{0,n}$  = Probability of exactly 0 defectives in a sample of size  $n$ .

$P_{1,n}$  = Probability of exactly 1 defectives in a sample of size  $n$ .

...

$P_{i,n}$  = Probability of exactly  $i$  defectives in a sample of size  $n$ .

According to the new algorithm the following events are mutually exclusive.

$$P_{0,n}; P_{1,n}P_{0,n}; P_{2,n}P_{0,n}^2; P_{3,n}P_{0,n}^3; \dots P_{i,n}P_{0,n}^i$$

Therefore, the probability of acceptance can be written as

$$P_a(p) = P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots P_{i,n}P_{0,n}^i$$

In general,

$$P_a(p) = \sum_{x:0}^i P_{x,n}P_{0,n}^x \tag{1}$$

### 3.2. Average sample number

Average sample number (ASN) is defined as the average number of sample units necessary for making a unique decision.

$$ASN = nP_0 + nP_1(nP_0) + nP_2(nP_0)^2 + \dots + nP_i(nP_0)^i$$

$$ASN = nP_0 + (1 + nP_1 + nP_2(nP_0) + \dots + nP_i(nP_0)^{i-1}) \tag{2}$$

### 3.3. Average outgoing quality

The expected quality of outgoing product following the implementation of the algorithm for a known value of incoming lot quality is termed as Average outgoing quality (AOQ).

$$AOQ = p\{P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots P_{i,n}P_{0,n}^i\} \tag{3}$$

### 3.4. Average total inspection

Average total inspection (ATI) is the average number of units inspected per lot based on the sample size for accepted lots and all inspected units in rejected lots.

$$ATI = ASN + (N - n)(1 - P_a(p)) \tag{4}$$

## 4. Conditions for application

- (1) Production process should be steady and continuous.
- (2) Lots are coming in a Queue such that chaining is possible.

## 5. Designing and construction of RChSP (0, i) plans through AQL

The OC function is defined as

$$P_a(p) = P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots P_{i,n}P_{0,n}^i$$

where

- $P_a(p)$  Probability of acceptance of lot with fraction defective  $p$
- $P_0$  Probability of finding Zero defective in a sample of size  $n$  units
- $P_1$  Probability of finding one defective in a sample of size  $n$  units
- $P_i$  Probability of finding “ $i$ ” defectives in a sample of size  $n$  units
- $i$  number of preceding samples

- (1) Let the AQL be denoted by  $p_1$  and have an associated probability of acceptance of 0.95 ( $\alpha = 0.05$ )
- (2) Substituting  $p = p_1$  in (1), we get

**Table 1. Values of “n” and i for known values of AQL, (P[AQL] = 0.95) and defectives d**

$P_1$	Values of n						
	$i = d = 1$	$i = d = 2$	$i = d = 3$	$i = d = 4$	$i = d = 5$	$i = d = 6$	$i = d = 7$
0.001	183	143	123	110	101	93	88
0.002	92	72	62	55	50	47	44
0.003	61	48	41	37	34	31	30
0.004	46	36	31	28	25	24	22
0.005	37	29	25	22	20	19	18
0.006	31	24	21	19	17	16	15
0.007	27	21	18	16	15	14	13
0.008	23	18	16	14	13	12	11
0.009	21	16	14	13	12	11	10
0.010	19	14	13	12	11	10	9

$$0.95 = P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots P_{i,n}P_{0,n}^i \tag{5}$$

The parameters  $n$  and  $i$  are obtained by solving Equation (5). Solving the non-parametric equation is tedious, hence an iterative program is written to find the parameters of RChsp(0, i) and the values are given in Table 1. On the improvement of designing, the RChsp(0, i) is being developed through LQL, AQL & LQL, IQL, Cross over point, MAPD and MIN TAN Quality levels to facilitate the quality control engineers.

### 6. Designing and construction of RChSP(0, 1) plans through LQL

- (1) Let the LQL be denoted by  $p_2$  and have an associated probability of acceptance of 0.10 ( $\beta = 0.10$ )
- (2) Substituting  $p = p_2$  in (1), we get

$$0.10 = P_{0,n} + P_{1,n}P_{0,n} + P_{2,n}P_{0,n}^2 + P_{3,n}P_{0,n}^3 + \dots P_{i,n}P_{0,n}^i \tag{6}$$

**Table 2. Comparison of OC Curves: CChSP(0, 1) and ChSP –1 and RChsp(0, 1)**

$p$ (fraction defectives)	$P_a$ (CChSP(0, 1)) ( $i = 1, n = 10$ )	$P_a$ (ChSP (0, 1)) ( $i = 1, n = 10$ )	$P_a$ (ChSP1) ( $i = 2, n = 10$ )	$P_a$ (RChsp (0, 1))
0.001	0.99985	0.99995	0.99985	0.99995
0.0015	0.99966	0.99989	0.99966	0.99966
0.002	0.99941	0.99980	0.99941	0.99941
0.0025	0.99909	0.99969	0.99909	0.99909
0.003	0.99869	0.99956	0.99869	0.99869
0.004	0.99771	0.99922	0.99771	0.99771
0.005	0.99647	0.99879	0.99647	0.99647
0.007	0.99324	0.99766	0.99324	0.99324
0.01	0.98671	0.99532	0.98671	0.98671
0.04	0.85005	0.93844	0.85005	0.85005
0.08	0.61085	0.80879	0.61085	0.61085
0.1	0.50321	0.73576	0.50321	0.50321
0.2	0.17191	0.40601	0.17191	0.17191

The parameters  $n$  and  $i$  can be obtained by solving Equation (6). A computer program is written to solve the above equations and hence tables are constructed. Comparisons between the new sampling inspection plans and existing ones are made and given in Table 2.

### 7. Illustration

Assume that a company is producing Litho solar cells. The production process turns out AQL,  $p_1 = 0.3\%$  which is a process average and 3 defectives are to be allowed in the current lot during sampling, then determine the Relational Chain Sampling plans RChSP ( $0, i$ ) whose OC curve passes through (AQL,  $1 - \alpha = 0.95$ ).

### 8. Solution

It is given that the producer risk is  $\alpha = 0.05$ , and AQL = 0.3% . The allowable defective  $d = 3$ , then from Table 1, we get  $n = 41$  and  $i = 3$ .

The parameters of RChSP( $0, 1$ ) whose OC curve passes through the point (0.003, 0.95) are  $n = 41$  and  $i = 3$ .

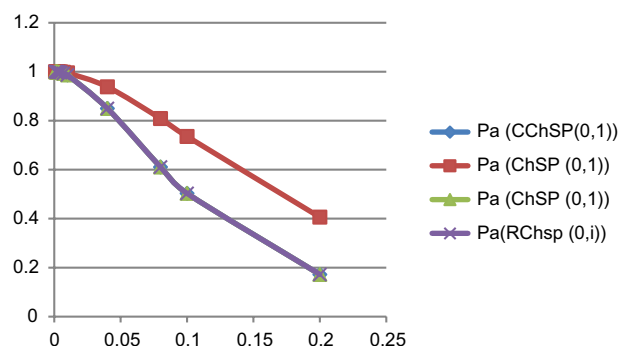
### 9. Sentencing a lot

- (1) Take a sample of size  $n = 41$ .
- (2) Find the number of non-conformities  $d$ .
- (3) If  $d = 0$ , accept the current lot.
- (4) If  $d = 1$ , accept the current lot, provided the preceding one lot is of zero defective.
- (5) If  $d = 2$ , accept the current lot, provided the preceding two lots are of zero defective.
- (6) If  $d = 3$ , accept the current lot, provided the preceding three lots are of zero defective.
- (7) If  $d \geq 4$ , reject the current lot, and sentence it.

### 10. Interpretation

It is evident that in case of RChsp( $0, 1$ ), the probability of acceptance is higher whenever the quality is maintained. However if the quality of the lot deteriorates then the OC values decreases drastically when compared with that of ordinary Chsp( $0, 1$ ). The OC curve shows better discrimination between good and bad lots (Figure 2 and Table 3).

**Figure 2. Comparison between RChsp( $0, 1$ ) with other chain sampling plans.**



**Table 3. Values of sample size  $n$  for the known fraction defectives and  $np$**

$np$	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
$P$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$	$n$
0.001	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000	4,500	5,000	5,500	6,000	6,500	7,000	7,500	8,000	8,500	9,000	9,500	10,000
0.002	250	500	750	1,000	1,250	1,500	1,750	2,000	2,250	2,500	2,750	3,000	3,250	3,500	3,750	4,000	4,250	4,500	4,750	5,000
0.003	167	333	500	667	833	1,000	1,167	1,333	1,500	1,667	1,833	2,000	2,167	2,333	2,500	2,667	2,833	3,000	3,167	3,333
0.004	125	250	375	500	625	750	875	1,000	1,125	1,250	1,375	1,500	1,625	1,750	1,875	2,000	2,125	2,250	2,375	2,500
0.005	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500	1,600	1,700	1,800	1,900	2,000
0.006	83	167	250	333	417	500	583	667	750	833	917	1,000	1,083	1,167	1,250	1,333	1,417	1,500	1,583	1,667
0.007	71	143	214	286	357	429	500	571	643	714	786	857	929	1,000	1,071	1,143	1,214	1,286	1,357	1,429
0.008	63	125	188	250	313	375	438	500	563	625	688	750	813	875	938	1,000	1,063	1,125	1,188	1,250
0.009	56	111	167	222	278	333	389	444	500	556	611	667	722	778	833	889	944	1,000	1,056	1,111
0.01	50	100	150	200	250	300	350	400	450	500	550	600	650	700	750	800	850	900	950	1,000
0.011	45	91	136	182	227	273	318	364	409	455	500	545	591	636	682	727	773	818	864	909
0.012	42	83	125	167	208	250	292	333	375	417	458	500	542	583	625	667	708	750	792	833
0.013	38	77	115	154	192	231	269	308	346	385	423	462	500	538	577	615	654	692	731	769
0.014	36	71	107	143	179	214	250	286	321	357	393	429	464	500	536	571	607	643	679	714
0.015	33	67	100	133	167	200	233	267	300	333	367	400	433	467	500	533	567	600	633	667

## 11. Conclusion

It is found that the OC function of new sampling plans well discriminate between good and bad lots. Consumer is also protected with new chain sampling method because the number of defectives in the sample is related with the preceding results of lots. The comparison between Relational chain sampling plans and ordinary chain sampling plans reveals that in relational chain sampling plans, the probability of acceptance of the lot decreases when the quality deteriorates. This ensures protection for the consumer. Hence, pressure is given to producer to maintain the prescribed quality. Moreover when  $i$  defectives are found then the new sampling plans converges with  $i$  times of  $Chsp(0, 1)$ . For practical reasons it is advisable to have smaller “ $i$ ” value. Since the newly developed Relational Chain sampling plans gives better protection for the consumer with minimum sample size and lesser inspection cost, one can implement these sampling plans for better quality control.

### Funding

The authors received no direct funding for this research.

### Author details

S. Deva Arul<sup>1</sup>

E-mail: [devaarul\\_s@rediffmail.com](mailto:devaarul_s@rediffmail.com)

Vijila Moses<sup>1,2</sup>

E-mail: [vijilamoses@karunya.edu](mailto:vijilamoses@karunya.edu)

<sup>1</sup> Department of Statistics, Government Arts College, Coimbatore, Tamil Nadu, India.

<sup>2</sup> Department of Mathematics, Karunya University, Coimbatore, India.

### Citation information

Cite this article as: Design and development of relational chain sampling plans [RChSP(0,  $i$ )] for attribute quality characteristics, S. Deva Arul & Vijila Moses, *Cogent Mathematics* (2017), 4: 1408232.

### References

- Balamurali, S., & Usha, M. (2013). Optimal designing of variables chain sampling plan by minimizing the average sample number. *International Journal of Manufacturing Engineering*, 2013.
- Clark, C. R. (1955). OC curves for Chsp -1. Chain sampling plans. *Industrial Quality Control*, 17(4), 10–12.
- Devaarul, S., & Rebecca Jebaseeli Edna, K. (2012). Two sided complete chain sampling plans for attribute quality characteristics (CChSP- 0, 1). *Karunya Journal of Research*, 3(1), 25–36.
- Dodge, H. F. (1955). Chain sampling inspection plan. *Industrial Quality Control*, 2, 10–13.
- Dodge, H. F., & Stephens, K. S. (1966). Some new chain sampling inspection. *Industrial Quality Control*, 23(2), 61–67.
- Dodge, H. F., & Stephens, K. S. (1974). An application of Markov chains for the evaluation of the operating characteristics of chain sampling inspection plans. *The QR Journal*, 1, 131–138.
- Frishman, F. (1960). An extended chain sampling inspection plan. *Industrial Quality Control*, 17(1), 10–12.
- ISO 2859-1. (1989, August 15). *Sampling procedures for inspection* (1st ed.). International Organization for Standardization.
- Kuralmani, V., & Govindaraju, K. (1993). Selection of conditional sampling plans for given AQL and LQL. *Journal of Applied Statistics*, 20(4), 467–479.  
<https://doi.org/10.1080/02664769300000047>
- Latha, M., & Jeyabharathi, S. (2014). Performance measures for Bayesian chain sampling plan using binomial distribution. *International Journal of Advanced Scientific and Technical Research*, 1(4), 156–161.
- Raju, C. (1992). On equivalence of OC functions of certain conditional sampling plans. *Communications in Statistics-Simulation and Computation*, 21(4), 961–969.  
<https://doi.org/10.1080/03610919208813061>
- Rebecca, J. E., & Joyce, J. (2013). Chain sampling plans for variable fraction non-conformities. *International Journal of Pure and Applied Mathematics*, 87(6), 827–833. ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version).
- Schilling, E. G. (1982). *Acceptance sampling in quality control*. New York, NY: Dekker.
- Soundarrajan, V. (1978). Procedures and tables for construction and selection of chain sampling plans (Chsp -1), part I & Part II. *Journal of Quality Technology*, 10(2, 3), 56–60, 99–103.
- Suresh, K. K., & Devaarul, S. (2002). Designing and selection of mixed sampling plan with chain sampling as attribute plan. *Quality Engineering*, 15(1), 155–160.  
<https://doi.org/10.1081/QEN-120006717>
- Vijayaraghavan, R., & Sakthivel, K. M. (2011). Chain sampling inspection plans based on Bayesian methodology. *Economic Quality Control*, 26(2), 173–187.



© 2017 The Author(s). This open access article is distributed under a Creative Commons Attribution (CC-BY) 4.0 license.

You are free to:

Share — copy and redistribute the material in any medium or format

Adapt — remix, transform, and build upon the material for any purpose, even commercially.

The licensor cannot revoke these freedoms as long as you follow the license terms.

Under the following terms:

Attribution — You must give appropriate credit, provide a link to the license, and indicate if changes were made.

You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use.

No additional restrictions

You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits.

