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Normal unisoft filters in R_0-algebras

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Abstract: In the present paper, the notions of normal uni-soft filters in R_0-algebras are introduced, and related properties are investigated. Furthermore, characterizations of a normal uni-soft filter are established, and a new normal uni-soft filter from old one is constructed. Finally, a condition for a uni-soft filter to be normal is given and a condensational property of a normal uni-soft filter is established.

Subjects: Advanced Mathematics; Applied Mathematics; Foundations & Theorems

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1. Introduction

To solve complicated problem in economics, engineering, and environment, we can not successfully use classical methods because of various uncertainties typical for those problems. Uncertainties can not be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in Molodtsov (1999). Maji, Roy, and Biswas (2002) and Molodtsov (1999) suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov (1999) introduced

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PUBLIC INTEREST STATEMENT

Soft set theory is an important mathematical tool to deal with uncertainties, fuzzy or vague objects and has vast applications in real life situations. In this paper, we presented an application of soft set theory in an algebraic structure, called an R_0-algebra. In fact, using the notion of uni-soft property, we introduced the notion of normal uni-soft filters in R_0-algebras, and investigated on several properties of them. We hope that this work will provide a deep impact on the upcoming research in this field and other soft algebraic studies to open up new horizons of interest and innovations. To extend these results, one can further study the union soft substructures of different algebras such as hemirings, MTL-algerbas, hyperalgebras and other mathematical branches. One may also apply this concept to study some applications in many fields like decision making, knowledge base systems, data analysis, etc.

the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji, Roy, and Biswas (2002) described the application of soft set theory to a decision making problem. Maji, Biswas, and Roy (2003) also studied several operations on the theory of soft sets. Chen, Tsang, Yeung, and Wang (2005) presented a new definition of soft set parametrization reduction, and compared this definition to the related concept of attributes reduction in rough set theory. R_0-algebras, which are different from BL-algebras, have been introduced by Wang (2000) in order to provide an algebraic proof of the completeness theorem of a formal deductive system (Wang, 1999). Furthermore, filter theories and some more important related concepts of R_0-algebras are discussed in Pei and Wang (2002), Hongjun and Guojun (2008), Hongjun and Bin (2011).

In this paper, we apply the notion of normal uni-soft property to the filter theory in R_0-algebras. We introduced the concept of normal uni-soft filters in R_0-algebras, and investigate related properties. Further, we establish characterizations of a normal uni-soft filter, and construct a new normal uni-soft filter from old one. Also, we provide a condition for an uni-soft filter to be normal. Finally, we construct a condensational property of a normal uni-soft filter.

2. Preliminaries

2.1. Basic results on R_0-algebras

Definition 2.1 (Wang, 2000) Let L be a bounded distributive lattice with order-reversing involution $\neg$ and a binary operation $\to$. Then $(L, \wedge, \vee, \neg, \to)$ is called an R_0-algebra if it satisfies the following axioms:

(R1) $x \to y = \neg y \to \neg x$,

(R2) $1 \to x = x$,

(R3) $(y \to z) \wedge ((x \to y) \to (x \to z)) = y \to z$,

(R4) $x \to (y \to z) = y \to (x \to z)$,

(R5) $x \to (y \vee z) = (x \to y) \vee (x \to z)$,

(R6) $(x \to y) \vee ((x \to y) \to (\neg x \vee y)) = 1$.

Let L be an R_0-algebra. For any $x, y \in L$, we define $x \odot y = \neg(x \to \neg y)$ and $x \oplus y = \neg x \to y$. It is proven that $\odot$ and $\oplus$ are commutative, associative and $x \oplus y = \neg(\neg x \odot \neg y)$, and $(L, \wedge, \vee, \odot, \to, 0, 1)$ is a residuated lattice. In the following, let x^n denote $x \odot x \odot \cdots \odot x$ where x appears n times for $n \in N$.

We refer the reader to the book (Iorgulescu, 2008) for further information regarding R_0-algebras.

Definition 2.2 (Pei & Wang, 2002) A nonempty subset F of L is called a *filter* of L if it satisfies

(F1) $1 \in F$,

(F2) $(\forall x \in F)(\forall y \in L)(x \to y \in F \Rightarrow y \in F)$.

LEMMA 2.3 *(Pei & Wang, 2002) Let F be a nonempty subset of L. Then F is a filter of L if and only if it satisfies*

(1) $(\forall x \in F)(\forall y \in L)(x \leq y \Rightarrow y \in F)$.

(2) $(\forall x, y \in F)(x \odot y \in F)$.

LEMMA 2.4 *(Pei & Wang, 2002) Let L be an R_0-algebra. Then the following properties hold:*

$$(\forall x, y \in L)(x \leq y \Leftrightarrow x \to y = 1), \tag{2.1}$$

$$(\forall x, y \in L)(x \leq y \to x), \tag{2.2}$$

$$(\forall x \in L)\left(\neg x = x \to 0\right), \tag{2.3}$$

$$(\forall x, y \in L)((x \to y) \vee (y \to x) = 1), \tag{2.4}$$

$$(\forall x, y \in L)(x \leq y \Rightarrow y \to z \leq x \to z,\ z \to x \leq z \to y), \tag{2.5}$$

$$(\forall x, y \in L)(((x \to y) \to y) \to y = x \to y), \tag{2.6}$$

$$(\forall x, y \in L)(x \vee y = ((x \to y) \to y) \wedge ((y \to x) \to x)), \tag{2.7}$$

$$(\forall x \in L)\left(x \odot \neg x = 0,\ x \oplus \neg x = 1\right), \tag{2.8}$$

$$(\forall x, y \in L)(x \odot y \leq x \wedge y,\ x \odot (x \to y) \leq x \wedge y), \tag{2.9}$$

$$(\forall x, y, z \in L)((x \odot y) \to z = x \to (y \to z)), \tag{2.10}$$

$$(\forall x, y \in L)(x \leq y \to (x \odot y)), \tag{2.11}$$

$$(\forall x, y, z \in L)(x \odot y \leq z \Leftrightarrow x \leq y \to z), \tag{2.12}$$

$$(\forall x, y, z \in L)(x \leq y \Rightarrow x \odot z \leq y \odot z), \tag{2.13}$$

$$(\forall x, y, z \in L)(x \to y \leq (y \to z) \to (x \to z)), \tag{2.14}$$

$$(\forall x, y, z \in L)((x \to y) \odot (y \to z) \leq x \to z). \tag{2.15}$$

2.2. Basic results on soft set theory

Soft set theory was introduced by Molodtsov (1999) and Çağman, Çitak, and Enginoğlu (2010).

In what follows, let U be an initial universe set and E be a set of parameters. We say that the pair (U, E) is a *soft universe*. Let $\mathscr{P}(U)$ (resp. $\mathscr{P}(E)$) denotes the power set of U (resp. E).

By analogy with fuzzy set theory, the notion of soft set is defined as follows:

Definition 2.5 (Çağman, et al. 2010; Molodtsov, 1999) A soft set of E over U (a soft set of E for short) is: any function $f_A{:}E \to \mathscr{P}(U)$, such that $f_A(x) = \emptyset$ if $x \notin A$, for $A \in \mathscr{P}(E)$, or, equivalently, any set

$$\mathscr{F}_A{:}= \left\{(x, f_A(x)) \mid x \in E,\ f_A(x) \in \mathscr{P}(U),\ f_A(x) = \emptyset \text{ if } x \notin A\right\},$$

for $A \in \mathscr{P}(E)$.

Definition 2.6 (Çağman et al., 2010) Let $\mathscr{F}_A$ and $\mathscr{F}_B$ be soft sets of E. We say that $\mathscr{F}_A$ is a *soft subset* of $\mathscr{F}_B$, denoted by $\mathscr{F}_A \tilde{\subseteq} \mathscr{F}_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.7 (Muhiuddin & Abdullah, 2015) For any non-empty subset A of E, a soft set $\mathscr{F}_A$ of E is said to satisfy the *uni-soft property* (it is also known as *uni-soft set*) if it satisfies:

$$(\forall x, y \in A)\left(x \hookrightarrow y \in A \Rightarrow f_A(x \hookrightarrow y) \subseteq f_A(x) \cup f_A(y)\right)$$

where $\hookrightarrow$ is a binary operation in E.

In our further discussion, $S(U, L)$ will denote the set of all soft sets of L over U where L is an R_0 -algebra unless otherwise specified.

Definition 2.8 (Muhiuddin & Abdullah, 2015) A soft set $\mathscr{F}_L \in S(U, L)$ is called a *filter of L based on the uni-soft property* (briefly, *uni-soft filter* of L) if it satisfies:

$$(\forall x \in L)\big(f_L(1) \subseteq f_L(x)\big), \tag{2.16}$$

$$(\forall x, y \in L)\big(f_L(y) \subseteq f_L(x \to y) \cup f_L(x)\big). \tag{2.17}$$

Definition 2.9 (Muhiuddin & Abdullah, 2015) Let $\mathscr{F}_L \in S(U, L)$ be a unisoft filter of L. Then for any $\delta \in \mathscr{P}(U)$, the *$\delta$-exclusive set* of $\mathscr{F}_L$ is defined by

$$e(\mathscr{F}_L;\delta) = \{x \in L \mid f_L(x) \subseteq \delta\}.$$

If $\mathscr{F}_L$ is a uni-soft filter of L, every δ-exclusive set $e(\mathscr{F}_L;\delta)$ is called an exclusive filter of L.

Lemma 2.10 *(Muhiuddin & Abdullah, 2015) Let $\mathscr{F}_L \in S(U, L)$. For a subset δ of U, define a soft set $\mathscr{F}_L^*$ of L by*

$$f_L^*: L \to \mathscr{P}(U),\ x \mapsto \begin{cases} f_L(x) & \text{if } x \in e(\mathscr{F}_L;\delta), \\ \tau & \text{otherwise} \end{cases}$$

where τ is a subset of U such that $\tau \supsetneq \bigcup_{x \in e(\mathscr{F}_L;\delta)} f_L(x)$. If $\mathscr{F}_L$ is a uni-soft filter of L, then so is $\mathscr{F}_L^*$.

Lemma 2.11 *(Muhiuddin & Abdullah, 2015) Let $\mathscr{F}_L \in S(U, L)$. Then $\mathscr{F}_L$ is a uni-soft filter of L if and only if the following assertion is valid:*

$$(\forall x, y, z \in L)\big(x \le y \to z \Rightarrow f_L(z) \subseteq f_L(x) \cup f_L(y)\big). \tag{2.18}$$

Lemma 2.12 *(Muhiuddin & Abdullah, 2015) Let $\mathscr{F}_L \in S(U, L)$ be a uni-soft filter of L. Then $\mathscr{F}_L$ is order-reversing, that is,*

$$(\forall x, y \in L)(x \le y \Rightarrow f_L(y) \subseteq f_L(x)).$$

3. Normal uni-soft filters

We begin with the following definitions and properties that will be needed in the sequel.

Definition 3.1 (Liu & Ren, 2009) A subset F of L is called a *normal filter* of L if it satisfies (F1) and

$$(\forall x, y \in L)(\forall z \in F)(z \to (y \to x) \in F \Rightarrow ((x \to y) \to y) \to x \in F). \tag{3.1}$$

Definition 3.2 A uni-soft filter $\mathscr{F}_L$ of L is said to be *normal* if the following assertion is valid:

$$(\forall x, y \in L)\big(f_L(((x \to y) \to y) \to x) \subseteq f_L(y \to x)\big). \tag{3.2}$$

Example 3.3 Let $L = \{0, a, b, c, 1\}$ be a set with the order $0 < a < b < c < 1$, and the following Cayley tables:

x	$\neg x$
0	1
a	c
b	b
c	a
1	0

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	b	1	1	1
c	a	a	b	1	1
1	0	a	b	c	1

Then $(L, \wedge, \vee, \neg, \rightarrow)$ is an R_0-algebra (see Lianzhen & Kaitai, 2005) where $x \wedge y = \min\{x, y\}$ and $x \vee y = \max\{x, y\}$.

Let $\mathscr{F}_L \in S(U, L)$ be given as follows:

$$\mathscr{F}_L = \{(0, \delta_1), (a, \delta_1), (b, \delta_1), (c, \delta_2), (1, \delta_2)\}$$

where δ_1 and δ_2 are subsets of U with $\delta_2 \subsetneq \delta_1$. Then $\mathscr{F}_L$ is a normal uni-soft filter of L.

We first consider characterizations of a normal uni-soft filter.

THEOREM 3.4 *Let $\mathscr{F}_L \in S(U, L)$. Then $\mathscr{F}_L$ is a normal uni-soft filter of L if and only if it satisfies the condition (2.16) and*

$$(\forall x, y, z \in L)\left(f_L(((x \rightarrow y) \rightarrow y) \rightarrow x) \subseteq f_L(z \rightarrow (y \rightarrow x)) \cup f_L(z)\right). \tag{3.3}$$

Proof Suppose that $\mathscr{F}_L$ is a normal uni-soft filter of L. Obviously, (2.16) is valid. For every $x, y, z \in L$, we have

$$f_L(((x \rightarrow y) \rightarrow y) \rightarrow x) \subseteq f_L(y \rightarrow x) \subseteq f_L(z \rightarrow (y \rightarrow x)) \cup f_L(z)$$

by (2.17) and (3.2).

Conversely, assume that $\mathscr{F}_L$ satisfies two conditions (2.16) and (3.3). If we take $y = 1$ in (3.3), then $f_L(x) \subseteq f_L(z) \cup f_L(z \rightarrow x)$ for all $x, z \in L$. Hence $\mathscr{F}_L$ is a uni-soft filter of L. Now if we put $z = 1$ in (3.3), then

$$\begin{aligned} f_L(((x \rightarrow y) \rightarrow y) \rightarrow x) &\subseteq f_L(1 \rightarrow (y \rightarrow x)) \cup f_L(1) \\ &= f_L(1 \rightarrow (y \rightarrow x)) = f_L(y \rightarrow x) \end{aligned}$$

by (R2) and (2.16). Therefore $\mathscr{F}_L$ is a normal uni-soft filter of L. □

THEOREM 3.5 *A soft set $\mathscr{F}_L \in S(U, L)$ is a normal uni-soft filter of L if and only if every nonempty δ-exclusive set of $\mathscr{F}_L$ is a normal filter of L for all $\delta \in \mathscr{P}(U)$.*

If $\mathscr{F}_L$ is a normal uni-soft filter of L, every δ-exclusive set $e(\mathscr{F}_L; \delta)$ is called a *normal exclusive filter* of L.

Proof Assume that $\mathscr{F}_L$ is a normal uni-soft filter of L and let $\delta \in \mathscr{P}(U)$ be such that $e(\mathscr{F}_L; \delta) \neq \emptyset$. Then there exists $a \in e(\mathscr{F}_L; \delta)$, and so $f_L(1) \subseteq f_L(a) \subseteq \delta$ by the condition (2.16). Thus $1 \in e(\mathscr{F}_L; \delta)$. Let $x, y, z \in L$ be such that $z \in e(\mathscr{F}_L; \delta)$ and $z \rightarrow (y \rightarrow x) \in e(\mathscr{F}_L; \delta)$. Then $f_L(z) \subseteq \delta$ and $f_L(z \rightarrow (y \rightarrow x)) \subseteq \delta$. Using (3.3), we have

$$f_L(((x \rightarrow y) \rightarrow y) \rightarrow x) \subseteq f_L(z \rightarrow (y \rightarrow x)) \cup f_L(z) \subseteq \delta$$

and thus $((x \rightarrow y) \rightarrow y) \rightarrow x \in e(\mathscr{F}_L; \delta)$. Therefore the nonempty δ-exclusive set of $\mathscr{F}_L$ is a normal filter of L for all $\delta \in \mathscr{P}(U)$.

Conversely, suppose that every nonempty δ-exclusive set $e(\mathscr{F}_L; \delta)$ of $\mathscr{F}_L$ is a normal filter of L for all $\delta \in \mathscr{P}(U)$. For any $x \in L$, let $f_L(x) = \delta$. Then $1 \in e(\mathscr{F}_L; \delta)$, which implies that $f_L(1) \subseteq \delta = f_L(x)$ for all $x \in L$. For any $x, y, z \in L$, let

$$f_L(z \rightarrow (y \rightarrow x)) \cup f_L(z) = \delta.$$

Then $z \in e(\mathscr{F}_L; \delta)$ and $z \rightarrow (y \rightarrow x) \in e(\mathscr{F}_L; \delta)$. Since $e(\mathscr{F}_L; \delta)$ is a normal filter of L, it follows from (3.1) that $((x \rightarrow y) \rightarrow y) \rightarrow x \in e(\mathscr{F}_L; \delta)$ and so that

$$f_L(((x \to y) \to y) \to x) \subseteq \delta = f_L(z \to (y \to x)) \cup f_L(z).$$

Using Theorem 3.4, we know that $\mathscr{F}_L$ is a normal uni-soft filter of L. □

Obviously, every normal uni-soft filter is a uni-soft filter, but the converse may not be true as seen in the following example.

Example 3.6 Let $L = [0, 1]$. For any $a, b \in L$, we define

$$\neg a = 1 - a,\ \ a \wedge b = \min\{a, b\},\ \ a \vee b = \max\{a, b\}$$

$$a \to b = \begin{cases} 1 & a \le b \\ \neg a \vee b & \text{otherwise.} \end{cases}$$

Then $(L, \wedge, \vee, \neg, \to)$ is an R_0-algebra (see Wang, 2000). Let $\mathscr{F}_L \in S(U, L)$ be given as follows:

$$\mathscr{F}_L = \{(1, \delta_2), (x, \delta_1) | x \in L \setminus \{1\}\}$$

where δ_1 and δ_2 are subsets of U with $\delta_2 \subsetneq \delta_1$. Then $\mathscr{F}_L$ is a uni-soft filter of L. But

$$f_L(((0.7 \to 0.4) \to 0.4) \to 0.7) = \delta_1 \nsubseteq \delta_2 = f_L(1 \to (0.4 \to 0.7)) \cup f_L(1),$$

and so $\mathscr{F}_L$ is not a normal uni-soft filter of L.

We provide a condition for a uni-soft filter to be normal.

THEOREM 3.7 *Let L be an R_0-algebra satisfying the following inequality:*

$$(\forall x, y \in L)((x \to y) \to y \le (y \to x) \to x). \tag{3.4}$$

Then every uni-soft filter of L is normal .

Proof Let $\mathscr{F}_L$ be a uni-soft filter of L. Using (2.5), (2.6) and (3.4), we have

$$y \to x = ((y \to x) \to x) \to x \le ((x \to y) \to y) \to x$$

for all $x, y \in L$. It follows from Lemma 2.12 that

$$f_L(((x \to y) \to y) \to x) \subseteq f_L(y \to x)$$

for all $x, y \in L$. Therefore $\mathscr{F}_L$ is a normal uni-soft filter of L. □

We make a new normal uni-soft filter from old one.

THEOREM 3.8 *Let $\mathscr{F}_L \in S(U, L)$. For a subset δ of U, define a soft set $\mathscr{F}_L^*$ of L by*

$$f_L^*: L \to \mathscr{P}(U),\ \ x \mapsto \begin{cases} f_L(x) & \text{if } x \in e(\mathscr{F}_L; \delta), \\ \tau & \text{otherwise} \end{cases}$$

where τ is a subset of U such that $\tau \supsetneq \bigcup_{x \in e(\mathscr{F}_L;\delta)} f_L(x)$. If $\mathscr{F}_L$ is a normal uni-soft filter of L, then so is $\mathscr{F}_L^*$.

Proof Assume that $\mathscr{F}_L$ is a normal uni-soft filter of L. Then $\mathscr{F}_L$ is a uni-soft filter of L, and so $\mathscr{F}_L^*$ is a uni-soft filter of L by Lemma 2.10. Let $x, y, z \in L$. If $z \in e(\mathscr{F}_L; \delta)$ and $z \to (y \to x) \in e(\mathscr{F}_L; \delta)$, then $((x \to y) \to y) \to x \in e(\mathscr{F}_L; \delta)$ because $e(\mathscr{F}_L; \delta)$ is a normal filter of L by Theorem 3.5. Hence

$$
\begin{aligned}
f_L^*(((x \to y) \to y) \to x) &= f_L(((x \to y) \to y) \to x)\\
&\subseteq f_L(z) \cup f_L(z \to (y \to x))\\
&= f_L^*(z) \cup f_L^*(z \to (y \to x)).
\end{aligned}
$$

If $z \notin e(\mathscr{F}_L;\delta)$ *or* $z \to (y \to x) \notin e(\mathscr{F}_L;\delta)$, *then* $f_L^*(z) = \tau$ *or* $f_L^*(z \to (y \to x)) = \tau$. *Thus*

$$
f_L^*(((x \to y) \to y) \to x) \subseteq \tau = f_L^*(z) \cup f_L^*(z \to (y \to x)).
$$

Therefore $\mathscr{F}_L^*$ is a normal uni-soft filter of L by Theorem 3.4. □

COROLLARY 3.9 *Let* $\mathscr{F}_L \in S(U, L)$. *For a subset* δ *of* U, *define a soft set* $\mathscr{F}_L^*$ *of* L *by*

$$
f_L^*: L \to \mathscr{P}(U),\ x \mapsto \begin{cases} f_L(x) & \text{if} x \in e(\mathscr{F}_L;\delta),\\ U & \text{otherwise.}\end{cases}
$$

If $\mathscr{F}_L$ is a normal uni-soft filter of L, then so is $\mathscr{F}_L^*$.

Proof Straightforward. □

We consider a condensational property of a normal uni-soft filter.

THEOREM 3.10 *Let* $\mathscr{F}_L$ *and* $\mathscr{G}_L$ *be two uni-soft filters of* L *such that* $\mathscr{G}_L \tilde{\subseteq} \mathscr{F}_L$ *and* $f_L(1) = g_L(1)$. *If* $\mathscr{F}_L$ *is normal, then so is* $\mathscr{G}_L$.

Proof Assume that $\mathscr{F}_L$ is a normal uni-soft filter of L. For any $x, y \in L$, let $a = y \to x$. Since $\mathscr{F}_L$ is a normal uni-soft filter of L, it follows from (R4), (3.2) and assumption that

$$
\begin{aligned}
g_L(1) &= f_L(1) = f_L(y \to (a \to x))\\
&\supseteq f_L((((a \to x) \to y) \to y) \to (a \to x))\\
&\supseteq g_L((((a \to x) \to y) \to y) \to (a \to x)),
\end{aligned} \tag{3.5}
$$

and so that

$$
\begin{aligned}
g_L(1) &= g_L((((a \to x) \to y) \to y) \to (a \to x))\\
&= g_L(a \to ((((a \to x) \to y) \to y) \to x)).
\end{aligned} \tag{3.6}
$$

Since $\mathscr{G}_L$ is a uni-soft filter of L, we have

$$
\begin{aligned}
g_L(a) &= g_L(a) \cup g_L(1)\\
&= g_L(a) \cup g_L(a \to ((((a \to x) \to y) \to y) \to x))\\
&\supseteq g_L((((a \to x) \to y) \to y) \to x).
\end{aligned} \tag{3.7}
$$

Using (R4) and (2.14), we have

$$
\begin{aligned}
1 &= x \to (a \to x) \le ((a \to x) \to y) \to (x \to y)\\
&\le ((x \to y) \to y) \to (((a \to x) \to y) \to y)\\
&\le ((((a \to x) \to y) \to y) \to x) \to (((x \to y) \to y) \to x).
\end{aligned} \tag{3.8}
$$

It follows from (3.7) and Lemma 2.11 that

$$
\begin{aligned}
g_L(y \to x) = g_L(a) &\supseteq g_L((((a \to x) \to y) \to y) \to x)\\
&= g_L(1) \cup g_L((((a \to x) \to y) \to y) \to x)\\
&\supseteq g_L(((x \to y) \to y) \to x).
\end{aligned} \tag{3.9}
$$

Therefore $\mathscr{G}_L$ is a normal uni-soft filter of L. □

4. Applications

Soft set theory, introduced by Molodtsov (1999), is an important mathematical tool to deal with uncertainties, fuzzy or vague objects and has vast applications in real life situations. Several possible applications of soft set theory in various directions are given in Molodtsov (1999).

In this paper, we presented an application of soft set theory in an algebraic structure, called an R_0-algebra. In fact, using the notion of uni-soft property, we introduced the notion of normal uni-soft filters in R_0-algebras, and investigated on several properties of them. We hope that this work will provide a deep impact on the upcoming research in this field and other soft algebraic studies to open up new horizons of interest and innovations. Indeed, this work may serve as a foundation for further study of soft R_0-algebras. To extend these results, one can further study the union soft substructures of different algebras such as hemirings, MTL-algerbas, hyperalgebras and other mathematical branches. One may also apply this concept to study some applications in many fields like decision making, knowledge base systems, data analysis, etc.

5. Conclusion

Using the notion of uni-soft property, we have introduced the concept of normal uni-soft filters in R_0-algebras, and investigated related properties. We have established characterizations of a normal uni-soft filter, and made a new normal uni-soft filter from old one. We have provided a condition for a uni-soft filter to be normal. Furthermore, we constructed a condensational property of a normal uni-soft filter. We can extend these results as follows:

(1) to develop strategies for obtaining more valuable results,

(2) to apply these notions and results for studying related notions in other (soft) algebraic structures,

(3) to study the notions of implicative uni-soft filters and Boolean uni-soft filters.

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