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Stability analysis of delayed neural networks with slope-bounded activation functions

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Abstract: This paper deals with the global asymptotic stability problem of delayed neural networks with unbounded activation functions and network parameter uncertainties. New stability criteria for global asymptotic stability of the delayed neural networks are derived by employing suitable Lyapunov functionals. These results reported in this paper can be regarded as generalizations of some existing stability results. The effectiveness and usefulness of the obtained results can be verified by comparing our results with the previously published results.

Subjects: Computer Mathematics; Non-Linear Systems; Dynamical Systems

Keywords: delayed neural networks; equilibrium point; Lyapunov functionals; robust stability

1. Introduction

In the past decades, there has been a steady increase in the interest on the applications of dynamical neural networks in solving various classes of engineering problems such as image and signal processing, associative memory design, combinatorial optimization, and pattern recognition. In fact, most of these practical applications need the process information to be in the form of stable states. Hence, in order to apply neural networks to solve these practical problems, the stability property of the equilibrium point for the designed neural networks will be crucially essential.

For a biological neural network or artificial neural network, time delays are sometimes unavoidable to be taken into account. For instance, in electronic circuits of neural networks, time delays will

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PUBLIC INTEREST STATEMENT

In this paper, a more general class of activation functions are presented and they are not required to be bounded, differentiable, and monotonically increasing. Different from existing results, the slope of this class of activation functions exist both upper and lower bounds, and they may be positive, negative, or zero. In addition, more information of the states, activation functions, and upper bounds of the delay derivative of the time varying delays are taken into consideration. A new Lyapunov functional is constructed and utilized to derive sufficient conditions to guarantee the global asymptotic stability of the neural network.

This paper should be of interest to a broad readership including those interested in neural networks.

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always occur during the signal procession and transmission, which may lead to oscillation and deteriorate the stability performance. In addition, due to the existence of external disturbances and parameter fluctuations, uncertainties are other critical issues that may destabilize the neural networks. Therefore, robust stability analysis of neural networks in the presence of time delays and uncertainties will be of theoretical and practical importance.

Traditionally, the activation functions of neural networks being considered are continuous, differentiable, monotonically increasing and bounded (see in Cai & Xiong, 2007; Oh-Min et al., 2013; Zeng & Wang, 2006; Zhang & Li, 2007), which limits their practical applications. Hence, in order to solve practical engineering problems such as those in electronic circuits (the input-output functions of amplifiers are always not monotonically increasing or continuously differentiable), non-monotonic or non-differentiable functions are more suitable to describe the neuron activations and have been extensively studied (see in Huang & Cao, 2011; Huang et al., 2013; Mathiyalagan, Sakthivel, & Anthoni, 2012; Qin, Fan, Yan, & Liu, 2014; Qin, Xue, & Wang, 2013; Zheng, Shan, & Wang, 2012). Moreover, in Balasubramaniam, Vembarasan, and Rakkiyappan (2011), Jian and Wang (2015), Qin and Xue (2009), sufficient conditions for robust stability of delayed neural network are established, in which the neuron activations belong to a set of discontinuous monotone increasing and unbounded functions. In Huang and Cao (2011), Qin et al. (2014), Vadivel, Sakthivel, Mathiyalagan, and Arunkumar (2013), without assuming the boundedness and global Lipschitz continuity of activations, sufficient conditions for global asymptotic stability of delayed neural networks are constructed. Many results about stability of neural networks and complex systems can be found in the literature, for example, see (Cao & Chen, 2004; Cao, Li, & Han, 2006; Chen & Xu, 2012; Faydasicok & Arik, 2012; Li, Chen, & Huang, 2007; Luo, Xu, Wang, Sun, & Xu, 2016; Xie, Xu, & Zhang, 2014; Xu & Teo, 2009; Xu, Xie, & Shi, 2016; Zhang, Yang, Xu, & Teo, 2013).

Motivated by the above discussions, in this paper, we will focus on investigating better sufficient conditions ensuring the existence, uniqueness and global asymptotic stability for delayed neural networks. The obtained results can be regarded as generalizations of the previously published corresponding results by the following improvements. (1) A more general class of activation functions is presented and they are not required to be bounded, differentiable, and monotonically increasing. Different from existing results in (Arik, 2014a, 2014b; Faydasicok & Arik, 2013), the slope of this class of activation functions exist both upper and lower bounds, and they may be positive, negative, or zero. (2) More information of the states, activation functions, and upper bounds of the delay derivative of the time varying delays are taken into consideration. A new Lyapunov functional is constructed and utilized to derive sufficient conditions to guarantee the global asymptotic stability of the neural network.

2. Problem description and preliminaries

In this paper, we will study the robust stability of the following delayed neural network model:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j \left(x_j(t) \right) + \sum_{j=1}^{n} b_{ij} f_j \left(x_j \left(t - \tau_{ij}(t) \right) \right) + u_i \tag{1}$$

where n the number of the neurons, $x_i(t)$ denotes the state of the neuron i and c_i is the charging rate for the neuron i. a_{ij} and b_{ij} denote the strengths of connectivity between neurons j and i at time t and $t - \tau_{ij}(t)$, respectively. u_i is the constant input to the i-th neuron. $f_j \left(x_j(t) \right)$ denotes the j-th neuron activation function. The delay parameters are time-varying and denoted by $\tau_{ij}(t)$.

The uncertainties in the network parameters $A = (a_{ij})$, $B = (b_{ij})$ and $C = \text{diag}(c_i > 0)$ can be formulated as follows:

$$C_I := \left\{ C = \text{diag}(c_i) : 0 < \underline{C} \le C \le \bar{C}, \text{i.e.}, 0 < \underline{c}_i \le c_i \le \bar{c}_i, \forall i \right\}$$

$$A_I := \left\{ A = \left(a_{ij}\right): \underline{A} \le A \le \bar{A}, \text{i.e.}, \underline{a}_{ij} \le a_{ij} \le \bar{a}_{ij}, \quad i,j = 1,2,\ldots,n \right\}$$

$$B_I := \left\{ B = \left(b_{ij}\right): \underline{B} \le B \le \bar{B}, \text{i.e.}, \underline{b}_{ij} \le b_{ij} \le \bar{b}_{ij}, \quad i,j = 1,2,\ldots,n \right\}. \tag{2}$$

The activation functions are assumed to be slope-bounded and satisfy the following condition:

$$\underline{k}_i \le \frac{f_i(x) - f_i(y)}{x - y} \le \bar{k}_i.$$

with $k_i = \max\left\{ \left|\underline{k}_i\right|, \left|\bar{k}_i\right| \right\}, i = i, \ldots, n, \forall x, y, \in R, x \ne y.$

This class of functions will be denoted by $f \in \kappa$. As described above, this class of activation functions do not require to be bounded, differentiable and monotonically increasing, and $\underline{k}_i$ and $\bar{k}_i$ may be positive, negative, or zero. Unlike the previously corresponding activation functions in Arik (2014a), Cao and Chen (2004), Cao et al. (2006), Faydasicok and Arik (2012), Li et al. (2007), whose activation functions are assumed to be monotonic or bounded. Thus, the condition proposed on the activation functions in this paper are weaker than those in Arik (2014a), Cao and Chen (2004), Cao et al. (2006), Faydasicok and Arik (2012), Li et al. (2007).

Lemma 1 (Arik, 2014c). *Let A be any real matrix defined by $A \in A_I := \left\{ A = \left(a_{ij}\right): \underline{A} \le A \le \bar{A}, \text{i.e.}, \underline{a}_{ij} \le a_{ij} \le \bar{a}_{ij}, \quad i,j = 1,2,\ldots,n \right\}$. Then, for any two vectors $x = \left(x_1, x_2, \ldots, x_n\right)^T \in R^n$ and $y = \left(y_1, y_2, \ldots, y_n\right)^T \in R^n$, the following inequality holds:*

$$2x^T A y \le \rho \sum_{i=1}^{n} x_i^2 + \frac{1}{\rho} \sum_{i=1}^{n} \alpha_i y_i^2.$$

where ρ is any positive constant, and $\alpha_i = \sum_{k=1}^{n}(\hat{a}_{ki} \sum_{j=1}^{n} \hat{a}_{kj})$, $\hat{a}_{ij} = \max\left\{ \left|\underline{a}_{ij}\right|, \left|\bar{a}_{ij}\right| \right\}, \quad i,j = 1,2,\ldots,n.$

Lemma 2 (Ozcan & Arik, 2014). *Let A be any real matrix defined by $A \in A_I := \left\{ A = \left(a_{ij}\right): \underline{A} \le A \le \bar{A}, \text{i.e.}, \underline{a}_{ij} \le a_{ij} \le \bar{a}_{ij}, \quad i,j = 1,2,\ldots,n \right\}$. Then, for any positive diagonal matrix $P = \text{diag}(p_i > 0)$ and for any vector $x = \left(x_1, x_2, \ldots, x_n\right)^T \in R^n$, the following inequality holds:*

$$x^T \left(PA + A^T P\right) x \le - \sum_{i=1}^{n} \beta_i x_i^2$$

where $\beta_i = \sum_{j=1}^{n} s_{ij}, i = 1, 2, \ldots, n$ with $s_{ii} = -2p_i \bar{a}_{ii}$ and $s_{ij} = -\max\left(\left|p_i \bar{a}_{ij} + p_j \bar{a}_{ji}\right|, \left|p_i \underline{a}_{ij} + p_j \underline{a}_{ji}\right|\right)$ for $i \ne j$.

3. Robust stability results

In this section, we will present sufficient conditions for robust asymptotic stability of equilibrium point of delayed neural network (1). We first shift the equilibrium point x^* of system (1) to the origin.

The transformation $z_i(.) = x_i(.) - x_i^*$ is used to put system (1) in the following form:

$$\frac{dz_i(t)}{dt} = -c_i z_i(t) + \sum_{j=1}^{n} a_{ij} g_j\left(z_j(t)\right) + \sum_{j=1}^{n} b_{ij} g_j\left(z_j\left(t - \tau_{ij}(t)\right)\right) \tag{3}$$

where $g_i\left(z_i(.)\right) = f_i\left(z_i(.) + x_i^*\right) - f_i(x_i^*), i = 1, 2, \ldots, n.$ Note that it can easily be verified that the function g_i satisfies the assumptions on f_i, that is,

$$\underline{k}_i \le \frac{g_i\left(z_i(t)\right)}{z_i(t)} \le \bar{k}_i$$

$\forall z_i(t) \in R, z_i(t) \neq 0$ and $g_i(0) = 0, \quad i = 1, 2, \ldots, n.$

Since $z(t) \to 0$ implies $x \to x^*$, it is thus necessary and sufficient to prove the stability of the transformed model (3) instead of considering stability of the original one (1).

THEOREM 1 *For the delayed neural network (18), if the network parameters satisfy (2) and $g \kappa$, the time-varying delays satisfy that $0 \leq \dot{\tau}_{ij}(t) \leq \eta < 1, \quad i, j = 1, 2, \ldots, n$. Then, the neural network (3) is globally asymptotically stable, if there exist a positive diagonal matrix $P = \mathrm{diag}(p_i > 0)$ and some positive constants ρ, δ such that*

$$\Phi_i = 2\underline{c}_i - \rho - \frac{1}{\rho}\alpha_i k_i^2 - p_M - \frac{n\mu}{1-\eta} + 2p_i \underline{c}_i \underline{k}_i + \beta_i k_i^2 - \delta p_M k_i^2 > 0, \quad i = 1, 2, \ldots, n, \tag{4}$$

where $\alpha_i = \sum_{k=1}^{n}(\hat{a}_{ki} \sum_{j=1}^{n} \hat{a}_{kj}), \hat{a}_{ij} = \max\left\{ \left|\underline{a}_{ij}\right|, \left|\bar{a}_{ij}\right| \right\}$ and $\beta_i = \sum_{j=1}^{n} s_{ij}$ with

$s_{ii} = -2p_i \bar{a}_{ii}$ and $s_{ij} = -\max\left(\left|p_i\bar{a}_{ij} + p_j\bar{a}_{ji}\right|, \left|p_i\underline{a}_{ij} + p_j\underline{a}_{ji}\right| \right)$ for $i \neq j, i, j = 1, 2, \ldots, n.$

Proof We employ the following positive definite Lyapunov functional:

$$V(t) = \sum_{i=1}^{n} z_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{\mu}{1-\eta}\right)\int_{t-\tau_{ij}(t)}^{t} z_i^2(s)ds + 2\sum_{i=1}^{n}\int_{0}^{z_i(t)} p_i g_i(s)ds + \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon\int_{t-\tau_{ij}(t)}^{t} g_i^2(z_i(s))ds \tag{5}$$

where μ, ε are some constants to be determined later.

Taking the time derivative of the functional (5) along the trajectories of (3), noting $\frac{1-\dot{\tau}_{ij}(t)}{1-\eta} \geq 1, i, j = 1, 2, \ldots, n$, we obtain

$$\dot{V}(t) = -2\sum_{i=1}^{n} c_i z_i^2(t) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}z_i(t)g_j(z_j(t)) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}z_i(t)g_j(z_j(t)) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}z_i(t)g_j(z_j(t-\tau_{ij}(t)))$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\mu}{1-\eta}z_i^2(t) - \sum_{i=1}^{n}\sum_{j=1}^{n}\mu z_i^2(t-\tau_{ij}(t)) - 2\sum_{i=1}^{n} c_i p_i z_i(t)g_i(z_i(t)) + 2\sum_{i=1}^{n}\sum_{j=1}^{n} p_i a_{ij}g_i(z_i(t))g_j(z_j(t))$$

$$+ 2\sum_{i=1}^{n}\sum_{j=1}^{n} p_i b_{ij}g_i(z_i(t))g_j(z_j(t-\tau_{ij}(t))) - \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon g_i^2(z_i(t-\tau_{ij}(t)))\cdot(1-\dot{\tau}_{ij}(t)) + \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon g_i^2(z_i(t)). \tag{6}$$

According to Lemma 1, for any positive constant ρ, we have

$$2\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}z_i(t)g_j(z_j(t)) \leq \rho\sum_{i=1}^{n} z_i^2(t) + \frac{1}{\rho}\sum_{i=1}^{n}\alpha_i k_i^2 z_i^2(t). \tag{7}$$

We note the following inequalities

$$2\sum_{i=1}^{n}\sum_{j=1}^{n} b_{ij}z_i(t)g_j(z_j(t-\tau_{ij}(t))) \leq \sum_{i=1}^{n} p_i z_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{n}{p_i}\hat{b}_{ij}^2 g_j^2(z_j(t-\tau_{ij}(t)))$$

$$\leq p_M\sum_{i=1}^{n} z_i^2(t) + \frac{1}{p_m}\|B\|_1^2\sum_{i=1}^{n}\Sigma_{j=1}^{n} g_i^2(z_i(t-\tau_{ij}(t))). \tag{8}$$

For $g \in \kappa$, since $\underline{k}_i \leq \frac{g_i(z_i(t))}{z_i(t)} \leq \bar{k}_i$, with $k_i = \max\left\{ \left|\underline{k}_i\right|, \left|\bar{k}_i\right| \right\}$ we have

$$-2\sum_{i=1}^{n} c_i p_i z_i(t)g_i(z_i(t)) \leq -2\sum_{i=1}^{n}\underline{c}_i p_i \underline{k}_i z_i^2(t) \tag{9}$$

In light of Lemma 2, we know that

$$2\sum_{i=1}^{n}\sum_{j=1}^{n}p_i a_{ij} g_i(z_i(t)) g_j\big(z_j(t)\big) \le -\sum_{i=1}^{n}\beta_i g_i^2(z_i(t)) \le -\sum_{i=1}^{n}\beta_i k_i^2 z_i^2(t). \tag{10}$$

Similar to (8), we also notice the following inequalities

$$2\sum_{i=1}^{n}\sum_{j=1}^{n}p_i b_{ij} g_i(z_i(t)) g_j\big(z_j\big(t-\tau_{ij}(t)\big)\big) \le \frac{\delta}{n}\sum_{i=1}^{n}\sum_{j=1}^{n}p_i g_i^2(z_i(t)) + \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{np_i}{\delta}b_{ij}^2 g_j^2\big(z_j\big(t-\tau_{ij}(t)\big)\big)$$

$$\le \sum_{i=1}^{n}\delta p_M k_i^2 z_i^2(t) + \frac{p_M}{\delta}\|\hat{B}\|_1^2 \sum_{i=1}^{n}\Sigma_{j=1}^{n} g_i^2\big(z_i\big(t-\tau_{ij}(t)\big)\big). \tag{11}$$

Substituting (7)–(11) to (6) yields

$$\dot{V}(t) \le \sum_{i=1}^{n}(-2c_i + \rho + \frac{1}{\rho}\alpha_i k_i^2 + p_M) z_i^2(t) + \frac{1}{p_m}\|B\|_1^2 \sum_{i=1}^{n}\Sigma_{j=1}^{n} g_i^2\big(z_i\big(t-\tau_{ij}(t)\big)\big)$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\mu}{1-\eta}z_i^2(t) - \mu\sum_{i=1}^{n}\sum_{j=1}^{n}z_i^2\big(t-\tau_{ij}(t)\big) - 2\sum_{i=1}^{n}p_i \underline{c}_i \underline{k}_i z_i^2(t) - \sum_{i=1}^{n}\beta_i k_i^2 z_i^2(t)$$

$$+ \sum_{i=1}^{n}\delta p_M k_i^2 z_i^2(t) + \frac{p_M}{\delta}\|\hat{B}\|_1^2 \Sigma_{i=1}^{n}\Sigma_{j=1}^{n} g_i^2\big(z_i\big(t-\tau_{ij}(t)\big)\big) + \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon \cdot k_i^2 z_i^2(t) \tag{12}$$

By choosing $\mu = \frac{1}{p_m}\|\hat{B}\|_1^2\|L\|_2^2 + \frac{p_M}{\delta}\|\hat{B}\|_1^2\|L\|_2^2$, $L = \mathrm{diag}(k_i > 0)$, then the above inequality (12) can be written as

$$\dot{V}(t) \le \sum_{i=1}^{n}(-2c_i + \rho + \frac{1}{\rho}\alpha_i k_i^2 + p_M + \frac{n\mu}{1-\eta} - 2p_i \underline{c}_i \underline{k}_i - \beta_i k_i^2 + \delta p_M k_i^2) z_i^2(t) + \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon \cdot k_i^2 z_i^2(t) \le$$

$$-\sum_{i=1}^{n}\Phi_i z_i^2(t) + \sum_{i=1}^{n}n\varepsilon \cdot k_i^2 z_i^2(t) \le -(\Phi_m - n\varepsilon\|L\|_2^2)z(t)_2^2 \tag{13}$$

where $\Phi_i = 2\underline{c}_i - \rho - \frac{1}{\rho}\alpha_i k_i^2 - p_M - \frac{n\mu}{1-\eta} + 2p_i \underline{c}_i \underline{k}_i + \beta_i k_i^2 - \delta p_M k_i^2$ and $\Phi_m = \min\{\Phi_i\}$. Let $\varepsilon < \frac{\Phi_m}{n\|L\|_2^2}$ in (13). Since $\Phi_i > 0$, so $\dot{V}(z(t)$ is negative definite for all $z(t) \ne 0$. Now consider the case when $z(t) = 0$, noted that $z(t) = 0$ implies that $g(z(t)) = 0$. Then, substituting it into (6), $\dot{V}(z(t))$ is of the form

$$\dot{V}(t) \le -\sum_{i=1}^{n}\sum_{j=1}^{n}\mu z_i^2\big(t-\tau_{ij}(t)\big) - \sum_{i=1}^{n}\sum_{j=1}^{n}\varepsilon g_i^2\big(z_i(t-\tau_{ij}(t)\big)\big(1-\dot{\tau}_{ij}(t)\big) \tag{14}$$

From the above inequality, we conclude that $\dot{V}(z(t)) < 0$ if there exists at least one nonzero $g_i(z_i(t-\tau_{ij}(t))$, implying that $\dot{V}(t) = 0$ if and only if $z_i(t) = z_i\big(t-\tau_{ij}(t)\big) = g(z_i(t)) = g\big(z_i(t-\tau_{ij}(t)\big) = 0$ for all i, j, and $\dot{V}(z(t)) < 0$, otherwise. In addition, $V(z(t))$ is radially unbounded since $V(t) \to \infty$ as $\|z(t)\| \to \infty$. Hence, the origin of system (3), equivalently the equilibrium point, is globally asymptotically stable under the conditions given in Theorem 2. This completes the proof.

4. Comparisons and examples

We now consider the following example to compare our results with those previous results given above:

Example 1 Assume that the networks parameters of the delayed neural network (1) are given as follows:

$$\underline{A} = \begin{bmatrix} -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}, \ \bar{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} c_m & 0 & 0 & 0 \\ 0 & c_m & 0 & 0 \\ 0 & 0 & c_m & 0 \\ 0 & 0 & 0 & c_m \end{bmatrix},$$

$\eta = 0$ and $k_1 = k_2 = k_3 = k_4 = 1$.

The matrices $A^*, A_*, B^*, B_*, \hat{A}, \hat{B}$ are obtained as follows:

$$A^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A_* = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

$$B^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B_* = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \hat{B} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and we can calculate

$$\sigma_1(A) = \sqrt{\| |A^{*T}A^*| + 2|A^{*T}|A_* + A_*^T A_* \|_2} = 2$$

$$\sigma_2(A) = \|A^*\|_2 + \|A_*\|_2 = 2$$

$$\sigma_3(A) = \sqrt{\|A^*\|_2^2 + \|A_*\|_2^2 + 2\|A_*^T|A^*|\|_2} = 2$$

$$\sigma_4(A) = \|\hat{A}\|_2 = 2$$

$$\sigma_1(B) = \sigma_1(B) = \sigma_1(B) = \sigma_1(B) = 1.41$$

$$\|\hat{B}\|_1 = \max_{1 \leq i \leq 4} \sum_{j=1}^{4} \left| |\hat{b}_{ji}| \right| = 1, \|\hat{B}\|_\infty = \max_{1 \leq i \leq 4} \sum_{j=1}^{4} \left| \hat{b}_{ij} \right| = 2$$

It implies that $\sigma_m(A) = 2, \sigma_m(B) = 1.41$.

Now let us apply the result of Theorem 1 to this example. In this case, we first note that

$$P = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ and we can calculate

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 4, P_M = 2, P_m = 1, \beta_1 = -8, \beta_2 = -7, \beta_3 = -4, \beta_4 = -5, \mu = 3.$

Let $\rho = 2, \delta = 1$, the parameters of this example Φ_i in Theorem 1 can be calculated as

$\Phi_1 = 6c_m - 19, \Phi_2 = 6c_m - 18,$

$\Phi_3 = 4c_m - 13, \Phi_4 = 4c_m - 14,$

in which the robust stability conditions of Theorem 1 are satisfied if $c_m > 3.5$.

In the case of applying the result of Theorem 11 in Xie et al. (2014) for this example, we obtain

$$\Psi = \begin{bmatrix} 4c_m - 12.32 & 0 & 0 & 0 \\ 0 & 4c_m - 12.32 & 0 & 0 \\ 0 & 0 & 2c_m - 12.32 & 0 \\ 0 & 0 & 0 & 2c_m - 12.32 \end{bmatrix}$$

from which it can be calculated that $\Psi > 0$ if and only if $c_m > 6.16$, meaning that the sufficient condition for robust stability is obtained when $c_m > 6.16$. In order to compare the result of Theorem 12 in Xie et al. (2014) for this example, we first get

$$S = \begin{bmatrix} -4 & -2 & 0 & -2 \\ -2 & -4 & -1 & 0 \\ 0 & -1 & -2 & -1 \\ -2 & 0 & -1 & -2 \end{bmatrix}$$, the matrix Θ is obtained in the form of

$$\Theta = \begin{bmatrix} 4c_m - 9.64 & -2 & 0 & -2 \\ -2 & 4c_m - 9.64 & -1 & 0 \\ 0 & -1 & 2c_m - 7.64 & -1 \\ -2 & 0 & -1 & 2c_m - 7.64 \end{bmatrix}$$

The choice $c_m > 4.52$ implies that $\Theta > 0$ which guarantees the global robust stability of system (1). When checking the constraint condition of Theorem 2 in Ozcan and Arik (2014) for the network parameters of this example, we obtain $\zeta_i = 4c_m - 14, \zeta_i = 4c_m - 13, \zeta_i = 2c_m - 10, \zeta_i = 2c_m - 11$. Hence, for the network parameter of this example, we note that the conditions of Theorem 2 in Ozcan and Arik (2014) are satisfied if $c_m > 5.5$. In the case of applying the result of Theorem 2 in (Faydasicok and Arik (2013), we obtain $\in = c_m - 4$ in which the robust stability conditions are satisfied if $c_m > 4$.

5. Conclusion
This paper has studied the robust stability problem of neural networks with time-varying delays. An appropriate Lyapunov functional is constructed to derive sufficient conditions to ensure their globally asymptotic stability. The obtained results cannot only be used to testify the dynamic behaviors of the equilibrium point, but also generalize the existing results.

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