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Marichev-Saigo-Maeda fractional calculus operators, Srivastava polynomials and generalized Mittag-Leffler function

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Abstract: The aim of this paper is to evaluate four theorems for generalized fractional integral and derivative operators, applied on the product of Srivastava polynomials and generalized Mittag-Leffler function. The results are expressed in terms of generalized Wright function. Further, we also point out their relevance with the known results.

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PUBLIC INTEREST STATEMENT

The Mittag-Leffler functions are very useful almost in all areas of applied Mathematics, that provides solutions to a number of problems formulated in terms of fractional order differential, integral and difference equations; therefore, it has recently become a subject of interest for many authors in the field of fractional calculus and its applications. In this paper, we have evaluated four theorems for generalized fractional integral and derivative operators, applied on the product of Srivastava polynomials and generalized Mittag-Leffler function and also point out their relevance with the known results.

1. Introduction

The Mittag-Leffler functions are important special functions, that provides solutions to number of problems formulated in terms of fractional order differential, integral and difference equations; therefore, it has recently become a subject of interest for many authors in the field of fractional calculus and its applications. For detailed account of fractional calculus operators along with their properties and applications, one may refer to the research monographs by Kilbas, Srivastava, and Trujillo (2006), Kiryakova (1994), Miller and Ross (1993), Srivastava and Saigo (1987), Srivastava and Saxena (2001) and recent papers Mishra and Agarwal (2016), Mishra, Agarwal, and Sen (2016), Mishra and Sen (2016), Mishra, Srivastava, and Sen (2016), Purohit (2013) and Purohit and Kalla (2011).

The Swedish mathematician Mittag-Leffler (1903) introduced the function $E_\alpha(z)$, defined by:

$$E_\alpha(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+1)}z^n,\quad (\alpha\in\mathbb{C});\Re(\alpha)>0 \tag{1}$$

A further, two-index generalization of this function was studied by Wiman (1905) as:

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+\beta)}z^n,\quad (\alpha,\beta\in\mathbb{C}) \tag{2}$$

where $\Re(\alpha)>0$ and $\Re(\beta)>0$.

Prabhakar (1971) introduced the generalization of Mittag-Leffler function $E^{\delta}_{\beta,\gamma}(z)$ in the form

$$E^{\delta}_{\beta,\gamma}(z)=\sum_{n=0}^{\infty}\frac{(\delta)_n}{\Gamma(\beta n+\gamma)n!}z^n, \tag{3}$$

where $\beta,\ \gamma,\ \delta\in\mathbb{C},\ \Re(\alpha)>0$. Further, it is an entire function of order $\left[Re(\beta)\right]^{-1}$ (see Prabhakar, 1971, p. 7).

Shukla and Prajapati (2007) (see also Srivastava & Tomovski, 2009) defined and investigated the function $E^{\gamma,q}_{\alpha,\beta}(z)$ as

$$E^{\gamma,q}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n\,!}, \tag{4}$$

where $\alpha,\ \beta,\ \gamma,\ \delta\in\mathbb{C},\Re(\alpha)>0,\ \Re(\beta)>0,\ \Re(\gamma)>0, q\in(0,\ 1)U\mathbb{N}$ and $(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}$ denotes the generalized Pochhammer symbol, which in particular reduces to

$$q^{qn}\prod_{r=1}^{q}\left(\frac{\gamma+r-1}{q}\right)_n.$$

It is remarked that certain much more general functions of the Mittag-Leffler type have already been investigated in the literature rather systematically and extensively, but for the purpose of this paper we use the function given by (4) only.

The generalized Wright function ${}_p\psi_q(z)$ defined for $z\in\mathbb{C}$ $a_i,\ b_j\in\mathbb{C}$, and $A_i,\ B_j\in\Re(A_i,\ B_j\neq 0;$ $i=1,\ 2,\ \dots,\ p;\ j=1,\ 2,\ \dots,q)$ is given by the series

$${}_p\psi_q(z)={}_p\psi_q\left[\begin{array}{c}(a_i,\ A_i)_{1,p}\\ (b_j,\ B_j)_{1,q}\end{array}|z\right]=\sum_{k=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+A_ik)\,z^k}{\prod_{j=1}^{q}\Gamma(b_j+B_jk)\,k!}, \tag{5}$$

where $\Gamma(z)$ is the Euler gamma function and the function (5) was introduced by Wright (1935) and is known as generalized Wright function, for all values of the argument z, under the condition:

$$\sum_{j=1}^{q} B_j - \sum_{i=1}^{p} A_i > -1. \tag{6}$$

For detailed study of various properties, generalization and application of Wright function and generalized Wright function, we refer to paper (for instance, see Wright, 1935, 1940, 1940).

The Srivastava polynomials defined by Srivastava (1968, p. 1, Equation (1)) in the following manner:

$$S_w^u[x] = \sum_{s=0}^{[w/u]} \frac{(-w)_{u.s}}{s!} A_{w.s}\, x^s, \quad w = 0, 1, 2, \ldots \tag{7}$$

where u is an arbitrary positive integer and the coefficients $A_{w.s}(w, s) \geq 0$ are arbitrary constants, real or complex.

On account of success of the Saigo operators (Saigo, 1978, 1979), in their study on various function spaces and their application in the integral equation and differential equations, Saigo and Maeda (1998) introduced the following generalized fractional and differential operators of any complex order with Appell function $F_3(\cdot)$ in the kernel, as follows:

Let $\alpha, \alpha', \beta, \beta', \gamma \in \mathbb{C}$ and $x > 0$, then the generalized fractional calculus operators (the Marichev-Saigo-Maeda operators) involving the Appell function, or Horn's F_3-function are defined by the following equations:

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \frac{x^{-\alpha}}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} t^{-\alpha'} \times F_3\left(\alpha, \alpha', \beta, \beta'; \gamma; 1-\frac{t}{x}, 1-\frac{x}{t}\right) f(t)dt, \ (\Re(\gamma) > 0), \tag{8}$$

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^k \left(I_{0+}^{\alpha,\alpha',\beta+k,\beta',\gamma+k} f\right)(x), \quad \left(\Re(\gamma) \leq 0; k = [-\Re(\gamma)+1]\right); \tag{9}$$

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty (t-x)^{\gamma-1} t^{-\alpha} \times F_3\left(\alpha, \alpha', \beta, \beta'; \gamma; 1-\frac{x}{t}, 1-\frac{t}{x}\right) f(t)dt, \quad (\Re(\gamma) > 0), \tag{10}$$

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(-\frac{\mathrm{d}}{\mathrm{d}x}\right)^k \left(I_{-}^{\alpha,\alpha',\beta,\beta'+k,\gamma+k} f\right)(x), \quad \left(\Re(\gamma) \leq 0; k = [-\Re(\gamma)+1]\right); \tag{11}$$

and

$$\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{0+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x) = \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^k \left(I_{0+}^{-\alpha',-\alpha,-\beta'+k,-\beta=\gamma+k} f\right)(x), \quad \left(\Re(\gamma) > 0; k = [\Re(\gamma)+1]\right); \tag{12}$$

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x) = \left(-\frac{\mathrm{d}}{\mathrm{d}x}\right)^k \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k} f\right)(x), \qquad \left(\Re(\gamma) > 0; k = [\Re(\gamma)+1]\right). \tag{13}$$

For the definition of the Appell function $F_3(\cdot)$ the interested reader may refer to the monograph by Srivastava and Karlsson (1985) (see Erdélyi, Magnus, Oberhettinger, and Tricomi (1953), Prudnikov, Brychkov, and Marichev (1992) and Samko, Kilbas, and Marichev (1993)).

Following Saigo and Maeda (1998), the image formulas for a power function, under operators (8) and (10), are given by:

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1}\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1} \times \Gamma\left[\begin{array}{c} \rho,\ \rho+\gamma-\alpha-\alpha'-\beta,\ \rho+\beta'-\alpha' \\ \rho+\beta',\ \rho+\gamma-\alpha-\alpha',\ \rho+\gamma-\alpha'-\beta \end{array}\right], \tag{14}$$

where $\Re(\rho) > \max\left\{0,\ \Re(\alpha+\alpha'+\beta-\gamma),\ \Re(\alpha'-\beta')\right\}$ and $\Re(\gamma) > 0$.

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} x^{\rho-1}\right)(x) = x^{\rho+\gamma-\alpha-\alpha'-1} \times \frac{\Gamma\left(1-\rho-\gamma+\alpha+\alpha'\right)\Gamma(1-\rho+\alpha+\beta'-\gamma)\Gamma(1-\rho-\beta)}{\Gamma(1-\rho)\Gamma(1-\rho+\alpha+\alpha'+\beta'-\gamma)\Gamma(1-\rho+\alpha-\beta)}, \tag{15}$$

where $\Re(\gamma) > 0,\ \Re(\rho) < 1+\min\left\{\Re(-\beta),\ \Re\left(\alpha+\alpha'-\gamma\right),\ \Re\left(\alpha+\beta'-\gamma\right)\right\}$.

Here, we used the symbol $\Gamma\left[\begin{array}{c}\cdots\\ \cdots\end{array}\right]$ representing the fraction of many Gamma functions.

The computations of fractional integrals and fractional derivatives of special functions of one and more variables are important from the point of view of the usefulness of these results in the evaluation of generalized integrals and generalized derivatives and the solution of differential and integral equations (for example see Baleanu, Kumar, and Purohit (2016), Kumar, Purohit, and Choi (2016), Nisar, Purohit, Abouzaid, Qurashi, and Baleanu (2016), Purohit, Kalla, and Suthar (2011), Purohit, Suthar, and Kalla (2012), Srivastava (1972, 2016), Suthar, Parmar, and Purohit, (2017), Tomovski, Hilfer, and Srivastava (2010), Tomovski, Pogány, and Srivastava (2014)). Motivated by these avenues of applications, here we establish four image formulas for the generalized Mittag-Leffler function (4), involving left- and right-sided operators of Marichev-Saigo-Meada fractional integral operators and fractional derivatives, in term of the generalized Wright function.

2. Main results

Throughout this paper, we assume that $a,\ \alpha,\ \alpha',\ \beta,\ \beta',\ \gamma,\ \delta,\ \rho,\ \mu,\ \eta \in \mathbb{C},\ \ \lambda > 0,$ such that $\Re(\delta) > 0,\ \Re(\mu) > 0,\ \Re(\eta) > 0,\ q \in (0,\ 1) \bigcup \mathbb{N}$. Further, let the constants satisfy the condition $a_i,\ b_j \in \mathbb{C}$, and $A_i,\ B_j \in \mathbb{R}(A_i,\ B_j \neq 0;i = 1,\ 2,\ \ldots,\ p;j = 1,\ 2,\ \ldots,\ q)$, such that the condition (6) is also satisfied.

2.1. Left-sided generalized fractional integration of product of polynomial and generalized Mittag-Leffler function

In this section, we establish image formulas for the product of Srivastava polynomial and generalized Mittag-Leffler function involving left-sided operators of Marichev-Saigo-Meada fractional integral operators (8), in term of the generalized Wright function. These formulas are given by the following theorems:

THEOREM 2.1 *Let* $\Re(\gamma) > 0,\ \Re(\lambda) > 0,\ \Re(\rho) > \max\left[0,\ \Re(\alpha+\alpha'+\beta-\gamma),\ \Re(\alpha'-\beta')\right]$, *then the generalized fractional integration* $I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ *of the product of generalized Mittag-Leffler function* $E_{\delta,\mu}^{\eta,q}(\cdot)$, *and* $S_n^m(.)$ *is given by*

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right) E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{\Gamma(\eta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}$$
$$\times A_{n,s}\left(\sigma x^{\xi}\right)^s {}_4\Psi_4\left[\begin{matrix}(\rho+\gamma-\alpha-\alpha'-\beta+\xi s,\ \lambda),\ (\rho+\beta'-\alpha'+\xi s,\ \lambda),\ (\rho+\xi s,\ \lambda),\ (\eta,\ q)\\ (\rho+\gamma-\alpha'-\beta+\xi s,\ \lambda),\ (\rho+\gamma-\alpha-\alpha'+\xi s,\ \lambda),\ (\rho+\beta'+\xi s,\ \lambda),\ (\mu,\ \delta)\end{matrix}\Bigg|ax^{\lambda}\right]. \tag{16}$$

Proof On using (4) and (7), writing the function in the series form, the left-hand side of (16), leads to

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}$$
$$\times A_{n,s}\left(\sigma t^{\xi}\right)^s\sum_{k=0}^{\infty}\frac{(\eta)_{qk}}{\Gamma(\mu+\delta k)\,k!}\left(at^{\lambda}\right)^k\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-\alpha-\alpha'+\gamma-1}\right)\right)(x), \tag{17}$$

Now, upon using the image formula (14), which is valid under the conditions stated with Theorem 2.1, we get

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right) E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)$$
$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma x^{\xi}\right)^s\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{\Gamma(\eta)}\sum_{k=0}^{\infty}\frac{\Gamma(\rho+\gamma-\alpha-\alpha'-\beta+\xi s+\lambda k)}{\Gamma(\rho+\gamma-\alpha'-\beta+\xi s+\lambda k)}$$
$$\times\frac{\Gamma(\rho+\beta'-\alpha'+\xi s+\lambda k)\,\Gamma(\rho+\xi s+\lambda k)\,\Gamma(\eta+qk)}{\Gamma(\rho+\gamma-\alpha-\alpha'+\xi s+\lambda k)\,\Gamma(\rho+\beta'+\xi s+\lambda k)\,\Gamma(\mu+\delta k)}\frac{\left((ax)^{\lambda}\right)^k}{k!}, \tag{18}$$

Interpreting the right-hand side of the above equation, in view of the definition (5), we arrive at the result (16).

On setting $n=0$, $A_{0,0}=1$ then $S_0^m[x]\to 1$ in (16), we obtained the following particular case of Theorem 2.1:

Corollary 2.1 *Let the conditions of Theorem 2.1 are satisfied, then the following formula holds ture*

$$\left(I_{0+}^{\alpha,\alpha',\beta\beta',\gamma}\left(t^{\rho-1}E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{\Gamma(\eta)}$$
$$\times_4\Psi_4\left[\begin{matrix}(\rho+\gamma-\alpha-\alpha'-\beta,\ \lambda),\ (\rho+\beta'-\alpha',\ \lambda),\ (\rho,\ \lambda),\ (\eta,\ q)\\ (\rho+\gamma-\alpha'-\beta,\ \lambda),\ (\rho+\gamma-\alpha-\alpha',\ \lambda),\ (\rho+\beta',\ \lambda),\ (\mu,\ \delta)\end{matrix}\Bigg|ax^{\lambda}\right]. \tag{19}$$

Remark 1 If we set $q=1$, in Corollary 2.1 , we arrive at the known result given by Chouhan, Khan, and Saraswat (2014, Equation (13)).
Now, we present some special cases of (19) as below:
For $\alpha=\alpha+\beta$, $\alpha'=\beta'=0$, $\beta=-\tau$, $\gamma=\alpha$, we obtain the following relationship

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\right)(x)=\left(I_{0+}^{\alpha,\beta,\tau}f\right)(x), \tag{20}$$

where the operator $I_{0+}^{\alpha,\beta,\tau}(\cdot)$ denotes the Saigo fractional integral operator (Saigo, 1978), which is defined by

$$\left(I_{0+}^{\alpha,\beta,\tau}f\right)(x)=\frac{x^{-\alpha-\tau}}{\Gamma(\alpha)}\int_0^x(x-t)_2^{\alpha-1}F_1(\alpha+\tau,\ -\eta;\alpha;1-t\,x)f(t)dt,\quad \Re(\alpha)>0. \tag{21}$$

Corollary 2.2 *Let* $\Re(\gamma)>0$, $\Re(\upsilon)>0$, $\Re(\rho)>\max\left[0,\ \Re(\beta-\tau)\right]$, *then there hold the following formula:*

$$\left(I_{0+}^{\alpha,\beta,\tau}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\frac{x^{\rho-\beta-1}}{\Gamma(\eta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma x^{\xi}\right)^s \times{}_3\psi_3\left[\begin{matrix}(\rho-\beta+\tau+\xi s,\ \lambda),\ (\rho+\xi s,\ \lambda),\ (\eta,\ q)\\ (\rho+\alpha+\tau+\xi s,\ \lambda),\ (\rho-\beta+\xi s,\ \lambda),\ (\mu,\ \delta)\end{matrix}\ \Bigg|ax^{\lambda}\right]. \tag{22}$$

Remark 2 If we set $q=1$, $\tau=\gamma$ and $n=0$, $A_{0,0}=1$ then $S_0^m[x]\to 1$ in Corollary 2.2, we arrive at the known result given by Ahmed (2014, Equation (3.1)).

2.2. Right-sided generalized fractional integration of product of polynomial and generalized Mittag-Leffler function

In this part, we establish image formulas for the product of Srivastava polynomial and generalized Mittag-Leffler function involving right-sided operators of Marichev-Saigo-Meada fractional integral operators (10), in term of the generalized Wright function. These formulas are given by the following theorems:

THEOREM 2.2 *For* $\Re(\gamma)>0$, $\Re(1-\gamma-\rho)<1+\min\left[\Re(-\beta),\ \Re(\alpha+\alpha'-\gamma),\ \Re(\alpha+\beta'-\gamma)\right]$, *we have*

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\frac{x^{-\rho-\alpha-\alpha'}}{\Gamma(\eta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!} \times A_{n,s}\left(\sigma x^{\xi}\right)^s{}_4\psi_4\left[\begin{matrix}(\alpha+\alpha'+\rho-\xi s,\ \lambda),\ (\alpha+\beta'+\rho-\xi s,\ \lambda),\ (\rho-\beta+\gamma-\xi s,\ \lambda),(\eta,\ q)\\ (\mu,\ \delta),\ (\alpha+\alpha'+\beta'+\rho-\xi s,\ \lambda),\ (\alpha-\beta+\rho+\gamma-\xi s,\ \lambda),\ (\rho+\gamma-\xi s,\ \lambda)\end{matrix}\ \Bigg|ax^{-\lambda}\right]. \tag{23}$$

Proof On using (4) and (7), the left-hand side of (23), can be written as:

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!} \times A_{n,s}\left(\sigma t^{\xi}\right)^s\sum_{k=0}^{\infty}\frac{(\eta)_{qk}}{\Gamma(\mu+\delta k)}\left(at^{-\lambda}\right)^k\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\rho-\alpha-\alpha'}\right)\right)(x), \tag{24}$$

which on using the image formula (15), arrive at

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x) =\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma x^{\xi}\right)^s\frac{x^{-\rho-\alpha-\alpha'}}{\Gamma(\eta)}\sum_{k=0}^{\infty}\frac{\Gamma(\alpha+\alpha'+\rho-\xi s+\lambda k)}{\Gamma(\alpha+\alpha'+\beta'+\rho-\xi s+\lambda k)} \times\frac{\Gamma(\alpha+\beta'+\rho-\xi s+\lambda k)\Gamma(\rho-\beta+\gamma-\xi s+\lambda k)\Gamma(\eta+qk)}{\Gamma(\alpha-\beta+\rho+\gamma-\xi s+\lambda k)\Gamma(\rho+\gamma-\xi s+\lambda k)\Gamma(\mu+\delta k)}\frac{\left((ax)^{-\lambda}\right)^k}{k!}, \tag{25}$$

Interpreting the right-hand side of the above equation, in view of the definition (5), we arrive at the result (23).

On setting $n=0$, $A_{0,0}=1$ then $S_0^m[x]\to 1$ in (23), we obtained the following particular case of Theorem 2.2.

COROLLARY 2.3 *The generalized fractional integration of generalized Mittag-Leffler function* $E_{\delta,\mu}^{\eta,q}(\cdot)$, *is given by*

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\frac{x^{-\rho-\alpha-\alpha'}}{\Gamma(\eta)} \times{}_4\psi_4\left[\begin{matrix}(\alpha+\alpha'+\rho,\ \lambda),\ (\alpha+\beta'+\rho,\ \lambda),\ (\rho-\beta+\gamma,\ \lambda),\ (\eta,\ q)\\ (\mu,\ \delta),\ (\alpha+\alpha'+\beta'+\rho,\ \lambda),\ (\alpha-\beta+\rho+\gamma,\ \lambda),\ (\rho+\gamma,\ \lambda)\end{matrix}\ \Bigg|ax^{-\lambda}\right],$$

provided $\Re(\gamma) > 0$, $\Re(1-\gamma-\rho) < 1 + \min\left[\Re(-\beta), \Re(\alpha+\alpha'-\gamma), \Re(\alpha+\beta'-\gamma)\right]$.

Remark 3 If we set $q = 1$, in Corollary 2.3, we arrive at the known result given by Chouhan et al. (2014, Equation (15)).
When we let $\alpha = \alpha+\beta$, $\alpha' = \beta' = 0$, $\beta = -\tau$, $\gamma = \alpha$, then we obtain the relationship

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\right)(x) = \left(I_{-}^{\alpha,\beta,\tau} f\right)(x), \tag{26}$$

where the Saigo fractional integral operator (Saigo, 1978) is defined by

$$\left(I_{-}^{\alpha,\beta,\tau} f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} t_2^{-\alpha-\beta} F_1(\alpha+\beta, -\tau; \alpha; 1-x\,t) f(t)dt. \tag{27}$$

COROLLARY 2.4 *If* $\Re(\alpha) > 0$, $\Re(\lambda) > 0$, $\Re(1-\gamma-\rho) < 1 + \min\left[\Re(-\beta), \Re(-\tau)\right]$, *then we have*

$$\begin{aligned}\left(I_{-}^{\alpha,\beta,\tau}\left(t^{-\gamma-\rho} S_n^m\left(\sigma\, t^{\xi}\right) E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x) &= \frac{x^{-\rho-\alpha-\beta}}{\Gamma(\eta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{m,s}}{s!} \\ &\times A_{n,s}\left(\sigma x^{\xi}\right)^s {}_3\Psi_3\left[\begin{matrix}(\alpha+\beta+\rho-\xi s,\, \lambda),\, (\rho+\tau+\alpha-\xi s,\, \lambda),\, (\eta,\, q) \\ (\mu,\, \delta),\, (2\alpha+\beta+\tau+\rho-\xi s,\, \lambda),\, (\rho+\alpha-\xi s,\, \lambda)\end{matrix}\,\middle|\, ax^{-\lambda}\right].\end{aligned} \tag{28}$$

Remark 4 If we set $q = 1$, $\tau = \gamma$ and $n = 0$, $A_{0,0} = 1$ then $S_0^m[x] \to 1$ in Corollary 2.4, we arrive at the known result given by Ahmed (2014, Equation (4.1)).

2.3. Left-sided generalized fractional differentiation of product of polynomial and generalized Mittag-Leffler function

Now, we shall establish image formulas for the product of Srivastava polynomial and generalized Mittag-Leffler function involving left-sided operators of Marichev-Saigo-Meada fractional differentiation operators (12) in term of the generalized Wright function. These formulas are given by the following theorems:

THEOREM 2.3 *The generalized fractional differentiation* $D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ *of the product of generalized Mittag-Leffler function* $E_{\delta,\mu}^{\eta,q}(\cdot)$ *and Srivastava polynomials* $S_n^m(\cdot)$ *is given by*

$$\begin{aligned}\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1} S_n^m\left(\sigma t^{\xi}\right) E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x) &= \frac{x^{\rho+\alpha+\alpha'-\gamma-1}}{\Gamma(\eta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{m,s}}{s!} \\ &\times A_{n,s}\left(\sigma x^{\xi}\right)^s {}_4\Psi_4\left[\begin{matrix}(\rho-\gamma+\alpha+\alpha'+\beta'+\xi s,\, \lambda),\, (\rho-\beta+\alpha+\xi s,\, \lambda),\, (\rho+\xi s,\, \lambda),\, (\eta,\, q) \\ (\rho-\gamma+\alpha+\beta'+\xi s,\, \lambda),\, (\rho-\gamma+\alpha+\alpha'+\xi s,\, \lambda),\, (\rho-\beta+\xi s,\, \lambda),\, (\mu,\, \delta)\end{matrix}\,\middle|\, ax^{\lambda}\right],\end{aligned} \tag{29}$$

where $\Re(\gamma) > 0$, $\Re(\lambda) > 0$, $\Re(\rho) > \max\left[0, \Re(\gamma-\alpha-\alpha'-\beta'), \Re(\beta-\alpha)\right]$.

Proof On using (4) and (7), writing the function in the series form, the left-hand side of (29), leads to

$$\begin{aligned}\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1} S_n^m\left(\sigma\, t^{\xi}\right) E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x) &= \sum_{s=0}^{[n/m]} \frac{(-n)_{m,s}}{s!} A_{n,s}\left(\sigma\, t^{\xi}\right)^s \\ &\times \sum_{k=0}^{\infty} \frac{(\eta)_{qk}}{\Gamma(\mu+\delta k)\, k!} \left(at^{\lambda}\right)^k \left(I_{0,+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}\left(t^{\rho-\alpha-\alpha'+\gamma-1}\right)\right)(x),\end{aligned} \tag{30}$$

Now, upon using the image formula (14), which is valid under the conditions stated with Theorem 2.3, we get

$$\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}$$
$$\times A_{n,s}\left(\sigma x^{\xi}\right)^{s}\frac{x^{\rho-\alpha-\alpha'+\gamma-1}}{\Gamma(\eta)}\sum_{k=0}^{\infty}\frac{\Gamma(\rho-\gamma+\alpha+\alpha'+\beta'+\xi s+\lambda k)}{\Gamma(\rho+\gamma+\alpha+\beta'+\xi s+\lambda k)} \tag{31}$$
$$\times\frac{\Gamma(\rho-\beta+\alpha+\xi s+\lambda k)\Gamma(\rho+\xi s+\lambda k)\Gamma(\eta+qk)}{\Gamma(\rho+\gamma+\alpha+\alpha'+\xi s+\lambda k)\Gamma(\rho-\beta+\xi s+\lambda k)\Gamma(\mu+\delta k)}\frac{\left((ax)^{\lambda}\right)^{k}}{k!},$$

Interpreting the right-hand side of the above equation, in view of the definition (5), we arrive at the result (29).

On setting $n=0$, $A_{0,0}=1$ then $S_0^m[x]\rightarrow 1$ in (29), we obtained the following particular case of Theorem 2.3.

COROLLARY 2.5 *Under the conditions* $\Re(\gamma)>0, \Re(\lambda)>0$ *and* $\Re(\rho)>$ *max* $\left[0, \Re(\gamma-\alpha-\alpha'-\beta'), \Re(\beta-\alpha)\right]$, *the following formula holds*

$$\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\frac{x^{\rho+\alpha+\alpha'-\gamma-1}}{\Gamma(\eta)}$$
$$\times{}_4\Psi_4\left[\begin{matrix}(\rho-\gamma+\alpha+\alpha'+\beta',\lambda),(\rho-\beta+\alpha,\lambda),(\rho,\lambda),(\eta,q)\\(\rho-\gamma+\alpha+\beta',\lambda),(\rho-\gamma+\alpha+\alpha',\lambda),(\rho-\beta,\lambda),(\mu,\delta)\end{matrix}\Bigg|ax^{\lambda}\right]. \tag{32}$$

Now, we present one more special case of (29), by making use of identity (20), as given below:

COROLLARY 2.6 *The following generalized fractional differentiation formula holds*

$$\left(D_{0+}^{\alpha,\beta,\tau}\left(t^{\rho-1}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{\lambda}\right]\right)\right)(x)=\frac{x^{\rho+\alpha+\alpha'-\gamma-1}}{\Gamma(\eta)}$$
$$\times\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma x^{\xi}\right)^{s}{}_3\Psi_3\left[\begin{matrix}(\rho+\alpha+\beta+\tau+\xi s,\lambda),\rho+\xi s,\lambda),(\eta,q)\\(\rho+\beta+\xi s,\lambda),(\rho+\tau+\xi s,\lambda),(\mu,\delta)\end{matrix}\Bigg|ax^{\lambda}\right], \tag{33}$$

where $\Re(\gamma)>0$, $\Re(\upsilon)>0$ and $\Re(\rho)>\max\left[0,\Re(\beta-\tau)\right]$.

Remark 5 If we set $q=1$, $\tau=\gamma$ and $n=0$, $A_{0,0}=1$ then $S_0^m[x]\rightarrow 1$ in Corollary 2.6, we arrive at the known result given by Ahmed (2014, Equation (5.1)).

2.4. Right-sided generalized fractional differentiation of product of polynomial and generalized Mittag-Leffler function

Here, we establish image formulas for the product of Srivastava polynomials and generalized Mittag-Leffler function involving right-sided operators of Marichev-Saigo-Meada fractional differentiation operators (13) in term of the generalized Wright function. These results are given as follows:

THEOREM 2.4 *If* $\Re(\gamma)>0$, $\Re(1-\gamma-\rho)<1+\min\left[\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\right]$, *then we have*

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}S_n^m\left(\sigma t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\frac{x^{-\rho+\alpha+\alpha'}}{\Gamma(\eta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}$$
$$\times A_{n,s}\left(\sigma x^{\xi}\right)_4^s\Psi_4\left[\begin{matrix}(\rho-\alpha-\alpha'-\xi s,\lambda),(\rho-\beta-\alpha'-\xi s,\lambda),(\rho+\beta'-\gamma-\xi s,\lambda),(\eta,q)\\(\mu,\delta),(\rho-\alpha-\alpha'-\beta-\xi s,\lambda),(\rho-\alpha'+\beta'-\gamma-\xi s,\lambda),(\rho-\gamma-\xi s,\lambda)\end{matrix}\Bigg|ax^{-\lambda}\right]. \tag{34}$$

Proof By using (4) and (7), the left-hand side of (34), can be written as

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}S_n^m\left(\sigma\,t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma\,t^{\xi}\right)^s \times\sum_{k=0}^{\infty}\frac{(\eta)_{qk}}{\Gamma(\mu+\delta\,k)}\left(at^{-\lambda}\right)^k\left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}\left(t^{-\rho+\alpha+\alpha'}\right)\right)(x), \tag{35}$$

which on using the image formula (15), arrive at

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}S_n^m\left(\sigma\,t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x) =\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!}A_{n,s}\left(\sigma x^{\xi}\right)^s\frac{x^{-\rho+\alpha+\alpha'}}{\Gamma(\eta)}\sum_{k=0}^{\infty}\frac{\Gamma(\rho-\alpha-\alpha'-\xi\,s+\lambda k)}{\Gamma(\rho-\alpha-\alpha'-\beta-\xi\,s+\lambda k)} \times\frac{\Gamma(\rho-\alpha'-\beta-\xi\,s+\lambda k)\Gamma(\rho+\beta'-\gamma-\xi\,s+\lambda k)\,\Gamma(\eta+qk)}{\Gamma(\rho-\alpha'+\beta'-\gamma-\xi\,s+\lambda k)\,\Gamma(\rho-\gamma-\xi\,s+\lambda k)\,\Gamma(\mu+\delta\,k)}\frac{\left((a\,x)^{-\lambda}\right)^k}{k!}, \tag{36}$$

Interpreting the right-hand side of the above equation, in view of the definition (5), we arrive at the result (34).

Further, on setting $n=0$, $A_{0,0}=1$ then $S_0^m[x]\to 1$ in (34), we obtained the following particular case of Theorem 2.4.

Corollary 2.7 *Let the conditions of Theorem 2.4 are satisfied, then the following formula holds*

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\frac{x^{-\rho+\alpha+\alpha'}}{\Gamma(\eta)} \times{}_4\psi_4\left[\begin{array}{l}(\rho-\alpha-\alpha's,\,\lambda),\,(\rho-\beta-\alpha',\,\lambda),\,(\rho+\beta'-\gamma,\,\lambda),\,(\eta,\,q)\\(\mu,\,\delta),\,(\rho-\alpha-\alpha'-\beta s,\,\lambda),\,(\rho-\alpha'+\beta'-\gamma s,\,\lambda),\,(\rho-\gamma,\,\lambda)\end{array}\middle|\,a\,x^{-\lambda}\right]. \tag{37}$$

Now, by using the identity (26), we present certain special cases of (34), as given below:

Corollary 2.8 *The generalized fractional differentiation formula associated with the product of generalized Mittag-Leffler function and Srivastava polynomials, is given by*

$$\left(D_{-}^{\alpha,\beta,\tau}\left(t^{\gamma-\rho}S_n^m\left(\sigma\,t^{\xi}\right)E_{\delta,\mu}^{\eta,q}\left[at^{-\lambda}\right]\right)\right)(x)=\frac{x^{-\rho+\alpha+\alpha'}}{\Gamma(\eta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{m,s}}{s!} \times A_{n,s}\left(\sigma\,x^{\xi}\right)^s{}_3\psi_3\left[\begin{array}{l}(\rho-\alpha-\beta-\xi\,s,\,\lambda),\,(\rho+\tau-\xi\,s,\,\lambda),\,(\eta,\,q)\\(\mu,\,\delta),\,(\rho-\alpha-\beta-\xi\,s,\,\lambda),\,(\rho-\alpha-\xi\,s,\,\lambda)\end{array}\middle|\,a\,x^{-\lambda}\right], \tag{38}$$

provided $\Re(\gamma)>0$, $\Re(\lambda)>0$, $\Re(1-\gamma-\rho)<1+\min\left[\Re(-\beta),\,\Re(-\tau)\right]$.

Remark 6 Finally, if we set $q=1$, $\tau=\gamma$ and $n=0$, $A_{0,0}=1$, hence, $S_0^m[x]\to 1$ in Corollary 2.8, we arrive at the known result given by Ahmed (2014, Equation (6.1)).

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