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PURE MATHEMATICS | RESEARCH ARTICLE

Some properties of m -isoclinism and ID^* -derivations in Filippov algebras

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Abstract: In this paper, we introduce the concept of m -isoclinism of Filippov algebras. And then we present a new class of subalgebra of $\text{Der}(A)$ which is denoted by $ID_m^*(A)$, i.e. the set of all $\alpha \in \text{Der}(A)$, such that image of its image is contained in the $(m + 1)^{\text{th}}$ term of lower central series of A , and it maps central elements to 0. As a consequence, we show that if A and B are two m -isoclinism Filippov algebras, then $ID_m^*(A) \cong ID_m^*(B)$.

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1. Introduction and preliminaries

The notion of (n -Lie) Filippov algebra was introduced in 1985 by Filippov (1985). Let F be a field. A Filippov algebra over F is an F -vector space A , together with n -ary multilinear and skew-symmetric operation $[x_1, \dots, x_n]$, which satisfies the following generalized Jacobi identity

$$[[x_1, \dots, x_n], y_2, \dots, y_n] = \sum_{i=1}^n [x_1, \dots, [x_i, y_2, \dots, y_n], \dots, x_n].$$

ABOUT THE AUTHORS

Aslan Doosti is a PhD student of pure mathematics algebra branch. The doctor Farshid Saeedi is my supervisor and the professor Mohammad Reza R. Moghaddam and Mrs doctor Soodabeh Tajnia are advisor teachers. I passed theoretical courses in the group theory and Lie algebras. Then, at the suggestion of supervisor continued my research work in the concepts isoclinism, m -isoclinism and derivations of (n -Lie) Filippov algebras and provided a new definition for m -isoclinism in Filippov algebras. Farshid Saeedi published numerous papers in the field of the groups theory and Lie algebras, especially about the concept of isoclinism of them. he has been considered the concept of Filippov algebras in recent years. Also, professor M.R.R. Moghaddam and S. Tajnia research in the field of the groups theory and Lie algebras. We are going to investigate the obtained results in my article for n -ary algebras. Furthermore, we're going to examine new definitions of derivation in Filippov and other n -ary algebras.

PUBLIC INTEREST STATEMENT

Algebraic structure is among the most important topics in mathematics. There are various algebraic structures such as groups, rings, Lie algebras, Leibniz algebras, Filippov algebras, etc. For an optimal usage, such structures need to be classified. The group theory is an algebraic structure which has been previously applied to develop and create other structures. There are many concepts in group theory, such as homomorphism, isomorphism, isoclinism, substructures such as subgroups, subalgebras and derivations, which can be applied for classification. Algebraic concepts have many applications in other scientific fields. In particular, groups and Lie algebras are used in physical applications. For example, Lie algebras are applied in quantum physics and the physics of symmetry.

Clearly when $n = 2$, we have Jacobi identity in Lie algebra.

A subspace B of Filippov algebra A is called Filippov subalgebra, if for elements x_1, \dots, x_n in B , $[x_1, \dots, x_n] \in B$. Let A_1, A_2, \dots, A_n be subalgebra of Filippov algebra A , then the subalgebra $[A_1, A_2, \dots, A_n]$, generated by all vectors $[x_1, \dots, x_n]$, where $x_i \in A_i$, for $i = 1, 2, \dots, n$, is the derived algebra of A and denoted by A^2 . If $A^2 = 0$ then A is called an abelian Filippov algebra. The subspace I of Filippov algebra A is an ideal of A , if $[I, A, \dots, A] \subseteq I$. Note that, if $[I, I, A, \dots, A] = 0$, then I is called an abelian ideal. The center of A is defined by

$$Z(A) = \{x \in A; [x, A, \dots, A] = 0\}.$$

The lower and upper central series of Filippov algebra A , are defined as follows

$$\dots \subseteq A^{m+1} \subseteq A^m \subseteq \dots \subseteq A^2 \subseteq A^1 = A$$

and

$$(0) \subseteq Z(A) = Z_1(A) \subseteq Z_2(A) \subseteq \dots \subseteq Z_m(A) \subseteq \dots$$

respectively, where $A^{m+1} = [A, A, \dots, A^m]$, and $Z_{m+1}(A)/Z_m(A) = Z(A/Z_m(A))$.

Note that for $n \geq 2$ and $m \geq 1$, The subalgebra A^{m+1} is generated by elements of the form $[x_1, x_2, \dots, x_{(n-1)}, [x_n, \dots, x_{2(n-1)}, [\dots, [\dots, x_{(m-1)(n-1)}, [x_{(m-1)(n-1)+1}, \dots, x_{m(n-1)+1}] \dots]]]$. Thus, for convenience, we put $t = (n-1)m + 1$ and use the form $[x_1, x_2, \dots, x_{(n-1)}, x_t]$, where $x_1, x_2, \dots, x_{(n-1)} \in A$ and $x_t \in A^m$. The linear map $\phi: A \rightarrow B$ is called homomorphism of Filippov algebras, if ϕ satisfies

$$\phi([x_1, \dots, x_n]) = [\phi(x_1), \dots, \phi(x_n)].$$

Moneyhun (1994), introduced the isoclinism Lie algebras analogue of the concept and contrary to the group theory cases. She showed that when two Lie algebras have the same finite dimensions then the two concept of isoclinism and isomorphism coincide. Salemkar and Mirzaei (2010) introduced the notion of the equivalence relation, m -isoclinism between Lie algebras and obtain some criterion under which Lie algebras are m -isoclinic. In the paper Saeedi and Veisi (2014), introduced the concept of isoclinism in Filippov algebras (To get more information see, Bioch, 1976; Darabi & Saeedi 2017; Hekster, 1986; Kasymov, 1987; Parvaneh & Moghaddam, 2011).

In the following, we introduce the concept of m -isoclinism of Filippov algebras.

Definition Let A and B be two Filippov algebras. Then A and B are m -isoclinic, if there exist isomorphisms $\varphi: A/Z_m(A) \rightarrow B/Z_m(B)$ and $\theta: A^{m+1} \rightarrow B^{m+1}$, such that the following diagram is commutative.

$$\begin{array}{ccc} \underbrace{\frac{A}{Z_m(A)} \times \dots \times \frac{A}{Z_m(A)}}_{t\text{-times}} & \longrightarrow & A^{m+1} \\ \downarrow \varphi^t & & \downarrow \theta \\ \underbrace{\frac{B}{Z_m(B)} \times \dots \times \frac{B}{Z_m(B)}}_{t\text{-times}} & \longrightarrow & B^{m+1} \end{array}$$

where the rule of horizontal maps is $(\bar{x}_1, \dots, \bar{x}_t) \rightarrow [x_1, \dots, x_t]$. Also,

$\theta([x_1, \dots, x_t]) = [y_1, \dots, y_t]$, such that for all $0 \leq i \leq t$, $y_i \in \varphi(x_i + Z_m(A))$ and $t = (n - 1)m + 1$. In this case the pair (φ, θ) is called an m -isoclinic from A to B and is denoted by $A \sim_m B$.

By the above definition, it is easy to show that:

- If $m = 1$ and $n = 2$, then we have isoclinism of Lie algebra, that is presented by Moneyhun (1994).
- If $n = 2$ and m is a natural number, then the concept of m -isoclinism of Lie algebras accrued which is due to Salemkar and Mirzaie (2010).
- If $m = 1$ and $n \geq 2$, then we have the definition of isoclinism of Filippov algebras that is presented by Saeedi and Veisi (2014), for the first time.

In order to state our results, we shall need the following lemma, that proof is the same as for Lie algebra (Moneyhun, 1994).

LEMMA 1.1 Assume that A and B be two m -isoclinism Filippov algebras with pair (φ, θ) . Then for all $x \in A^{m+1}$,

- (i) $\varphi(x + Z_m(A)) = \theta(x) + Z_m(B)$.
- (ii) $\theta([x, x_2, \dots, x_t]) = [\theta(x), y_2, \dots, y_t]$, $x_i \in A$ and $y_i \in B$, for $i = 1, \dots, t$.

2. On ID_m^* -derivations of Filippov algebras

The notion of derivations and generalized derivations are very important in the study of n -ary algebras and many researchers have written articles about this notion. The concept of δ -derivations was introduced by Filippov (1998). This notion was developed in Lie, prime Lie and Malcev algebras by Filippov (1999, 2000) and simple, classical Lie and Jordan superalgebras were studied by Kaygorodov (2007, 2009, 2010), he described $(n + 1)$ -ary derivations of simple n -ary algebras and generalized derivations algebra of semisimple Filippov algebras over an algebraically closed field of characteristic zero in Kaygorodov (2011, 2014a, 2014b). Also, the concept of generalized derivations algebras of Lie algebras was introduced by Leger and Luks (2000), they show that the Quasi-derivation algebra of a Lie algebra can be embedded in to the derivation algebra of a larger Lie algebra. Quasi-derivations, derivation of Lie superalgebras, generalized derivations of color Lie algebra, Hom-Lie superalgebras and Ternary derivations were investigated in Shestakov (2012), Shestakov (2014), Leger and Luks (2000), Chen, Ma, and Ni (2013), Zhou and Fan (2016), Zhou, Chen, and Ma (2014) and Zhang and Zhang (2010). Saeedi and Sheikh-Mohseni (2016) and Sheikh-Mohseni, Saeedi, and Badrkhani-Asl (2015), defined a new subalgebra of set of all derivations of Filippov algebra A and denoted by $ID(A)$, that is the set of all derivations of A , such that its image is contained in the derived subalgebra of A . They studied the relationship between specific type of subalgebras of set of all derivations of Lie and Filippov algebras with concept of isoclinism.

In the following, we're going to generalize the results of Saeedi, Sheikh-Mohseni and Badrkhani-Asl to the new case of subalgebra of $Der(A)$ and study their relation with the concept of m -isoclinic.

Let A be a Lie algebra over an arbitrary field F . We denote $Der(A)$, the vector space of all derivations of A . For each $x \in A$, the map $ad_x: A \rightarrow A$, given by $ad_x(y) = [x, y]$ is a derivation called the inner derivation corresponding to x . The set of all inner derivations of A , denoted by $IDer(A)$, is an ideal of $Der(A)$. Also, denoted $Der_z(A)$, the set of all derivations of A mapping A into the center of A and $Der_c(A)$, the set of all derivation $\alpha \in Der(A)$ such that, $\alpha(x) \in [x, A]$. It should be remarked that, Sheikh-Mohseni et al. (2015), gave necessary and sufficient conditions under which $Der_c(A) = Der_z(A)$.

A derivation of a Filippov algebra A is a F -linear transformation $\alpha: A \rightarrow A$ satisfying in the following equation

$$\alpha([x_1, \dots, x_n]) = \sum_{i=1}^n [x_1, \dots, \alpha(x_i), \dots, x_n],$$

for all $x_1, \dots, x_n \in A$.

The set of all derivations under the following commutator operator

$$(\alpha_1, \alpha_2) \rightarrow [\alpha_1, \alpha_2] := (\alpha_1 \circ \alpha_2) - (\alpha_2 \circ \alpha_1)$$

is a Lie algebra and denoted by $\text{Der}(A)$. For $x_1, \dots, x_{(n-1)} \in A$, the map $ad_{x_1, \dots, x_{(n-1)}}: A \rightarrow A$, given by

$$ad_{x_1, \dots, x_{(n-1)}}(x_n) = [x_1, \dots, x_n]$$

is called a left multiplication defined by elements $x_1, \dots, x_{(n-1)}$, which is a derivation. The set of all linear combinations of left multiplications is denoted by $ad(A)$. Analogues to Lie algebra cases, we denote by $\text{Der}_z(A)$, the set of all central derivations of A any by $\text{Der}_c(A)$ the set of all derivations $\alpha \in \text{Der}(A)$, such that, $\alpha(x) \in [x, A, \dots, A]$.

Saeedi and Sheikh-Mohseni (2016), defined a new subalgebra of $\text{Der}(A)$ containing $\text{Der}_c(A)$ and denoted by $\text{ID}(A)$, i.e. the set of all derivations of A , such it's image is contained in the derived subalgebra of A . Also they denoted by $\text{ID}^*(A)$, the set of all $\alpha \in \text{ID}(A)$, such that it maps central elements to 0. Moreover they proved that, if A and B are two isoclinic Filippov algebras, then $\text{ID}^*(A) \cong \text{ID}^*(B)$ and $\text{Der}_c(A) \cong \text{Der}_c(B)$.

In this paper, we define two new subalgebras of $\text{Der}(A)$, The first subalgebra denoted by $\text{Der}_c^m(A)$, it is the set of all derivations of A , such that image α is contained in $[x, A, A, \dots, A^m]$, for all $x \in A$, i.e.

$$\text{Der}_c^m(A) = \{ \alpha \in \text{Der}(A) : \alpha(x) \in [x, A, A, \dots, A^m], \forall x \in A \}$$

and the second subalgebra denoted by $\text{ID}_m^*(A)$, the set of all $\alpha \in \text{Der}(A)$, such that image of it's contained in the $(m+1)$ th term of lower central series of A , and it maps central elements to 0. Moreover we proof that, if A and B are two m -isoclinic Filippov algebras, then we have $\text{ID}_m^*(A) \cong \text{ID}_m^*(B)$ and $\text{Der}_c^m(A) \cong \text{Der}_c^m(B)$, which is generalization of Saeedi and Sheikh-Mohseni (2016). It is easy to show that for every $m \geq 1$, $\text{ID}_m^*(A)$ and $\text{Der}_c^m(A)$ are subalgebra of $\text{Der}(A)$ and we have

$$\begin{aligned} \dots &\subseteq \text{Der}_c^{m+1}(A) \subseteq \text{Der}_c^m(A) \subseteq \dots \subseteq \text{Der}_c(A) \subseteq \text{Der}(A) \\ \dots &\subseteq \text{ID}_{m+1}^*(A) \subseteq \text{ID}_m^*(A) \subseteq \dots \subseteq \text{ID}^*(A) \subseteq \text{Der}(A) \end{aligned}$$

Moreover,

$$\text{Der}_c^m(A) \subseteq \text{ID}_m^*(A) \subseteq \text{ID}^*(A).$$

In the following, we give a useful lemma.

LEMMA 2.1 Let A is a Filippov algebra. Then for some $m \geq 1$, $\text{Der}_c^m(A)$ is subalgebra of $\text{Der}(A)$.

Proof Let $\alpha_1, \alpha_2 \in \text{Der}_c^m(A)$, $x \in A$ and put $\alpha_1(x) = [x, l_{i,1}, \dots, l_{i,(n-1)}]$ and $\alpha_2(x) = [x, l_{j,1}, \dots, l_{j,(n-1)}]$ for some elements $l_{i,k}, l_{j,k}$ in A , for $k = 1, \dots, (n-2)$ and $l_{i,(n-1)}, l_{j,(n-1)}$ in A^m , then

$$\begin{aligned}
 \alpha_1, \alpha_2 &= \alpha_1(\alpha_2(x)) - \alpha_2(\alpha_1(x)) \\
 &= \alpha_1([x, l_{j,1}, \dots, l_{j,(n-1)}]) - \alpha_2([x, l_{i,1}, \dots, l_{i,(n-1)}]) \\
 &= [\alpha_1(x), l_{j,1}, \dots, l_{j,(n-1)}] + \sum_{k=1}^{(n-1)} [x, l_{j,1}, \dots, \alpha_1(l_{j,k}), \dots, l_{j,(n-1)}] \\
 &\quad - [\alpha_2(x), l_{i,1}, \dots, l_{i,(n-1)}] - \sum_{k=1}^{(n-1)} [x, l_{i,1}, \dots, \alpha_2(l_{i,k}), \dots, l_{i,(n-1)}] \\
 &= [x, l_{i,1}, \dots, l_{i,(n-1)}, l_{j,1}, \dots, l_{j,(n-1)}] + \sum_{k=1}^{(n-1)} [x, l_{j,1}, \dots, \alpha_1(l_{j,k}), \dots, l_{j,(n-1)}] \\
 &\quad - [x, l_{j,1}, \dots, l_{j,(n-1)}, l_{i,1}, \dots, l_{i,(n-1)}] - \sum_{k=1}^{(n-1)} [x, l_{i,1}, \dots, \alpha_2(l_{i,k}), \dots, l_{i,(n-1)}] \\
 &= [x, \alpha_1(l_{j,1}), l_{j,2}, \dots, l_{j,(n-1)}] + \dots + [x, l_{j,1}, \dots, \alpha_1(l_{j,(n-1)})]
 \end{aligned}$$

and hence $[\alpha_1, \alpha_2] \in \text{Der}_c^m(A)$. □

LEMMA 2.2 Let A is a Filippov algebra. Then for some $m \geq 1$, $ID_m^*(A)$ is subalgebra of $\text{Der}(A)$.

Proof The proof is the same proof of Lemma 1.1. □

In the following, we show that if A and B are two m -isoclinic (n -Lie) Filippov algebras, then $ID_m^*(A)$ and $ID_m^*(B)$ are isomorphic.

THEOREM 2.3 Let A and B be two finite dimensional non-abelian m -isoclinic n -Lie algebras. Then $ID_m^*(A) \cong ID_m^*(B)$.

Proof Let A and B are m -isoclinism, so there exist isomorphisms $\varphi: A/Z_m(A) \rightarrow B/Z_m(B)$ and $\theta: A^{m+1} \rightarrow B^{m+1}$ such that $\theta([x_1, \dots, x_t]) = [y_1, \dots, y_t]$, where $\varphi(x_i + Z_m(A)) = (y_i + Z_m(B))$ for $i = 1, 2, \dots, t$. Let $\alpha \in ID_m^*(A)$ and $y \in B$. Now define a map $\tau_\alpha: B \rightarrow B$ by $\tau_\alpha(y) = \theta(\alpha(x))$, where $\varphi^{-1}(y + Z_m(B)) = x + Z_m(A)$. It is not hard to see that τ_α is well-defined and maps central elements to 0.

It is easy to show that the map τ_α is a linear transformation. Let $y_1, \dots, y_n \in B$ and $\varphi^{-1}(y_i + Z_m(B)) = x_i + Z_m(A)$, for $i = 1, \dots, n$. Then

$$\tau_\alpha([y_1, \dots, y_n]) = \theta(\alpha[x_1, \dots, x_n]) = \sum_{j=1}^n \theta[x_1, \dots, \alpha(x_j), \dots, x_n]$$

By using Lemma 1.1, we have

$$\tau_\alpha([y_1, \dots, y_n]) = \sum_{j=1}^n [y_1, \dots, \theta(\alpha(x_j)), \dots, y_n] = \sum_{j=1}^n [y_1, \dots, \tau_\alpha(y_j), \dots, y_n]$$

Moreover, τ_α is a derivation of B and map's central elements of B to 0. So τ_α is a ID^* -derivation of B .

Now define the map $\tau: ID_m^*(A) \rightarrow ID_m^*(B)$ by $\tau(\alpha) = \tau_\alpha$. And we show that is a Lie homomorphism. It is easy to see that τ is a well-defined linear transformation. Let $\alpha_1, \alpha_2 \in ID_m^*(A)$, $y \in B$ and $\varphi^{-1}(y + Z_m(B)) = x + Z_m(A)$. Then

$$\begin{aligned}
 \tau([\alpha_1, \alpha_2])(y) &= \tau_{[\alpha_1, \alpha_2]}(y) = \theta([\alpha_1, \alpha_2](x)) \\
 &= \theta(\alpha_1(\alpha_2(x)) - \alpha_2(\alpha_1(x))) \\
 &= \theta(\alpha_1(\alpha_2(x))) - \theta(\alpha_2(\alpha_1(x)))
 \end{aligned}$$

On the other hand, we have

$$\begin{aligned} \tau_{\alpha_1}, \tau_{\alpha_2}(y) &= \tau_{\alpha_1}(\tau_{\alpha_2}(y)) - \tau_{\alpha_2}(\tau_{\alpha_1}(y)) \\ &= \tau_{\alpha_1}(\theta(\alpha_2(x))) - \tau_{\alpha_2}(\theta(\alpha_1(x))) \\ &= \theta(\alpha_1(\alpha_2(x))) - \theta(\alpha_2(\alpha_1(x))) \end{aligned}$$

Hence τ is a Lie homomorphism. The pair $(\varphi^{-1}, \theta^{-1})$ is a m -isoclinism of B to A . So given by the same method we can show that for each $\beta \in \text{ID}_m^*(B)$, then $\sigma_\beta \in \text{ID}_m^*(A)$ and the map $\sigma: \text{ID}_m^*(B) \rightarrow \text{ID}_m^*(A)$ by $\sigma(\beta) = \sigma_\beta$ is a Lie homomorphism. It is clear that, $\tau_{\sigma_\beta} = \beta$ and $\sigma_{\tau_\alpha} = \alpha$, for each $\alpha \in \text{ID}_m^*(A)$ and $\beta \in \text{ID}_m^*(B)$. Thus $\sigma\tau = 1 = \tau\sigma$, which implies that the homomorphism from $\text{ID}_m^*(A)$ to $\text{ID}_m^*(B)$ defined above is a Lie isomorphism. So $\text{ID}_m^*(A) \cong \text{ID}_m^*(B)$. \square

As a consequence of above theorem, we conclude the following corollary, analogous to work of Saeedi and Sheikh-Mohseni (2016), that extends the result to m -isoclinic case in Filippov algebras.

COROLLARY 1.5 Let A and B are two finite dimensional non-abelian m -isoclinic Filippov algebras. Then $\text{Der}_c^m(A) \cong \text{Der}_c^m(B)$.

Proof Let $A \sim_m B$, so there exist isomorphisms $\varphi: A/Z_m(A) \rightarrow B/Z_m(B)$ and $\theta: A^{m+1} \rightarrow B^{m+1}$, such that $\theta([x_1, \dots, x_t]) = [y_1, \dots, y_t]$ where $\varphi(x_i + Z_m(A)) = (y_i + Z_m(B))$, for $i = 1, 2, \dots, t$. Let $\alpha \in \text{Der}_c^m(A)$, and $y \in B$, where $\varphi^{-1}(y + Z_m(B)) = x + Z_m(A)$, Then there exists elements $x_{i,1}, \dots, x_{i,(m(n-1))}$ in A ($i = 1, \dots, k$), such that

$$\begin{aligned} \alpha(x) &= \sum_{i=1}^k [x, x_{i,1}, \dots, x_{i,(1(n-1)-1)}, [x_{i,(n-1)}, \dots, x_{i,(2(n-1)-1)}, [x_{i,(2(n-1))}, \dots, x_{i,(3(n-1)-1)}, \\ &\quad [x_{i,(3(n-1))}, \dots, \nu[\dots, x_{i,((m-1)(n-1)-1)}, [x_{i,((m-1)(n-1))}, \dots, x_{i,(m(n-1))}] \dots]]] \end{aligned}$$

Now the same as in Theorem 2.3 and according to the mapping α , we define a map $\tau_\alpha: B \rightarrow B$. Then for every $y \in B$, we have

$$\begin{aligned} \tau_\alpha(y) &= \theta(\alpha(x)) = \theta\left(\sum_{i=1}^k [x, x_{i,1}, \dots, x_{i,(1(n-1)-1)}, [x_{i,(n-1)}, \dots, x_{i,(2(n-1)-1)}, [x_{i,(2(n-1))}, \dots, \right. \\ &\quad \left. x_{i,(3(n-1)-1)}, [x_{i,(3(n-1))}, \dots, [\dots, x_{i,((m-1)(n-1)-1)}, [x_{i,((m-1)(n-1))}, \dots, x_{i,(m(n-1))}] \dots]]]\right) \\ &= \sum_{i=1}^k \theta([x, x_{i,1}, \dots, x_{i,(1(n-1)-1)}, [x_{i,(n-1)}, \dots, x_{i,(2(n-1)-1)}, [x_{i,(2(n-1))}, \dots, x_{i,(3(n-1)-1)}, \\ &\quad [x_{i,(3(n-1))}, \dots, [\dots, x_{i,((m-1)(n-1)-1)}, [x_{i,((m-1)(n-1))}, \dots, x_{i,(m(n-1))}] \dots]]] \\ &= \sum_{i=1}^k [y, y_{i,1}, \dots, y_{i,(1(n-1)-1)}, [y_{i,(n-1)}, \dots, y_{i,(2(n-1)-1)}, [y_{i,(2(n-1))}, \dots, y_{i,(3(n-1)-1)}, \\ &\quad [y_{i,(3(n-1))}, \dots, [\dots, y_{i,((m-1)(n-1)-1)}, [y_{i,((m-1)(n-1))}, \dots, y_{i,(m(n-1))}] \dots]]] \end{aligned}$$

where $\varphi(x_{i,j} + Z_m(A)) = (y_{i,j} + Z_m(B))$ for all $i = 1, \dots, k$ and $j = 1, \dots, m(n-1)$. Clearly τ_α is a well-defined linear transformation and implies that $\tau_\alpha \in \text{Der}_c^m(B)$. Now we define a map $\tau: \text{Der}_c^m(A) \rightarrow \text{Der}_c^m(B)$ by $\tau(\alpha) = \tau_\alpha$. That is a Lie homomorphism. So $\tau(\text{Der}_c^m(A)) \subseteq \text{Der}_c^m(B)$. On the other hand, we can define the homomorphism σ the same as in Theorem 2.3. Since $\tau\sigma = 1$, that implies $\text{Der}_c^m(B) \subseteq \tau(\text{Der}_c^m(A))$. Moreover $\tau(\text{Der}_c^m(A)) = \text{Der}_c^m(B)$, which implies that $\text{Der}_c^m(A) \cong \text{Der}_c^m(B)$. \square

Future work: We are going to investigate the obtained results in this article for n -ary algebras. It seems that we can obtain same results by defining a new n -bracket and also give the new definition for subalgebras of algebra derivations in n -ary algebras.

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