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*Corresponding author: Poom Kumam, Science Laboratory Building, Faculty of Science, KMUTT-Fixed Point Theory and Applications Research Group (KMUTT-FPTA), Theoretical and Computational Science Center (TaCS), King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand; Science Laboratory Building, Faculty of Science, KMUTT Fixed Point Research Laboratory, Department of Mathematics, Room SCL 802 Fixed Point Laboratory, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand
E-mails: poom.kum@kmutt.ac.th, poom.kumam@mail.kmutt.ac.th

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APPLIED & INTERDISCIPLINARY MATHEMATICS | RESEARCH ARTICLE

A method for solving a fuzzy transportation problem via Robust ranking technique and ATM

Darunee Hunwisai^{1,2} and Poom Kumam^{1,2*}

Abstract: In this paper, we introduce the method for solving fuzzy transportation problem (FTP) using Robust's ranking technique for the representative value of the fuzzy number. In addition we are using allocation table method (ATM) to find an initial basic feasible solution (IBFS) for the FTP. Moreover, this method is also good optimal solution in the literature and illustrated with numerical examples.

Subjects: Science; Mathematics & Statistics; Applied Mathematics

Keywords: fuzzy transportation problem; fuzzy number; ranking technique

1. Introduction

The transportation problem is a special kind of optimization problem. Transportation problem aims at finding the least total transportation cost of goods in order to satisfy demand at destinations using available supplies at the sources.

In usual, transportation problems are solved with the hypothesis that values of supplies and demands and the transportation costs are specified in a precise way. In the real world, in many cases, the decision-maker has no crisp information about the coefficients belonging to the transportation problem. In this situation, the corresponding elements defining the problem can be formulated by means of fuzzy set, and the fuzzy transportation problem appears in a natural way. The basic transportation model was first developed by Hitchcock (1941). Dantzig (1951) applied linear programming to solve the transportation problem. The concept of decision-making in fuzzy environment was proposed by Bellman and Zadeh (1970). Several authors have carried out an examination about FTP (Basirzadeh (2011); Chanas and Kucht (1996); Gani & Razak, (2006); Kaur and Kumar (2011); Li, Huang, Da, & Hu, Li et al. (2008); Lin (2009); Oheigeartaigh, Oheigeartaigh (1982); Pandian and Natrajan, (2010a); Zimmermann (1978). Chang (1981) suggested the concept of the preference function of an alternative. Yager (1981) studied a procedure for ordering fuzzy subsets of the unit interval. Chen

ABOUT THE AUTHORS

Darunee Hunwisai is currently a PhD student of Associate Professor Dr Poom Kumam. Her field of interest is fuzzy optimization and fixed point theory.

Poom Kumam, works as an associate professor in the Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi, Thailand. He has expertise in Fixed Point Theory and its Applications. He has published more than 350 research papers in reputed national and international journals.

PUBLIC INTEREST STATEMENT

Recently, ATM is one of the useful techniques as preliminary solving tools for transportation problems. Owing to the unstable of world economic, all expenditures in each businesses are considered.

Logistic cost factor is one of the success keys for business benefit. Therefore, the objective of this research is to find the minimum cost of the total transportation under price fluctuation condition.

In this work, four key success factors; robust ranking technique, ATM, modified distribution method and fuzzy theory were implemented. Finally, we found that the solution of fuzzy transportation was the reliable tool with high accuracy and precision. This tool was comfortable and practical for transportation cost prediction.



Poom Kumam

(1985) studied ranking fuzzy numbers with maximizing set and minimizing set. Dinagar and Palanivel (2009) investigated the transportation problem in fuzzy environment. Pandian and Natarajan (2010a) introduced a new algorithm to find a fuzzy optimal solution for fuzzy transportation problems. Shanmugasundari and Ganesan (2013) studied a novel approach for the fuzzy optimal solution of fuzzy transportation problems. Kaur and Kumar (2012) presented a new approach for solving fuzzy transportation problems using generalized trapezoidal fuzzy numbers (TrFNs). Srinivas and Ganesan (2015) obtained the optimal solution for intuitionistic fuzzy transportation problem via revised distribution method. Nagarajan and Solairaju (2010) found computing improved fuzzy optimal hungarian assignment problem with fuzzy costs under Robust ranking techniques. Ahmed, Khan, Uddin and Ahmed (2016) introduced the allocation table method (ATM) for finding an IBFS.

In this paper, we introduce the method for solving fuzzy transportation problem (FTP) using Robust ranking technique for the representative value of the fuzzy number. In addition, we are using ATM by Ahmed et al. (2016) to find IBFS for the FTP and improve basic feasible solution by modified distribution method (MODIM) for find optimal solution. Moreover, the method is illustrated with numerical examples.

The rest of this paper is organized as follows: In Section 2, deals with some definitions and operations on TrFN from literature. In Section 3, a method to find IBFS and optimal solution for FTP is discussed. In Section 4, we give examples to illustrate to finding IBFS and the optimal solution for the FTP. Finally, Section contains the conclusion.

2. Preliminaries

In this section, we summarize some basic concepts of fuzzy set defined by Zadeh (1965), notation, definitions, and operations of TrFN which are used throughout the paper.

2.1. The definitions and operations of fuzzy number

Definition 2.1 Zadeh (1965) Let X be a nonempty set of the universe. A fuzzy set A in X denote by $\tilde{A} = \{x, \mu_{\tilde{A}}(x) | x \in X\}$ where the function

$$\mu_{\tilde{A}}: X \rightarrow [0, 1]$$

$\mu_{\tilde{A}}(x)$ is the degree of membership of element x in fuzzy set \tilde{A} . Thus, $\mu_{\tilde{A}}(x)$ is valued on the unit interval.

Definition 2.2 A fuzzy number \tilde{A} is

- (i) subset of the real line.
- (ii) continuous membership function.
- (iii) convex, that is for any $x, y \in \mathbb{R}$ and $\lambda \in [0, 1]$.

$$\mu_{\tilde{A}}(\lambda x + (1 - \lambda)y) \geq \min(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)).$$

- (iv) normal, that is there exist at least one $x \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x) = 1$.

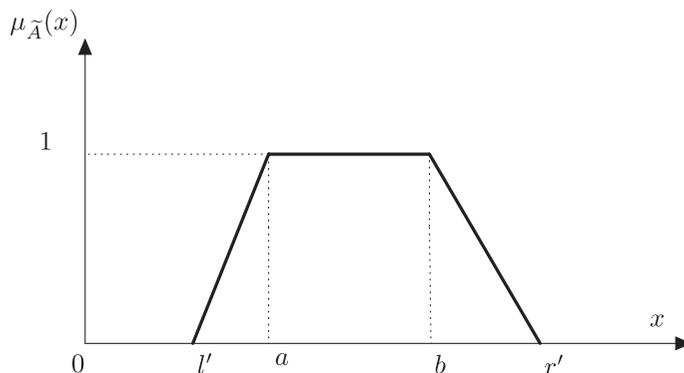
Definition 2.3 Let \tilde{A} and \tilde{B} be any fuzzy numbers and let $\xi \in \mathbb{R}$ be any real number. Then, the sum of two fuzzy numbers and the scalar product of ξ and \tilde{A} are defined by the membership functions

$$\mu_{\tilde{A} + \tilde{B}}(z) = \sup_{z=v+w} \min \{ \mu_{\tilde{A}}(v), \mu_{\tilde{B}}(w) \},$$

$$\mu_{\xi \tilde{A}}(z) = \max \left\{ \sup_{z=\xi v} \mu_{\tilde{A}}(v), 0 \right\},$$

where we set $\sup\{\emptyset\} = -\infty$

Figure 1. A TrFN $\tilde{A} = (l', a, b, r')$



Definition 2.4 Dubois and Prade (1980) A TrFN $\tilde{A} = (l', a, b, r')$ is a special fuzzy set in \mathbb{R} , whose membership function is defined as follow (Figure 1):

$$\mu_{\tilde{A}}(x) = \begin{cases} (x - l') / (a - l') & \text{if } l' \leq x \leq a \\ 1 & \text{if } a \leq x \leq b \\ (r' - x) / (r' - b) & \text{if } b \leq x \leq r' \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where $l' \leq a \leq b \leq r'$.

Remark 2.5 From Definition 2.4 A TrFN $\tilde{A} = (l', a, b, r')$ if $a = b = p$ then $\tilde{A} = (l', p, r')$ that is $\tilde{A} = (l', p, r')$ is a triangular fuzzy number (TFN), which is particular case of TrFN. In a similar way, the arithmetic operations of TFN and TrFN are defined as follows.

Definition 2.6 Let $\tilde{A} = (l'_1, a_1, b_1, r'_1)$ and $\tilde{B} = (l'_2, a_2, b_2, r'_2)$ be two TrFNs with $\xi \neq 0$ be any real number. Then, the arithmetic operations of TrFNs are defined as follows:

$$\tilde{A} + \tilde{B} = (l'_1 + l'_2, a_1 + a_2, b_1 + b_2, r'_1 + r'_2), \quad (2)$$

$$\tilde{A} - \tilde{B} = (l'_1 - r'_2, a_1 - b_2, b_1 - a_2, r'_1 - l'_2), \quad (3)$$

$$\tilde{A}\tilde{B} = \begin{cases} (l'_1 l'_2, a_1 a_2, b_1 b_2, r'_1 r'_2) & \text{if } \tilde{A} > 0, \tilde{B} > 0 \\ (l'_1 r'_2, a_1 b_2, b_1 a_2, r'_1 l'_2) & \text{if } \tilde{A} < 0, \tilde{B} > 0 \\ (r'_1 r'_2, b_1 b_2, a_1 a_2, l'_1 l'_2) & \text{if } \tilde{A} < 0, \tilde{B} < 0, \end{cases} \quad (4)$$

$$\tilde{A}/\tilde{B} = \begin{cases} (l'_1/r'_2, a_1/b_2, b_1/a_2, r'_1/l'_2) & \text{if } \tilde{A} > 0, \tilde{B} > 0 \\ (r'_1/r'_2, b_1/b_2, a_1/a_2, l'_1/l'_2) & \text{if } \tilde{A} < 0, \tilde{B} > 0 \\ (r'_1/l'_2, b_1/a_2, a_1/b_2, l'_1/r'_2) & \text{if } \tilde{A} < 0, \tilde{B} < 0, \end{cases} \quad (5)$$

$$\xi/\tilde{A} = \begin{cases} (\xi l'_1, \xi a_1, \xi b_1, \xi r'_1) & \text{if } \xi > 0 \\ (\xi r'_1, \xi b_1, \xi a_1, \xi l'_1) & \text{if } \xi < 0 \end{cases} \quad (6)$$

and

$$\tilde{A}^{-1} = (1/r'_1, 1/b_1, 1/a_1, 1/l'_1) \quad \text{if } \tilde{A} \neq 0. \quad (7)$$

Definition 2.7 The λ -cut set of TrFN $\tilde{A} = (l', a, b, r')$ is a crisp subset of \mathbb{R} , which is defined as follow:

$$\tilde{A}_\lambda = \{x | \mu_{\tilde{A}}(x) \geq \lambda\},$$

where $\lambda \in [0, 1]$.

From Definition 2.7, λ -cut set of any TrFN \tilde{A} is $\tilde{A}_\lambda = [\tilde{A}_\lambda^L, \tilde{A}_\lambda^U]$ if we get crisp interval by λ -cut operation λ -cut interval for TrFN \tilde{A} is written below.

For all $\lambda \in [0, 1]$, and from Equation (1)

$$\frac{l'_\lambda - l'}{a - l'} = \lambda \quad \text{and} \quad \frac{r' - b_\lambda}{r' - b} = \lambda$$

such that

$$\tilde{A}_\lambda^L = l' + (a - l')\lambda \quad \text{and} \quad \tilde{A}_\lambda^U = r' - (r' - b)\lambda$$

therefore, we get

$$\tilde{A}_\lambda = [l' + (a - l')\lambda, r' - (r' - b)\lambda]. \tag{8}$$

2.2. Robust's ranking technique

In this section, we introduced the concept of Robust's ranking Yager (1981) and we give the notation of the Robust's ranking index.

Definition 2.8 Yager (1981) Let \tilde{A} be a convex fuzzy number the Robust's ranking index is defined by

$$R(\tilde{A}) = \frac{1}{2} \int_0^1 [\tilde{A}_\lambda^L, \tilde{A}_\lambda^U] d\lambda, \tag{9}$$

where $[\tilde{A}_\lambda^L, \tilde{A}_\lambda^U]$ is the λ -level cut of the fuzzy number \tilde{A} .

Since Definitions 2.4 and 2.8 and Equation (8) such that

$$\begin{aligned} R(\tilde{A}) &= \frac{1}{2} \int_0^1 [\tilde{A}_\lambda^L, \tilde{A}_\lambda^U] d\lambda \\ &= \frac{1}{2} \int_0^1 [(l' + (a - l')\lambda) + (r' - (r' - b)\lambda)] d\lambda. \end{aligned} \tag{10}$$

In this paper, we give index $R(\tilde{A})$ is the representative value of the fuzzy number \tilde{A} . And we give index $R(\tilde{C}_{ij})$ is the representative value of the fuzzy cost \tilde{C}_{ij} .

3. Fuzzy transportation problem

The mathematical formulation of the FTP is of the following form (Table 1):

$$\begin{aligned} \text{Minimize } \tilde{z} &= \sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij} \\ \text{subject to } \sum_{j=1}^n x_{ij} &\leq \alpha_i, \quad i = 1, 2, \dots, m \\ \sum_{i=1}^m x_{ij} &\geq \beta_j, \quad j = 1, 2, \dots, n \\ x_{ij} &\geq 0 \quad \text{for all } i \text{ and } j, \end{aligned}$$

Table 1. The fuzzy transportation table

	1	2	...	N	α_i
1	\tilde{c}_{11}	\tilde{c}_{12}	\dots	\tilde{c}_{1n}	α_1
2	\tilde{c}_{21}	\tilde{c}_{22}	\dots	\tilde{c}_{2n}	α_2
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
β_j	β_1	β_2	\dots	β_n	$\sum_{i=1}^m \alpha_i = \sum_{j=1}^n \beta_j$

where

\tilde{c}_{ij} is the fuzzy cost of transportation one unit of the goods from i th source to the j th destination.

x_{ij} is the quantity transportation from i th source to the j th destination.

α_i is the total availability of the goods at i th source.

β_j is the total demand of the goods at j th destination.

$\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} x_{ij}$ is total fuzzy transportation cost.

If $\sum_{i=1}^m \alpha_i = \sum_{j=1}^n \beta_j$, then FTP is said to be balanced.

If $\sum_{i=1}^m \alpha_i \neq \sum_{j=1}^n \beta_j$, then FTP is said to be unbalanced.

3.1. Algorithm to find an initial basic feasible solution (IBFS)

In this section, we use an allocation table set up to find the solution for FTP. This method is called ATM and the algorithm is illustrated as follows:

Step-1: Establish the formulated fuzzy linear programming problem into the tabular form known as fuzzy transportation table (FTT). The fuzzy cost of transportation to be put on the table allocation. And we use the approximate fuzzy cost of TP by Robust’s ranking technique.

Step-2: Examine that the FTP is balanced or unbalanced, if unbalanced, make it balanced.

Step-3: Choose minimum odd cost (can be an integer/decimal/fractional number odd) from every cost cells of FTT. If there is no odd cost in cost cells of the FTT, continue by dividing every cost cells by 2 until get at least an odd value in cost cells.

Step-4: Form a new table which revises is to be known as allocation table by keeping the minimum odd cost in the respective cost cell/cells as it was/were, and subtract selected minimum odd cost only from each of the odd cost valued cells of the FTT. Now every the cell values are to be called as allocation cell value in allocation table.

Step-5: At first start with selected minimum odd cost in allocation table in Step-4. Delete the row(availability) or column(demand) that has been allocated to complete.

Step-6: Now specify the minimum allocation cell value and allocate minimum of availability/demand at the place of selected allocation cell value in the allocation table. In the event of same allocation cell values, select the allocation cell value where minimum allocation can be made. Afresh in the event of same allocation in the allocation cell values, choose the minimum cost cell which is corresponding to the cost cells of FTT formed in Step-1. Afresh if the allocations and the cost cells are

equal, in such case choose the nearer cell to the minimum of demand/availability which is to be allocated. Now Delete the row(availability) or column(demand) that has been allocated to complete.

Step-7: Repeat Step-6 as far as the demand and availability are depleted.

Step-8: Finally, from the FTT, we compute the total fuzzy transportation cost.

3.2. Modified distribution method (MODIM) for finding optimal solution

In this section, we find the best solution for FTP using a modified method of distribution. Algorithm of MODIM is illustrated as follow:

Step-1: Find IBFS by proposed ATM.

Step-2: Compute dual variables R_i and K_j for all row and column, respectively, satisfying $R_i + K_j = C_{ij}$, set $R_1 = 0$.

Step-3: Calculate the improvement index value for unoccupied cells by the equation $E_{ij} = C_{ij} - R_i - K_j$.

Step-4: Consider valued of E_{ij} .

case (i) IBFS is fuzzy optimal solution, if $E_{ij} \geq 0$ for every unoccupied cells.

case (ii) IBFS is not fuzzy optimal solution, for at least one $E_{ij} < 0$. Go to step 5.

Step-5: Choose the unoccupied cell for the most negative value of E_{ij} .

Step-6: We construct the closed loop below.

At first, start the closed loop with choose the empty cell and move vertically and horizontally with corner cells occupied and come back to choose the empty cell to complete the loop. Use sign “+” and “-” at the corners of the closed loop, by assigning the “+” sign to the selected empty cell first.

Step-7: Look for the minimum allocation value from the cells which have “-” sign. After that, allocate this value to the choose empty cell and subtract it to the other occupied cell having “-” sign and add it to the other occupied cells having “+” sign.

Step-8: Allocation in Step-7 will result an improved basic feasible solution (BFS).

Step-9: Test the optimality condition for improved BFS. The process is complete when $E_{ij} \geq 0$ for all the empty cell.

4. Numerical example

Example 4.1 Consider fuzzy transportation problem with three sources that is S_1, S_2, S_3 and three destinations D_1, D_2, D_3 . The cost of transporting one unit of the goods from i th source to the j th destination given whose elements are trapezoidal fuzzy numbers, and is shown in Table 2. Find out the minimum cost of total fuzzy transportation.

Since $\sum_{i=1}^3 \alpha_i = \sum_{j=1}^3 \beta_j = 125$, the FTP is balanced.

From Table 2, the cost of transporting one unit of the goods from i th source to the j th destination are TrFNs. We use Robust’s ranking technique form Equation (10) for calculating the membership function of the TrFN.

Table 2. Data of the Example 4.1: The fuzzy transportation table

Source	D_1	D_2	D_3	Supply (α_i)
S_1	(3, 5, 7, 14)	(2, 4, 8, 13)	(3, 5, 9, 15)	35
S_2	(2, 5, 8, 10)	(3, 6, 9, 12)	(4, 7, 10, 16)	40
S_3	(3, 6, 8, 13)	(4, 8, 10, 15)	(5, 9, 13, 15)	50
Demand(β_j)	45	55	25	125

Table 3. The fuzzy transportation table after ranking

Source	D_1	D_2	D_3	Supply
S_1	7.25	6.75	8	35
S_2	6.25	7.5	9.25	40
S_3	7.5	9.25	10.5	50
Demand	45	55	25	125

Let us consider Table 2 where the element (3, 5, 7, 14) is TrFN. The λ -cut of the TrFN (3, 5, 7, 14) is $[\tilde{c}_{11\lambda}^L, \tilde{c}_{11\lambda}^U] = [3 + 2\lambda, 14 - 7\lambda]$. Therefore, we obtain

$$\begin{aligned}
 R(\tilde{c}_{11}) &= R(3, 5, 7, 14) \\
 &= \frac{1}{2} \int_0^1 [\tilde{c}_{11\lambda}^L, \tilde{c}_{11\lambda}^U] d\lambda \\
 &= \frac{1}{2} \int_0^1 (17 - 5\lambda) d\lambda \\
 &= 7.25.
 \end{aligned}$$

Similarly, the Robust's ranking indexes for the fuzzy costs \tilde{c}_{ij} are calculated as: $R(\tilde{c}_{12}) = 6.75, R(\tilde{c}_{13}) = 8, R(\tilde{c}_{21}) = 6.25, R(\tilde{c}_{22}) = 7.5, R(\tilde{c}_{23}) = 9.25, R(\tilde{c}_{31}) = 7.5, R(\tilde{c}_{32}) = 9.25, R(\tilde{c}_{33}) = 10.5$. We put all $R(\tilde{c}_{ij})$ in Table 3.

From Table 3, it is found that the FTP is balanced. Thus, move to step-3.

Step-3, minimum odd cost is 6.25 in cost cell (2,1) among all the cost cells of Table 3.

According to step-5, minimum of supply/demand is 40 that is allocation in cell (2,1). After allocating this value, it is found that the supply is satisfied. For which S_2 row is to be exhausted.

After step-5, only S_1 and S_3 rows are to be considered. Where 6.75 is the lowest cell value in cells (1,1), (1,2), and (3,1). Among these three cells 5 is the lowest allocation can be made in cells (1,1) and (3,1). In the case, we choose cell (1,1) because 7.25 is the lowest cell value in cells (1,1) and (3,1). For which D_1 column is to be exhausted.

Next, consider cells (1,3) and (3,3) we found that 25 is the lowest allocation of cost cells (1,3) and (3,3). Thus, select cell (1,3) because 8 is the lowest cell value in cells (1,3) and (3,3). For which D_3 column is to be exhausted

Now complete the allocation by allocating 5 and 50, respectively, to the cell (1,2) and (3,2). All these allocations are made according to step-6 and step-7 of the proposed algorithm.

Table 4. By the first iteration of allocation cell, we have

Source	D_1	D_2	D_3	Supply
S_1	7.25	6.75	8	35
S_2	6.25 40	7.5	9.25	40
S_3	7.5	9.25	10.5	50
Demand	45	55	25	125

Table 5. Finally, allocation of various cells are in the allocation table

Source	D_1	D_2	D_3	α_i
S_1	7.25 5	6.75 5	8 25	35
S_2	6.25 40	7.5	9.25	40
S_3	7.5	9.25 50	10.5	50
β_j	45	55	25	125

Table 6. Improved basic feasible solution

Source	D_1	D_2	D_3	α_i
S_1	7.25	6.75 10	8 25	35
S_2	6.25	7.5 40	9.25	40
S_3	7.5 45	9.25 5	10.5	50
β_j	45	55	25	125

After that, transfer this allocation to the FTT. The first allocation is shown in Table 4 and the final allocation is shown in Table 5.

Therefore, IBFS is $x_{11} = 5, x_{12} = 5, x_{13} = 25, x_{21} = 40, x_{32} = 50$.

Finally, total fuzzy transportation cost is
 $(5 \times 7.25 + 5 \times 6.75 + 25 \times 8 + 40 \times 6.25 + 50 \times 9.25) = 982.5$.

Now we apply MODIM, to compute the optimal solution. Algorithm of MODIM is shown in Section 3.2. We will show only the results of improved basic feasible solution is shown in Table 6 and calculation minimum fuzzy transportation cost is given below.

Hence, optimal solution is $x_{12} = 10, x_{13} = 25, x_{22} = 40, x_{31} = 45, x_{32} = 5$ and minimum fuzzy transportation cost is $(10 \times 6.75 + 25 \times 8 + 40 \times 7.5 + 45 \times 7.5 + 5 \times 9.25) = 951.25$

Example 4.2 Consider fuzzy transportation problem with four sources that is S_1, S_2, S_3, S_4 and three destinations D_1, D_2, D_3 . The cost of transportation one unit of the goods from i th source to the j th destination given whose elements are trapezoidal fuzzy numbers and is shown in the Table 7. Find out the minimum cost of total fuzzy transportation.

Since $\sum_{i=1}^4 \alpha_i = \sum_{j=1}^3 \beta_j = 120$, the FTP is balanced.

From Table 7, the cost of transporting one unit of the goods from i th source to the j th source to the j th destination are TrFNs. Therefore, we are using Robust's ranking technique for calculating the membership function of the TrFN.

Table 7. Data of the Example 4.2: The fuzzy transportation table

Source	D_1	D_2	D_3	Supply (α_i)
S_1	(2, 5, 8, 15)	(2, 3, 4, 7)	(3, 7, 9, 15)	25
S_2	(3, 6, 9, 12)	(4, 7, 9, 11)	(4, 8, 10, 13)	35
S_3	(3, 7, 10, 16)	(5, 6, 12, 16)	(4, 6, 8, 14)	50
S_4	(3, 4, 6, 9)	(4, 5, 7, 9)	(5, 8, 11, 13)	10
Demand(β_j)	30	40	50	120

Table 8. The fuzzy transportation table after ranking

Source	D_1	D_2	D_3	Supply (α_i)
S_1	7.5	4	8.5	25
S_2	7.5	7.75	8.75	35
S_3	9	9.75	8	50
S_4	5.5	6.25	9.25	10
Demand(β_j)	30	40	50	120

The Robust's ranking indexes for the fuzzy costs \tilde{c}_{ij} are calculated as:
 $R(\tilde{C}_{11}) = 7.5, R(\tilde{C}_{12}) = 4, R(\tilde{C}_{13}) = 8.5, R(\tilde{C}_{21}) = 7.5, R(\tilde{C}_{22}) = 7.75, R(\tilde{C}_{23}) = 8.75, R(\tilde{C}_{31}) = 9,$
 $R(\tilde{C}_{32}) = 9.75, R(\tilde{C}_{33}) = 8, R(\tilde{C}_{41}) = 5.5, R(\tilde{C}_{42}) = 6.25, R(\tilde{C}_{43}) = 9.25.$

We put all $R(\tilde{C}_{ij})$ in Table 8.

From Table 8, it is found that the FTP is balanced. Thus, move to step-3.

Step-3, minimum odd cost is 5.5 in cost cell (4,1) among all the cost cells of Table 8.

According to step-5, minimum of supply/demand is 10 that is allocation in cell (4,1). After allocating this value, it is found that the supply is satisfied. For which S_4 row is to be exhausted.

After step-5, only S_1, S_2 and S_3 rows are to be considered. Where 4 is the lowest cell value in the cells (1,1), (1,2) and (2,1). Among these three cells 20 is the lowest allocation can be made in cells (1,1) and (2,1). We choose cell (2,1), thus minimum allocation 20 is allocated in cell (2,1). For which D_1 column is to be exhausted.

Next, consider cells (2,2) and (2,3), in both of these cells 15 is the minimum allocation can be make. But in between these two cells, it is found that 7.75 is the lowest cell value in the cells (2,2) and (2,3). Thus, minimum allocation 15 is allocated in cell (2,2). For which S_2 row is to be exhausted.

After that, consider cells (1,2) and (1,3), in both of these cells 25 is the minimum allocation can be make. But in between these two cells, it is found that the minimum cost cell appears in the cell (1, 2); therefore, the minimum allocation 25 is allocated in cell (1, 2). For which D_2 column and S_1 row are to be exhausted.

Now complete the allocation by allocating 50 to the cell (3,3). All these allocations are made according to step-6 and step-7 of the proposed algorithm.

After that, transfer this allocation to the FTT. The final allocation is shown in Table 9.

Form Table 9, IBFS is $x_{12} = 25, x_{21} = 20, x_{22} = 15, x_{33} = 50, x_{41} = 10,$ and the fuzzy transportation cost is $(5 \times 6.75 + 20 \times 7.5 + 15 \times 7.75 + 50 \times 8 + 10 \times 5.5) = 821.25.$

Table 9. IBFS according to ATM

Source	D_1	D_2	D_3	α_i
S_1	7.5	4 25	8.5	25
S_2	7.5 20	7.75 15	8.75	35
S_3	9	9.75	8 50	50
S_4	5.5 10	6.25	9.25	10
β_j	30	40	50	120

Table 10. Improved basic feasible solution

Source	D_1	D_2	D_3	α_i
S_1	7.5	4 25	8.5 0	25
S_2	7.5 20	7.75 15	8.75	35
S_3	9	9.75	8 50	50
S_4	5.5 10	6.25	9.25	10
β_j	30	40	50	120

Now we apply MODIM, to compute the optimal solution. Algorithm of MODIM as shown in Section 3.2. We will show only the results of improved basic feasible solution shown in Table 10 and calculation minimum fuzzy transportation cost below.

Form Table 10, optimal solution is $x_{12} = 25$, $x_{13} = 0$, $x_{21} = 20$, $x_{22} = 15$, $x_{33} = 50$, $x_{41} = 10$, and the minimum fuzzy transportation cost is $(5 \times 6.75 + 20 \times 7.5 + 15 \times 7.75 + 50 \times 8 + 10 \times 5.5) = 821.25$.

5. Conclusion

In this paper, the ATM is used to find an IBFS. Using λ -cut set and Robust's ranking technique for the representative value of the fuzzy number based on the both demand and availability are real numbers. In addition, the cost is always TrFNs. Moreover, we improve the basic feasible solution by MODIM to find the optimal solution. This method can be used to solve TP and FTP which is TFN and TrFN. Therefore, this method can be applied to solve the real-life transportation problem.

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Author details

Darunee Hunwisai^{1,2}

E-mail: darunee.3790@mail.kmutt.ac.th

Poom Kumam^{1,2}

E-mails: poom.kum@kmutt.ac.th, poom.kumam@mail.kmutt.ac.th

ORCID ID: <http://orcid.org/0000-0002-5463-4581>

¹ Science Laboratory Building, Faculty of Science, KMUTT-Fixed Point Theory and Applications Research Group (KMUTT-FPTA), Theoretical and Computational Science Center (TaCS), King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand.

² Science Laboratory Building, Faculty of Science, KMUTTFixed Point Research Laboratory, Department of Mathematics, Room SCL 802 Fixed Point Laboratory, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thrung Khru, Bangkok 10140, Thailand.

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Authors' contributions

All authors have equal contributions. All authors read and approved final manuscript.

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