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## APPLIED & INTERDISCIPLINARY MATHEMATICS | RESEARCH ARTICLE

# A mathematical model and optimization of total production cost and quality for a deteriorating production process

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**Abstract:** Proper production planning can considerably improve a producer's competitiveness in delivering low-cost products with high quality and short-cycle time. In a deterioration process, the quality may change during a production run, leading to early product failures. To mitigate the impact of deterioration such as process shifting, the optimal resetting cycle before the next setup must be determined, as well as the initial settings at the beginning of a production run. The aim of this study is to propose a mathematical model that consists of a new quality loss function and adds maintenance costs to the ordering and inventory holding costs commonly considered in the classical model. The model is applied to a real-world case study to demonstrate the effect of initial settings of the mean quality characteristic. The results demonstrate the superiority of the proposed approach in terms of total production cost, as compared with the economic production quantity model.



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### PUBLIC INTEREST STATEMENT

Recently, the integration of production, maintenance, and quality control has attracted much attention. The deteriorating process and their maintenance have a great importance in reliability engineering. The paper proposed a new economic production quantity model to consider the production cycle time and quality characteristic to minimize total expected production cost that consists of quality loss and maintenance costs. We used the model to calculate total production cost in a spring production line in Iran. To investigate the initial effect of the mean quality characteristic, the computational analysis on a real world has been applied. The results illustrate the total production cost has been reduced from the proposed approach compared with the traditional economic production quantity model.

**Subjects:** Applied Mathematics; Mathematical Modeling; Mathematics & Statistics; Science

**Keywords:** mathematical model; production planning; deteriorating process

### 1. Introduction

Due to fast globalization, many manufacturers encounter tight competition in the market place. Consumers can find goods with cheaper prices, better quality, and faster delivery; consequently, production time and quality is increasingly important for competitiveness. Research has focused on production problems with constant processing time and non-deteriorated items; however, in many real industrial applications, processing times and values of products may change due to deterioration and wear phenomena such as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or loss of marginal value of commodities that result in decreasing usefulness (Wee, 1993). The process control and reset for deteriorating processes are challenging for production. Production planning should be adjusted according to the time control process to meet the production targets.

Hall and Eillon (1963) discussed optimal production policies with two criteria: (a) maximum rate of production and (b) minimum cost. These optimal policies analyzed for processes show a linear trend in a variable dimension produced by the process with a known distribution. Gibra (1967) combined the costs of resetting and nonconforming products to provide an economic model balancing resetting and not resetting. Padmanabhan and Vrat (1995) proposed different economic order quantity (EOQ) models with a known and constant deterioration rate of the production process. Chen and Lin (2002) developed a dynamic model to solve a deteriorating process problem with normally distributed shelf life products. Sana, Goyal, and Chaudhuri (2004) determined the optimal number of production cycles of deteriorating products with a linear time-varying demand. Tadj, Bounkhel, and Benhadid (2006) modeled the problem of controlling the production rate of deteriorating items for a production–inventory system. Lodree and Uzochukwu (2008) derived an inventory model for a deteriorating product rate with known probability distribution.

A production process starts with an in-control policy and acceptable quality for producing conforming products. When the process starts to produce non-conforming items, preventive/regular maintenance activity should be applied to promptly mitigate and reset deterioration. Alfares, Khursheed, and Noman (2005) proposed a model considering deterioration for both the produced items and the production equipment. This model integrates several realistic aspects, considering item and process deterioration, different demands and production rates, quality, maintenance and inspection. Radhoui, Rezg, and Chelbi (2009) extended a joint quality control and preventive maintenance policy for a randomly failing production system producing conforming and non-conforming units, but not including a deteriorating process that transitions from conforming to nonconforming. Wee and Widyadana (2013) developed a production model for deteriorating items with stochastic preventive maintenance time and rework using the first in first out (FIFO) rule.

Ollila and Malmipuro (1999) present in their study that maintenance has a major impact on product quality.

Kazaz and Sloan (2013) discuss how the critical ratios may be combined in order to determine the optimal policy, together with the trade-offs that involve production profits, maintenance costs, and the effect on the process condition; however, cost of poor quality is not considered. Hou and Lin (2014) developed an economic production quantity (EPQ) model with finite production rate and derived an expression for the expected total annual cost function when maintenance actions and capital investment in set-up cost reduction are adopted; however, this is not considered in deterioration production process. Khan, Jaber, and Ahmad (2014) represent a simple but integrated mathematical model for determining an optimal vendor–buyer inventory policy considering quality inspection errors at the buyer’s end and learning in production at the vendor’s end.

Taguchi, Elsayed, and Hsiang (1989) proposed that customers become increasingly dissatisfied as performance departs farther from the target, which constitutes, a loss suffered by society. This loss can involve delay, waste scrap, etc. He suggested a quadratic curve to represent a customer's dissatisfaction with a product's performance (quality characteristic/metric/KPI). The Taguchi Loss Function (TLF) uses both the process average and the variation as critical measures of quality. A few studies are available for integrating preventive maintenance with combining quality loss functions. Al-Fawzan and Rahim (2001) proposed a loss function, which included quadratic off-target costs, and time-based costs of maintenance and salvage value, but production policies were not considered. Chen and Jin (2006) proposed a quality-oriented-maintenance model to minimize the total production cost including product quality loss function. Muchiri, Pintelon, Gelders, and Martin (2011) developed a conceptual framework that provided guidelines for choosing maintenance performance indicators based on loss function, through alignment of manufacturing objectives. Chen and Huang (2011) presented optimization models for determining the optimum process mean under different quality characteristic to minimize the manufacturing cost and the quality loss. Pandey, Kulkarni, and Vrat (2012) developed an integrated model for joint optimization of preventive maintenance interval and control parameters incorporating the TLF. Chung, Lin, and Srivastava (2014) proposed an inventory model for deteriorating items in which comprehensive analytic methods of mathematics explore the functional behaviors of APV(T); however, it is not considered loss quality function.

Ben-Daya and Makhdoum (1998) studied the effect of various preventive maintenance policies on the joint optimization of the EPQ and the economic design of control chart for a deteriorating process. Subsequently, Ben-Daya (2002) developed an integrated model for the joint optimization of the EPQ and the economic design of an  $\bar{x}$ -control chart. Chung and Hou (2003) proposed a model to determine an optimal run time for a deteriorating production system under allowable shortage with EPQ. Liao, Chen, and Sheu (2009) integrated maintenance and production programs with the EPQ model for an imperfect process involving a deteriorating production system with increasing hazard rate. Pan, Jin, and Wang (2011) developed an extensive EPQ model considering an imperfect three-state production process. Pan, Jin, Wang, and Cang (2012) then presented an EPQ model, which considers quality-related and maintenance-related costs in addition to the storage and ordering costs already considered in the classic model. The objective of their proposed model was to minimize the total production cost with the determination of EPQ and design parameters of control chart subjected to quality loss and other process costs; but in this model, a deteriorating process is not considered. Wen, Ershun, Ying, and Wenzhu (2014) integrated predictive maintenance into EPQ model in which an autoregressive integrated moving average (ARIMA) time series model is adopted to predict a system's health indicator. Shah, Patel, and Shah (2015) analyzed an EPQ model with a time-varying trend in demand while the units are subject to a constant rate of deterioration.

A significant portion of the literature focused on the cost minimization using inventory models, while neglecting impact of the quality characteristic.

The rest of paper is organized as follows: Section 2 contains problem definition and represents mathematical modeling. Section 3 includes numerical example and sensitivity analysis. Finally, conclusion and future research are given in Section 4.

## 2. Problem definition

In this research, we propose a model to consider the production cycle time and quality characteristic so that the total expected manufacturing cost is minimized. It consists of a new quality loss function and maintenance costs other than ordering and inventory holding costs already considered in the classic model. Therefore, the purpose of this study develops a new EPQ model. The objective of the proposed model is to minimize total production cost per production cycle while simultaneously determining the interval between sampling, the sample size and maintenance decision theory. In this paper, we assumed that the process is subject to a random deterioration from an in-control state to an out-of-control state with an arbitrary distribution and, thus, produces some proportion of non-conforming items.

## 2.1. Mathematical modeling

The following notations and model formulation are explained throughout the paper.

$\mu$ : Initial qualitative characteristic mean (spring constant) when the process is in control

$\mu^*$ : Optimum qualitative characteristic mean (spring constant) process (decision variable)

$T$ : Total production time (decision variable)

$K$ : Random part of loss function ( $K = \frac{A}{\Delta^2}$ ,  $A$ : quality loss index based on control chart,  $\Delta$ : tolerance limit of control chart,  $\Delta = Z_{\alpha/2}\sigma$ )

$I$ : Setup cost

$\alpha$ : Type I error

$\theta$ : Failure rate

$P$ : Yearly production rate

$D$ : Yearly demand rate

$h$ : Holding cost per unit item per time

$\tau$ : Interval time between two assignable causes

$\varepsilon$ : Random part ( $\varepsilon \sim N(0, \sigma^2)$ )

$y(t, \tau)$ : Process quality characteristic function ( $y(t, \tau) = \mu(t, \tau) + \varepsilon$ )

$\mu(t, \tau)$ : Quality characteristic mean.  $\mu(t, \tau) = \begin{cases} \mu, & \text{if } t \leq \tau \\ \mu + w(t, \tau), & \text{if } t > \tau \end{cases}$

If  $t \leq \tau$  the process is in-control, otherwise the process is out of control

$w(t, \tau)$ : Deviation function

$\xi$ : Target value

$M(t)$ : Cost of maintenance at time  $t(2e^{0.03t})$

$S(t)$ : Sinking value at time  $t(150e^{-0.9t})$

$ETC$ : Expected annual total cost function

$E(L(t))$ : Expected value of quality loss function per unit time  $t$

### Assumptions

- Quality Loss function is quadratic and nonlinear.
- Lost sales cost per unit time is infinite.
- Upside deviation in sales is considered.
- At the end of a production period, part of a machine must be fixed or replaced.
- Interval between two assignable causes ( $\tau$ ) follows an exponential distribution, i.e.  $\mu = \frac{1}{\lambda}$ .
- Maintenance cost increases based on deviation per unit time.
- Machine value gradually decreases over time.
- Deviation function follows a Gamma distribution.  $W(t, \tau) = \theta(t - \tau)$ ,  $W(G(\theta, \lambda))$ .

A new expected annual total cost (ECT) function proposed, which consists of quality loss function cost, maintenance cost, sinking value, inventory holding cost, and setup cost. The production system is assumed to use the EPQ. Production processed during the  $T$  time. Expected quality loss function cost is:

$$P \int_0^T E(L(t)) dt \tag{1}$$

Since the loss function depends on  $\tau$ , the expected value of quality loss function per unit time ( $t$ ) is conditional and it reformulated as:

$$L(t, \tau) = K(y(t, \tau) - \xi)^2 \tag{2}$$

Quality loss function reformulated as:

$$L(t, \tau) = K(y(t, \tau) - \xi)^2 \tag{3}$$

Expected value of quality loss function reformulated as:

$$E(L(t, \tau)) = \begin{cases} KE[(\mu + \varepsilon - \xi)^2] & t \leq \tau \\ KE[(\mu + w(t, \tau) + \varepsilon - \xi)^2] & t < \tau \end{cases} \tag{4}$$

*Definition 1* Let  $X$  be a numerically valued random variable with expected value  $\mu = E(X)$ . Then the variance of  $X$ , denoted by  $V(X)$ , is

$$V(X) = E((X - \mu)^2) \rightarrow E[X^2] = V[X] + (E[X])^2 \tag{5}$$

Using the above definition, (4) can be rewritten as:

$$E(L(t, \tau)) = \begin{cases} KV[(\mu + \varepsilon - \xi)] + K(E[(\mu + \varepsilon - \xi)])^2 & t \leq \tau \\ KV[(\mu + w(t, \tau) + \varepsilon - \xi)] + K(E[(\mu + w(t, \tau) + \varepsilon - \xi)])^2 & t < \tau \end{cases} \tag{6}$$

Using (1) and (6),  $E(L(t, \tau))$  reformulated as:

$$E[L(t)] = \begin{cases} (KV[(\mu + \varepsilon - \xi)] + K(E[(\mu + \varepsilon - \xi)])^2) \times e^{-\lambda t} + (KV[(\mu + w(t, \tau) + \varepsilon - \xi)] \\ + K(E[(\mu + w(t, \tau) + \varepsilon - \xi)])^2) \times (1 - e^{-\lambda t}) \end{cases} \tag{7}$$

The deviation function follows a Gamma distribution.  $W(t, \tau) = \theta(t - \tau)$ ,  $W(G(\theta, \lambda))$ ,  $V(G(\theta, \lambda)) = \frac{\theta}{\lambda^2}$ ,  $E(G(\theta, \lambda)) = \frac{\theta}{\lambda}$  and can be modeled as:

$$E[L(t)] = [KV[(\mu + \varepsilon - \xi)] + K(E[(\mu + \varepsilon - \xi)])^2 \times e^{-\lambda t} + KV[(\mu + \theta(t - \tau) + \varepsilon - \xi)] \\ + K(E[(\mu + \theta(t - \tau) + \varepsilon - \xi)])^2 \times 1 - e^{-\lambda t}] \tag{8}$$

Substituting  $V[(\mu + \varepsilon - \xi)] = \sigma^2$  it formulated as:  
 $E[(\mu + \varepsilon - \xi)] = \mu - \xi$

$$E[L(t)] = \left[ K(\sigma^2 + (\mu - \xi)^2)e^{-\lambda t} + K\left(\sigma^2 + \left(\frac{\theta}{\lambda^2}\right) + \left(\mu + \theta t - \left(\frac{\theta}{\lambda}\right) - \xi\right)^2\right) 1 - e^{-\lambda t} \right] \tag{9}$$

The quality loss function from Equation (9) obtained for the total production time. Maintenance cost increases over time. The cumulative form is:

$$\int_0^T M(t) dt \tag{10}$$

Sinking value is decreasing over time and denoted as  $S(t)$ .

Set up cost assumed to be:

$$\frac{DI}{PT} \tag{11}$$

Inventory holding cost in EPQ formulated as:

$$\frac{hQ}{2} \left( \frac{1-D}{P} \right) = \frac{h(P-D)T}{2} \tag{12}$$

The ETC cost consists of quality loss function cost, maintenance cost, sinking value, inventory holding cost, and setup cost. The total cost per unit time modeled as follows:

$$ETC = \frac{DI}{PT} + \frac{h(P-D)T}{2} + \frac{D}{PT} \left[ \int_0^T M(t)dt - S(t) + P \left[ \frac{K}{\lambda} (\sigma^2 + (\mu - \varepsilon)^2)(1 - e^{-\lambda t}) + \frac{K}{\lambda^3} e^{-\lambda t} (\lambda^2 \sigma^2 + \theta)(1 + e^{-\lambda t}(1 - \lambda t)) \right] + K \int_0^T \left( \mu + \theta t - \left( \frac{\theta}{\lambda} \right) - \xi \right)^2 (1 - e^{-\lambda t}) dt \right] \tag{13}$$

The optimal solution derived when:

$$\frac{dETC}{dT} = 0 \tag{14}$$

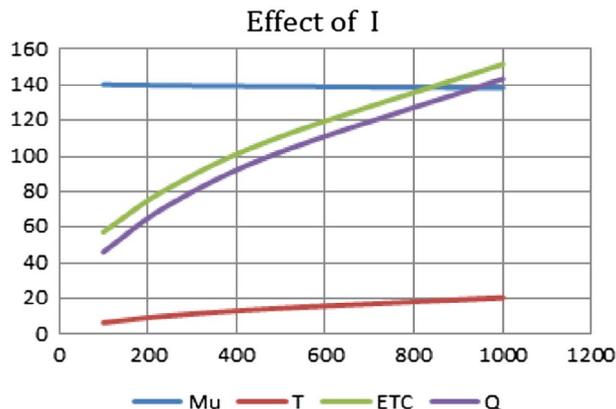
Total optimum production time ( $T^*$ ) can be calculated from (14). EPQ ( $Q^*$ ) obtained:

$$Q^* = PT^* \tag{15}$$

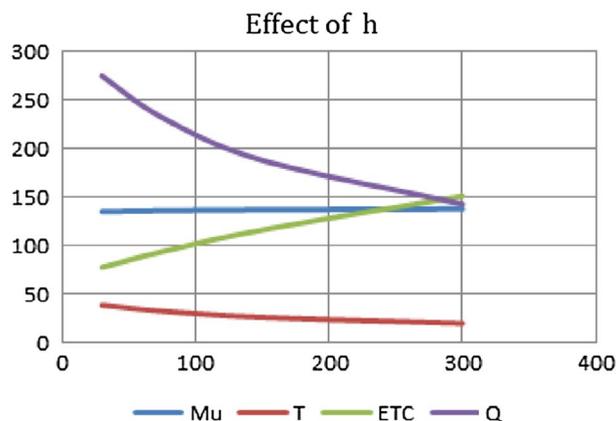
**Table 1. Comparison of the results between the proposed and the previous model**

Operation number	$\theta$	Proposed model	Classic EPQ model	Deference of two models
		ETC* (1)	ETC* (2)	$\Delta ETC^* (2)-(1)$
1	0.03	14,349	29,277	14,928
2	0.035	14,580	29,854	15,274
3	0.04	14,852	30,416	15,564
4	0.045	15,166	31,102	15,936
5	0.05	15,497	31,999	16,502
6	0.055	15,876	33,254	17,378
7	0.06	16,284	34,582	18,298
8	0.065	16,728	36,055	19,327
9	0.07	17,129	37,788	20,659
10	0.075	17,630	39,610	21,980
11	0.08	18,138	41,262	23,124
12	0.085	18,646	43,013	24,367
13	0.09	19,183	44,972	25,789
14	0.095	19,720	47,211	27,491
15	0.1	20,317	50,919	30,602

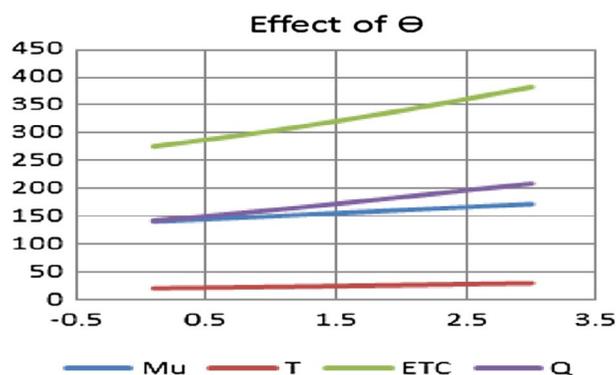
**Figure 1. The setup cost effect on total production cost.**



**Figure 2. The inventory holding cost effect on total production cost.**



**Figure 3. The failure rate effect on total production cost.**



### 3. Numerical example and sensitivity analysis

A real-world example and sensitivity analysis illustrates the proposed model, using a case study of a spring production line in Iran. One of the most common types of springs is the compressing spring.

The spring constant is the qualitative characteristic. Let  $\sigma = 1$  when the process is control,  $p = 70,000$  annual production rate,  $D = 50,000$  annual demand rate,  $I = \$1,000$  per unit time,  $h = \$300$  per unit per unit time,  $\theta = 0.03$ ,  $\lambda = 0.25$ ,  $M(t) = 2e^{0.03t}$ ,  $S(t) = 150e^{-0.9t}$ ,  $A = 6.14 \times 10^{-4}$ ,  $\xi = 140$ ,  $\alpha = 0.05$ . MATLAB 2013 software used for the calculations; the optimal total cost was calculated as \$14,349, with the optimal qualitative characteristic mean process calculated as 138.43.

The results are compared with the classic EPQ model, in which the optimal total cost was \$29,277. The comparison to the previous model is displayed in Table 1.

The results demonstrate more optimal financial results from the proposed approach compared with the EPQ model. The sensitivity analysis is performed by changing parameters ( $I, h, \theta$ ). Figure 1 shows that the effect of  $I$  on total production cost (ETC) and EPQ ( $Q$ ). With the rising costs of setup time, production cycle and total production cost (ETC) are increasing.

Figure 2 shows that the increase in inventory holding costs reduced total cost. This means that inverse relationships exist between inventory holding cost and total production cost.

Figure 3 shows that increasing the  $\theta$  (failure rate) leads to increase in production cycle time and total production cost. In fact, the failure rate representing the slope of the line is the deviation.

The effects of the parameters on the optimal values were determined. We observed impact of loss quality cost on the total production cost. This represents importance of a combination between production planning and control process.

#### 4. Conclusion

In this study, an EPQ model is proposed considering the production cycle time and quality characteristic to minimize the total expected manufacturing cost that consists of quality loss and maintenance costs other than ordering and holding costs already considered in the classic model. We used this model to calculate total production cost in a spring production line in Iran. To investigate the initial effect of the mean quality characteristic, the computational analysis is applied on a real-world case study. The results demonstrate improved financial results from the proposed approach compared with the EPQ model. Proposed model decreased total production cost. The sensitivity analysis from our study shows that the total production cost is sensitive to loss quality cost. This represents the importance of a combination between production planning and control process. Future research may extend this model in an uncertain environment.

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