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Availability and profit analysis of a two-unit cold standby system for general distribution

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Abstract: This paper discussed a two-unit cold standby system by considering the concepts of degradation, inspection, preventive maintenance (PM), and priority. The unit works as a reduced capacity after its repair and is called degraded unit. There is a single repairman who visits the system immediately. The repairman inspects the failed degraded unit to see the feasibility of repair. If the repair of the degraded unit is not possible, it is replaced by new unit. PM will provide to the system when both the units are degraded and available for use. Priority for operation and repair is provided to new unit over-degraded unit. Various reliability characteristics such as mean time to system failure (MTSF), availability, busy period of the repairman, expected number of visits by the repairman, and profit of the system have been obtained using regenerative point graphical technique. For particular case, the results for MTSF, availability, and profit function are obtained considering exponential, Rayleigh, and Weibull distributions for all random variables.

Subjects: Applied Mathematics; Engineering & Technology; Mathematics & Statistics; Science; Statistics & Probability; Technology

Keywords: degradation; inspection; preventive maintenance; regenerative point graphical technique; general distribution

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PUBLIC INTEREST STATEMENT

Reliability analysis of a system is the study of various aspects related to system such as design, safety, risk, failure. It helps to design the system for better performance or product to perform adequately under the specified time and stated conditions. To meet the requirement of urbanization it is essential to maintain any system or product which remains sustainable for a longer period. Reliability analysis can play an important role in this regard. Considerable research work has been done by many researchers in the direction of cold standby redundant systems with constant rates but not much work has been done with general rates with RPGT in this direction. Keeping above points under consideration, this work has been carried out to analyze a two-unit cold standby system for general distribution using RPGT technique. This work will help to study this system in more general situations and will be helpful in designing the systems which are compatible in general situations.

1. Introduction

Complexity of an industrial system depends on the configuration and design of the system. In present scenario of industrial growth, complexity of the system is increasing day by day. The complexity accounts for reliability of such systems. Redundancy is one of the effective techniques to enhance reliability of system. Redundancy is the duplication of critical components or functions of a system with the sole aim to increase reliability of system or to make system fail safe. A large number of researchers including Srinivasan and Gopalan (1973), Murari and Goyal (1984) and Singh and Mishra (1994) examined the effect of redundancy on reliability of industrial system in various forms. These researchers assumed that units work as a new after its repair. But the working ability and competency of a repaired unit depend more or less on the standard of the repair mechanism exercised. If the unit is repaired by an ordinary repairman, it may work with reduced efficiency and it becomes degraded unit. Mokaddis, Labib, and Ahmed (1997) studied a reliability model of a two-unit warm standby system subject to degradation. Kumar, Kadyan, and Malik (2010) discussed a two-unit parallel system subject to degradation after repair. Sometimes, it is observed that repair of the degraded unit turns out to be infeasible due to extensive use and high cost of maintenance. To avoid such problems, inspection is carried out to see the feasibility of repair or replacement of failed degraded unit. Malik, Chand, and Singh (2008) discussed the operating systems under the concept of inspection. In addition to it, if priority is given to new unit for operation and repair over degraded unit then it causes the enhancement of mean time to system failure (MTSF), availability, and profit of the system. Chander (2005) discussed the reliability models in which priority is given to operation as well as repair. Kumar, Kadyan, and Malik (2012) discussed a two-unit cold standby system subject to degradation, inspection, and priority. Moreover, performance of the system can be increased considerably if preventive maintenance is provided to the system. Goel, Sharma, and Gupta (1986), Said, Salah, and Sherbeny (2005) and Garg and Kadyan (2016) analyzed the reliability models using the concept of PM.

To avoid massive computation and long methodology used in regenerative point technique Gupta and Singh (2007, 2008) analyzed various stochastic systems using a new technique called RPGT for determining MTSF, availability of the system, busy period of the repairman, and expected number of visits by the repairman and profit of the system model with the assumption that failure and repair rates are constant. But in actual practice the variables associated with the reliability models are not bound with the constant failure and repair rates. So, to obtain the more reliable and flexible information about the reliability models, we consider all rates associated with model are general. Keeping above in mind, here we discussed a two-unit cold standby system by considering the concepts of degradation, inspection, PM, and priority. The unit works as a reduced capacity after its repair and is called degraded unit. There is a single repairman who visits the system immediately. The repairman inspects the failed degraded unit to see the feasibility of repair. If the repair of the degraded unit is not possible, it is replaced by new unit. PM will provide to the system when both the units are degraded and available to use. Priority for operation and repair is provided to new unit over degraded unit. Various reliability characteristics such as MTSF, availability, busy period of the repairman, expected number of visits by the repairman, and profit of the system have been obtained by using RPGT. For particular case, the results for MTSF, availability and profit function are obtained considering exponential, Rayleigh, and Weibull distributions for all random variables.

1.1. Notations

N_o/D_0	The unit is new/degraded and operative
N_{cs}/D_{cs}	The new/degraded unit in cold standby
N_{Fur}/N_{FUR}	New unit is failed and under repair/under continuous repair from previous state
D_{Fui}/D_{FUI}	Degraded unit is failed and under inspection/under continuous inspection from previous state
D_{fwi}/D_{FWI}	Degraded unit is failed and waiting for inspection/waiting for inspection continuously from previous state

D_{Fur}/D_{Fwr}	Degraded unit is failed and under repair/waiting for repair
D_{pm}/D_{PM}	Degraded unit is under PM/under continuous PM from previous state
$f(t)/F(t)$, $f_1(t)/F_1(t)$	p.d.f./c.d.f. for failure time distribution of new unit/degraded unit
$g(t)/G(t)$, $g_1(t)/G_1(t)$,	p.d.f./c.d.f. of repair time of the new failed unit/degraded unit
$g_2(t)/G_2(t)$	p.d.f./c.d.f. of PM time of the degraded unit
$w(t)/W(t)$	p.d.f./c.d.f. of time by which unit undergoes for PM
$h(t)/H(t)$	p.d.f./c.d.f. of inspection time of the degraded unit
$W_i(t)$	Probability that the server is busy in a particular job in regenerative state *i* at epoch *t* without passing through any other regenerative state, given that the system entered regenerative state *i* at $t = 0$
p/q	Probability that repair of the degraded system is feasible/not feasible
μ_i	The mean sojourn time spent in state *i* before visiting any other state $\mu_i = \int_0^\infty R_i(t)dt$
μ_i^1	The total unconditional time spent before transiting to any other regenerative state, given the system is in regenerative state *i* at $t = 0$, $\mu_i^1 = \mu_i$, if the system transits only to regenerative state from state *i*
$\overline{cycle}$	A circuit formed through un-failed states
$k - cycle$	A circuit whose terminals are at the regenerative point *k*. The circuit may be formed through regenerative/non-regenerative states/and failed states
$k - \overline{cycle}$	A circuit whose terminals are at the regenerative point *k* and formed through only regenerative/non-regenerative but un-failed states only
$p_{i,j}$	Steady-state transition probability from a regenerative state *i* to the regenerative state *j* without visiting any other state

1.2. State specification

$$
\begin{array}{llll}
S_0 = (N_0,\ N_{cs}) & S_1 = (N_o,\ N_{Fur}) & S_2 = (N_{wr},\ N_{FUR}) & S_3 = (N_0,\ D_{cs}) \\
S_4 = (D_o,\ N_{Fur}) & S_5 = (D_{Fwi},\ N_{FUR}) & S_6 = (D_0,\ D_{cs}) & S_7 = (D_o,\ D_{Fui}) \\
S_8 = (D_0,\ D_{pm}) & S_9 = (D_{Fwi},\ D_{FUI}) & S_{10} = (D_{fwi},\ D_{PM}) & S_{11} = (D_{Fwi},\ D_{FUR}) \\
S_{12} = (N_o,\ D_{Fui}) & S_{13} = (N_o,\ D_{Fur}) & S_{14} = (N_{Fur},\ D_{Fwi}) & S_{15} = (N_{Fur},\ D_{Fwr}) \\
S_{16} = (D_o, D_{Fur}) & S_{17} = (D_{Fur}, D_{FWI}) & &
\end{array}
$$

The states S_0, S_1, S_3, S_4, S_6, S_7, S_8, S_{12}, S_{13}, S_{14}, S_{15}, and S_{16} are regenerative states, while the others i.e., S_2, S_5, S_9, S_{10}, S_{11}, and S_{17} are non-regenerative states.

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1.3. Transition probabilities and mean sojourn times

$$p_{01} = \int_0^\infty f(t)dt = p_{34} \qquad p_{12} = \int_0^\infty f(t)\overline{G(t)}dt \qquad p_{13} = \int_0^\infty g(t)\overline{F(t)}dt$$

$$p_{24} = \int_0^\infty g(t)dt = p_{57} = p_{14,7} = p_{15,16} \qquad p_{45} = \int_0^\infty f_1(t)\overline{G(t)}dt \qquad p_{46} = \int_0^\infty g(t)\overline{F_1(t)}dt$$

$$p_{67} = \int_0^\infty f_1(t)\overline{W(t)}dt \qquad p_{68} = \int_0^\infty w(t)\overline{F_1(t)}dt \qquad p_{73} = \int_0^\infty qh(t)\overline{F_1(t)}dt$$

$$p_{79} = \int_0^\infty f_1(t)\overline{H(t)}dt \qquad p_{7,16} = \int_0^\infty ph(t)\overline{F(t)}dt \qquad p_{86} = \int_0^\infty g_2(t)\overline{F_1(t)}dt$$

$$p_{8,10} = \int_0^\infty f_1(t)\overline{G_2(t)}dt \qquad p_{9,12} = \int_0^\infty qh(t)dt \qquad p_{9,17} = \int_0^\infty ph(t)dt$$

$$p_{10,7} = \int_0^\infty g_2(t)dt \qquad p_{11,7} = \int_0^\infty g_1(t)dt = p_{17,7} \qquad p_{12,0} = \int_0^\infty qh(t)\overline{F(t)}dt$$

$$p_{12,13} = \int_0^\infty ph(t)\overline{F(t)}dt \qquad p_{12,14} = \int_0^\infty f(t)\overline{H(t)}dt \qquad p_{13,3} = \int_0^\infty g_1(t)\overline{F(t)}dt$$

$$p_{13,15} = \int_0^\infty f(t)\overline{G_1(t)}dt \qquad p_{16,6} = \int_0^\infty g_1(t)\overline{F_1(t)}dt \qquad p_{16,11} = \int_0^\infty f_1(t)\overline{G_1(t)}dt$$

From the above it is verified that

$$\begin{aligned} p_{01} &= p_{24} = p_{34} = p_{57} = p_{10,7} = p_{11,7} = p_{14,7} = p_{15,16} = p_{17,7} = p_{12} + p_{13} = p_{45} + p_{46} \\ &= p_{67} + p_{68} = p_{73} + p_{79} + p_{7,16} = p_{86} + p_{8,10} = p_{9,12} + p_{9,17} = p_{12,0} + p_{12,13} + p_{12,14} \\ &= p_{13,3} + p_{13,15} = p_{16,6} + p_{16,11} = 1 \end{aligned}$$

The mean sojourn times μ_i in states S_i are given as follows:

$$\mu_0 = \int_0^\infty \overline{F(t)}dt = \mu_3 \qquad \mu_1 = \int_0^\infty \overline{F(t)G(t)}dt \qquad \mu_2 = \int_0^\infty \overline{G(t)}dt$$

$$\mu_4 = \int_0^\infty \overline{F_1(t)G(t)}dt \qquad \mu_5 = \int_0^\infty \overline{G(t)}dt \qquad \mu_6 = \int_0^\infty \overline{F_1(t)W(t)}dt$$

$$\mu_7 = \int_0^\infty \overline{F_1(t)H(t)}dt \qquad \mu_8 = \int_0^\infty \overline{G_2(t)F_1(t)}dt \qquad \mu_9 = \int_0^\infty \overline{H(t)}dt$$

$$\mu_{10} = \int_0^\infty \overline{G_2(t)}dt \qquad \mu_{11} = \int_0^\infty \overline{G_1(t)}dt \qquad \mu_{12} = \int_0^\infty \overline{F(t)H(t)}dt$$

$$\mu_{13} = \int_0^\infty \overline{F(t)G_1(t)}dt \qquad \mu_{16} = \int_0^\infty \overline{F_1(t)G_1(t)}dt$$

2. Mean time to system failure

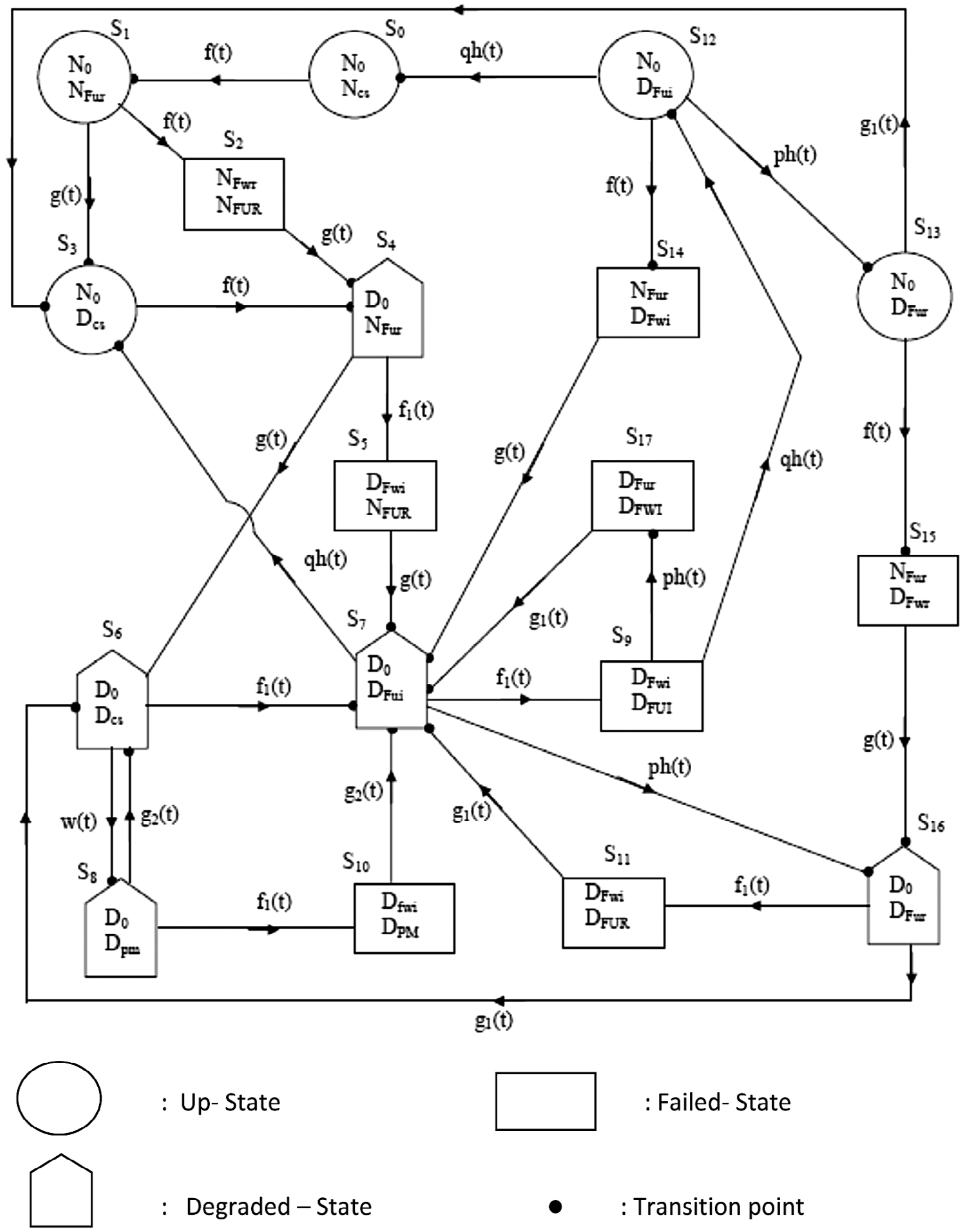
The regenerative un-failed states visited by the system before transiting to any failed states are: $i = 0, 1, 2$, and 4. The MTSF is given by

$$\text{MTSF} = \left[\sum_{i, s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r(sff)} i)\} \cdot \mu_i}{\prod_{k_1 \neq 0} \{1 - \sum pr(k_1 - \overline{\text{cycle}})\}} \right\} \right] \div \left[1 - \sum_{s_r} \left\{ \frac{\{pr(0 \xrightarrow{s_r(sff)} 0)\}}{\prod_{k_2 \neq 0} \{1 - \sum pr(k_2 - \overline{\text{cycle}})\}} \right\} \right]$$

where i: a regenerative un-failed state to which the system can transit before any failures from the initial state "0" (at time $t = 0$); k_1: a regenerative state visited along the path $(0 \xrightarrow{s_r(sff)} i)$, at which a $k_1 - \overline{cycle}$ is formed through regenerative un-failed states; k_2: a regenerative state visited along the path $(0 \xrightarrow{s_r(sff)} 0)$, at which a $k_2 - \overline{cycle}$ is formed through regenerative un-failed states.

From Figure 1, regenerative un-failed states to which the system can transit before entering any failed state are 0, 1, 3, 4, 6, 7, 8, and 16, $k_1 = 3, 6$.

Figure 1. State transition diagram.



$$\text{MTSF} = \mu_0 + \mu_1 + p_{13}(\mu_3 + \mu_4)/A + p_{13}p_{46}(\mu_6 + p_{68}\mu_8)/AB + p_{13}p_{46}p_{67}(\mu_7 + p_{7,16}\mu_{16})/AB$$

where

$A = 1 - p_{46}p_{67}p_{73}/B$ and $B = 1 - (p_{68}p_{86} + p_{67}p_{7,16}p_{16,6})$

3. Steady-state availability of the system

Total fraction of time for which the system is available is given by the formula (using RPGT) is

$$A_0 = \left[\sum_{j,s_r}\left\{\frac{pr(0 \xrightarrow{s_r} j) f_j \mu_j}{\underset{k_1 \neq 0}{\pi}\{1 - \sum pr(k_1 - cycle)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(0 \xrightarrow{s_r} i)\} \cdot \mu_i^1}{\underset{k_2 \neq 0}{\pi}\{1 - \sum pr(k_2 - cycle)\}}\right\}\right]$$

where j: an available state (which may be down/or reduced state) reachable from the state "0"; i: a regenerative state; k_i (≠0): a regenerative point at which a k_i-cycle is formed (may be through non-regenerative/failed states). k_1 is a regenerative state visited along the path ($0 \xrightarrow{s_r} j$) and k_1 can be equal to j. And k_2 is a regenerative state visited along the path ($0 \xrightarrow{s_r} i$) and k_2 can be equal to "i".

From Figure 1, the regenerative states at which the system is in up states are j = 0, 1, 3, 4, 6, 7, 8, 12, 13, and 16 with $f_j = 1$ and $k_1 = k_2 = 3, 6, 7$

$$A_0 = N/D$$

$$N = (\mu_0 + \mu_1) + (p_{13}\mu_3 + \mu_4)/A_1 + (\mu_6 + p_{68}\mu_8)\,(p_{46} + p_{45}p_{16,\,6}(p_{7,\,16} + p_{79}p_{9,\,12}p_{12,\,13}p_{13,\,15})/C)/A_1 * B_1 + (\mu_7 + p_{79}p_{9,\,12}(\mu_{12} + p_{12,\,13}\mu_{13} + p_{12,\,13}p_{13,\,15}\mu_{16}) + p_{7,\,16}\mu_{16})\,(p_{45} + (p_{46}p_{67})/B_1)/A_1 * C$$

$$D = (\mu_0 + \mu_1^1) + (p_{13}\mu_3 + \mu_4^1)/A_1 + (\mu_6 + p_{68}\mu_8^1)(p_{46} + p_{45}p_{16,\,6}(p_{7,\,16} + p_{79}p_{9,\,12}p_{12,\,13}p_{13,\,15})/C)/A_1 * B_1 \; (\mu_7^1 + p_{79}p_{9,\,12}(\mu_{12} + p_{12,\,13}\mu_{13} + p_{12,\,13}p_{13,\,15}\mu_{16}^1) + p_{7,\,16}\mu_{16})(p_{45} + (p_{46}p_{67})/B_1)/A_1 * C + p_{79}p_{9,\,12}(p_{12,\,14}\mu_{14} + p_{12,\,13}p_{13,\,15}\mu_{15})((p_{45} + (p_{46}p_{67})/B_1)/A_1 * C)$$

where

$$A_1 = 1 - (p_{73}(p_{46}p_{67}/B_1 + p_{45} + p_{46}p_{68}p_{8,\,10}/B_1))/C$$

$$B_1 = A_2 - p_{16,\,6}A_2(p_{7,\,16} + p_{79}p_{9,\,12}p_{12,\,13}p_{13,\,15})/C$$

$$C = 1 - (p_{79}p_{9,\,17} + p_{7,\,16}p_{16,\,11} + p_{79}p_{9,\,12}(p_{12,\,14} + p_{12,\,13}p_{13,\,15}p_{16,\,11})$$

$$A_2 = 1 - p_{68}p_{86}$$

4. Busy period analysis

The busy period of the server doing any given job, is given by the formula (using RPGT) is

$$B_0 = \left[\sum_{j,\,s_r}\left\{\frac{\{pr(0 \longrightarrow s_r j)\}.W_j}{\prod_{k_{1\neq 0}}\{1 - \sum pr(k_1 - cycle)\}}\right\}\right] \div \left[\sum_{i,\,s_r}\left\{\frac{\{pr(0 \xrightarrow{s_r} i)\}\cdot \mu_i^1}{\pi_{k_2 \neq 0}\{1 - \sum pr(k_2 - cycle)\}}\right\}\right]$$

where j: a reachable regenerative state (reachable from the state "0") which may be partially or a totally failed state where the server is busy doing any given job like repair/replacement/inspection/giving instructions, etc.); i: a regenerative state.

4.1. Due to repair of the unit

The regenerative states where the repairman is busy in doing repair of the unit are 1, 4, 13, 14, 15, and 16

$$B_1 = W_1 + W_4/A_1 + (p_{79}p_{9,\,12}(p_{12,\,14}W_{14} + p_{12,\,13}(W_{13} + p_{13,\,15}W_{15} + p_{13,\,15}p_{15,\,16}W_{16})) + p_{7,\,16}W_{16}((p_{45} + (p_{46}p_{67})/B_1)/A_1 * C))$$

D is already solved.

4.2. Due to inspection of the degraded unit

The regenerative states where the repairman is busy in doing inspection of the unit are 7 and 12

$$B_2 = (W_7 + p_{79}p_{9,\,12}W_{12})((p_{45} + (p_{46}p_{67})/B_1)/A_1 * C)$$

4.3. Due to PM of the degraded unit
The regenerative states where the repairman is busy in doing preventive maintenance of the unit is 8

$$B_3 = p_{68}(p_{46} + p_{45}p_{16,6}(p_{7,16} + p_{79}p_{9,12}p_{12,13}p_{13,15})/C)/A_1 * B_1 * W_8$$

5. Expected number of visits
The expected number of the visits of the server/replacements, etc. is given by the formula (using RPGT) is

$$V_0 = \left[\sum_{j,s_r}\left\{\frac{pr\,(0 \xrightarrow{s_r} j)}{\prod_{k_{1\neq 0}}\{1-\sum pr(k_1-cycle)\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(0 \xrightarrow{s_r} i)\}\cdot \mu_i^1}{\prod_{k_{2\neq 0}}\{1-\sum pr(k_2-cycle)\}}\right\}\right]$$

where j: a reachable regenerative state (reachable from the state "0"), where the server visits afresh may be after staying, exiting, absenting, and then again entering the system. In the later case, "j" appears as an interior point on the path $(0 \xrightarrow{s_r} j)$/replacement takes place (afresh) at the regenerative state j along the path $(0 \xrightarrow{s_r} j)$; i: a regenerative state; k_i (≠0) A regenerative state visited along a path (may be an interior or a terminal point of the path) at which a k_i-cycle is formed (which may be formed through non-regenerative or failed states). k_1 is a regenerative state visited along the path $(0 \xrightarrow{s_r} j)$ and $k_1 \neq j$. k_2 is a regenerative state visited along the path $(0 \xrightarrow{s_r} i)$ and k_2 can be equal to "i."

5.1. Due to the repair of the unit
The regenerative states where the repairman visits for the repair of the unit are 1, 4, 13, 14, 15, and 16

$$\begin{aligned} E_1 =& 1 + 1/A_1 + (p_{79}p_{9,12}(p_{12,14} + p_{12,13}(1 + p_{13,15} + p_{13,15}p_{15,16})) \\ & + p_{7,16}((p_{45} + (p_{46}p_{67})/B_1)/A_1 * C)) \end{aligned}$$

D is already solved.

5.2. Due to inspection of the degraded unit
The regenerative states where the repairman visit for doing inspection of the unit are 7 and 12

$$E_2 = (1 + p_{79}p_{9,12})\,((p_{45} + (p_{46}p_{67})/B_1)/A_1 * C))$$

5.3. Due to PM of the degraded unit
The regenerative states where the repairman visit for doing preventive maintenance of the unit is 8

$$E_3 = p_{68}(p_{46} + p_{45}p_{16,6}(p_{7,16} + p_{79}p_{9,12}p_{12,13}p_{13,15})/C)/A_1 * B_1$$

6. Profit analysis
Any manufacturing industry is basically a profit-making organization and no organization can survive for long without minimum financial returns for its investment. There must be an optimal balance between the reliability aspect of a product and its cost. The major factors contributing to the total cost are availability, busy period of the repairman, and expected number of visits by the repairman. The cost of these individual items varies with reliability or MTSF. In order to increase the reliability of the products, we would require a correspondingly high investment in the research and development activities. The production cost also would increase with the requirement of greater reliability.

The revenue and cost function leads to the profit function of a firm/organization, as the profit is excess of revenue over the cost of production. The profit function in time t is given by

$P(t)$ = expected revenue in (0, t] − expected total cost in (0, t]

In general, the optimal policies can more easily be derived for an infinite time span or compared to a finite time span. The profit per unit time, in infinite time span is expressed as follows:

$$\lim_{t\to\infty} \frac{P(t)}{t}$$

i. e. profit per unit time = total revenue per unit time−total cost per unit time. Considering the various costs, the profit equation is given as follows:

$$P = C_0A_0 - C_1B_1 - C_2B_2 - C_3B_3 - C_4E_1 - C_5E_2 - C_6E_3$$

where P = Profit per unit time incurred to the system; C_0 = Revenue per unit up time of the system; A_0 = Total fraction of time for which the system is operative; C_1 = Cost per unit time for which repairman is busy due to repair of the unit; B_1 = Total fraction of time for which repairman is busy due to repair of the unit; C_2 = Cost per unit time for which repairman is busy due to inspection of degraded unit; B_2 = Total fraction of time for which repairman is busy due to inspection of degraded unit; C_3 = Cost per unit time for which repairman is busy due to preventive maintenance of degraded unit; B_3 = Total fraction of time for which repairman is busy due to preventive maintenance of degraded unit; C_4 = Cost per visit by the repairman due to repair of the unit; E_1 = Expected number of visit per unit time for the repairman due to repair of the unit; C_5 = Cost per visit by the repairman due to inspection of degraded unit; E_2 = Expected number of visit per unit time for the repairman due to inspection of degraded unit; C_6 = Cost per visit by the repairman due to preventive maintenance of degraded unit; E_3 = Expected number of visit per unit time for the repairman due to preventive maintenance of degraded unit.

7. Results and discussion

To show the importance of results and characterize the behavior of MTSF, availability, and profit of the system, here we assume that the distribution of failure times of units, repair times of the units, inspection times and PM time, as Weibull distribution with two parameter. Probability density function of Weibull distribution with two parameters is given by

$$f(t) = k\lambda(\lambda t)^{k-1}\exp(-\lambda t)^k, \quad \lambda > 0 \text{ and } t \geq 0$$

When k = 1, it follows exponential distribution and when k = 2, it follows Rayleigh distribution

Let

$$\begin{array}{ll} f_1(t) = k\lambda_1(\lambda_1 t)^{k-1}\exp(-\lambda_1 t)^k & g(t) = kr(rt)^{k-1}\exp(-rt)^k \\ g_1(t) = kr_1(r_1 t)^{k-1}\exp(-r_1 t)^k & g_2(t) = kr_2(r_2 t)^{k-1}\exp(-r_2 t)^k \\ h(t) = k\theta(\theta t)^{k-1}\exp(-\theta t)^k & w(t) = k\beta(\beta t)^{k-1}\exp(-\beta t)^k \end{array}$$

The numerical results for MTSF, availability, and profit functions are obtained in Tables 1–9 by considering Weibull, exponential, and Rayleigh distributions for all random variables. Behaviour of MTSF, availability, and profit of a two-unit cold standby system with respect to failure rate of new unit for Weibull, exponential, and Rayleigh distributions are shown in Tables 1–9 by taking the values of different parameters λ_1 = 0.13, r = 1.6, r_1 = 2.1, r_2 = 3, θ = 2, β = 20, p = 0.6, q = 0.4, C_1 = 5,000, C_2 = 450, C_3 = 150, C_4 = 100, C_5 = 75, C_6 = 50, and C_7 = 25. Tables 1–3 show the trend of MTSF for Weibull, exponential, and Rayleigh distributions, respectively. From Tables 1–3, it is observed that MTSF will decrease with the increase of failure rates of new unit (λ) and degraded unit (λ_1), and increase with the increase of repair rate of new unit (r), preventive maintenance rate (r_2), and inspection rate (θ) of degraded unit. It is also found that value of MTSF is higher in the case when random variables follow Weibull distribution. In Tables 4–9, we calculate the numerical results of availability and profit, respectively, with different distributions as Weibull, exponential, and Rayleigh, and these tables show that the availability and profit go on decreasing with the increase of failure rates of new unit (λ) and degraded unit (λ_1). It means that the system will become more available and profitable as management of industry pay more attention toward decreasing the failure rates of new unit and degraded unit. Availability and profit increase with the increase of repair rate (r), preventive maintenance rate (r_2), and inspection rate (θ) of degraded unit. Values of availability and profit are higher for Rayleigh distribution. and they depict the higher value in case of Rayleigh distribution. It is also observed that system becomes more available to use and profitable if the degraded unit at its failure is replaced by new one. It means that when we provide inspection to degraded unit then the system becomes more advantageous because inspection is undertaken to ensure the feasibility of the repair or replacement of degraded unit. Moreover, it will help to save the time and cost of the producer as well as the consumer.

Table 1. Effect of failure rate of new unit and other parameters for Weibull distribution on MTSF

MTSF						
λ	**Weibull distribution**					
	$\lambda_1 = 0.13$	**$\lambda_1 = 0.15$**	**$r = 1.8$**	**$r_2 = 5$**	**$\theta = 12$**	**$p = 0.4, q = 0.6$**
0.01	387.285	387.260	387.891	387.563	387.291	387.327
0.02	191.794	191.770	192.153	191.874	191.800	191.835
0.03	127.092	127.068	127.346	127.172	127.098	127.131
0.04	94.911	94.888	95.1054	94.920	94.917	94.950
0.05	75.686	75.664	75.841	75.721	75.692	75.725
0.06	62.918	62.895	63.044	62.929	62.923	62.956
0.07	53.827	53.805	53.932	53.933	53.833	53.864
0.08	47.029	47.007	47.117	47.067	47.034	47.066
0.09	41.755	41.734	41.830	41.872	41.761	41.792
0.1	37.546	37.525	37.610	37.726	37.552	37.582

Table 2. Effect of failure rate of new unit and other parameters for exponential distribution on MTSF

MTSF						
λ	**Exponential distribution**					
	$\lambda_1 = 0.13$	**$\lambda_1 = 0.15$**	**$r = 1.8$**	**$r_2 = 5$**	**$\theta = 12$**	**$p = 0.4, q = 0.6$**
0.01	200.621	200.615	200.657	200.631	200.622	200.637
0.02	100.617	100.611	100.653	100.627	100.618	100.633
0.03	67.280	67.274	67.315	67.289	67.281	67.295
0.04	50.609	50.604	50.644	50.619	50.611	50.625
0.05	40.605	40.600	40.640	40.615	40.607	40.621
0.06	33.935	33.930	33.969	33.945	33.937	33.951
0.07	29.170	29.164	29.203	29.179	29.171	29.185
0.08	25.595	25.589	25.628	25.604	25.596	25.610
0.09	22.813	22.808	22.846	22.823	22.815	22.829
0.1	20.588	20.583	20.620	20.597	20.589	20.603

Table 3. Effect of failure rate of new unit and other parameters for Rayleigh distribution on MTSF

MTSF						
λ	Rayleigh distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	178.353	178.353	178.426	178.361	178.355	178.355
0.02	89.727	89.726	89.799	89.735	89.729	89.729
0.03	60.182	60.182	60.254	60.190	60.185	60.185
0.04	45.408	45.408	45.480	45.416	45.411	45.410
0.05	36.542	36.542	36.613	36.550	36.544	36.544
0.06	30.630	30.630	30.701	30.638	30.633	30.632
0.07	26.406	26.406	26.476	26.414	26.409	26.408
0.08	23.237	23.237	23.307	23.245	23.239	23.239
0.09	20.771	20.771	20.840	20.779	20.774	20.774
0.1	18.798	18.797	18.866	18.806	18.800	18.800

Table 4. Effect of failure rate of new unit and other parameters for Weibull distribution on availability

Availability						
λ	Weibull distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	0.995043	0.994830	0.995379	0.995187	0.996051	0.995638
0.02	0.989780	0.989341	0.990487	0.990063	0.991877	0.990986
0.03	0.984403	0.983731	0.985493	0.984818	0.987609	0.986213
0.04	0.978982	0.978076	0.980458	0.979523	0.983296	0.981382
0.05	0.973559	0.972416	0.975418	0.974218	0.978968	0.976528
0.06	0.968158	0.966781	0.970397	0.968930	0.974642	0.971676
0.07	0.962799	0.961188	0.965411	0.963677	0.970333	0.966844
0.08	0.957495	0.955652	0.960471	0.958474	0.966051	0.962042
0.09	0.952256	0.950184	0.955587	0.953330	0.961802	0.957282
0.1	0.947089	0.944790	0.950766	0.948253	0.957594	0.952571

Table 5. Effect of failure rate of new unit and other parameters for exponential distribution on availability

Availability						
λ	Exponential distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	0.999293	0.999200	0.999374	0.999297	0.999364	0.999339
0.02	0.998587	0.998401	0.998751	0.998596	0.998732	0.998678
0.03	0.997883	0.997603	0.998129	0.997895	0.998102	0.998018
0.04	0.997180	0.996808	0.997510	0.997196	0.997475	0.997359
0.05	0.996480	0.996015	0.996893	0.996499	0.996852	0.996701
0.06	0.995781	0.995224	0.996278	0.995804	0.996231	0.996045
0.07	0.995085	0.994436	0.995667	0.995111	0.995615	0.995390
0.08	0.994392	0.993651	0.995058	0.994420	0.995001	0.994738
0.09	0.993702	0.992870	0.994453	0.993733	0.994391	0.994087
0.1	0.993015	0.992093	0.993851	0.993048	0.993786	0.993439

Table 6. Effect of failure rate of new unit and other parameters for Rayleigh distribution on availability

Availability						
λ	Rayleigh distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	0.999975	0.999967	0.999981	0.999976	0.999976	0.999976
0.02	0.999952	0.999936	0.999963	0.999953	0.999953	0.999952
0.03	0.999928	0.999905	0.999946	0.999931	0.999930	0.999929
0.04	0.999906	0.999875	0.999929	0.999909	0.999908	0.999907
0.05	0.999883	0.999845	0.999912	0.999888	0.999886	0.999885
0.06	0.999861	0.999816	0.999895	0.999866	0.999865	0.999863
0.07	0.999840	0.999787	0.999879	0.999845	0.999844	0.999842
0.08	0.999818	0.999759	0.999863	0.999825	0.999823	0.999821
0.09	0.999797	0.999731	0.999847	0.999804	0.999803	0.999800
0.1	0.999777	0.999704	0.999831	0.999784	0.999783	0.999780

Table 7. Effect of failure rate of new unit and other parameters for Weibull distribution on profit

Profit						
λ	Weibull distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	4,970.234	4,969.169	4,972.255	4,970.870	4,975.649	4,973.779
0.02	4,938.825	4,936.626	4,943.051	4,940.060	4,950.045	4,945.991
0.03	4,906.839	4,903.479	4,913.327	4,908.637	4,923.946	4,917.576
0.04	4,874.667	4,870.133	4,883.426	4,876.992	4,897.630	4,888.882
0.05	4,842.534	4,836.823	4,853.546	4,845.351	4,871.261	4,860.111
0.06	4,810.585	4,803.700	4,823.818	4,813.861	4,844.946	4,831.394
0.07	4,778.923	4,770.869	4,794.332	4,782.624	4,818.762	4,802.826
0.08	4,747.619	4,738.407	4,765.153	4,751.716	4,792.766	4,774.477
0.09	4,716.729	4,706.369	4,736.330	4,721.191	4,767.000	4,746.400
0.1	4,686.291	4,674.796	4,707.900	4,691.090	4,741.500	4,718.634

Table 8. Effect of failure rate of new unit and other parameters for exponential distribution on profit

Profit						
λ	Exponential distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	4,992.230	4,991.755	4,992.933	4,992.985	4,992.755	4,992.779
0.02	4,984.519	4,983.571	4,985.926	4,984.633	4,985.566	4,985.603
0.03	4,976.869	4,975.450	4,978.980	4,976.944	4,978.434	4,978.474
0.04	4,969.282	4,967.391	4,972.095	4,970.321	4,971.360	4,971.394
0.05	4,961.759	4,959.398	4,965.271	4,962.564	4,964.344	4,964.365
0.06	4,954.300	4,951.472	4,958.511	4,955.874	4,957.388	4,957.386
0.07	4,946.908	4,943.614	4,951.813	4,948.254	4,950.492	4,950.461
0.08	4,939.583	4,935.824	4,945.179	4,941.703	4,943.656	4,943.589
0.09	4,932.325	4,928.105	4,938.609	4,935.223	4,936.882	4,936.773
0.1	4,925.136	4,920.456	4,932.104	4,928.814	4,930.170	4,930.012

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Table 9. Effect of failure rate of new unit and other parameters for Rayleigh distribution on profit

Profit

λ	Rayleigh distribution					
	$\lambda_1 = 0.13$	$\lambda_1 = 0.15$	$r = 1.8$	$r_2 = 5$	$\theta = 12$	$p = 0.4, q = 0.6$
0.01	4,996.330	4,996.285	4,996.670	4,996.430	4,996.425	4,996.396
0.02	4,992.734	4,992.646	4,993.403	4,992.832	4,992.920	4,992.865
0.03	4,989.211	4,989.080	4,990.198	4,990.437	4,989.483	4,989.403
0.04	4,985.756	4,985.583	4,987.051	4,986.342	4,986.111	4,986.007
0.05	4,982.367	4,982.154	4,983.962	4,983.564	4,982.802	4,982.676
0.06	4,979.042	4,978.788	4,980.927	4,979.409	4,979.552	4,979.405
0.07	4,975.777	4,975.483	4,977.944	4,976.030	4,976.359	4,976.193
0.08	4,972.571	4,972.238	4,975.012	4,973.131	4,973.222	4,973.037
0.09	4,969.420	4,969.049	4,972.129	4,970.478	4,970.138	4,969.936
0.1	4,966.324	4,965.915	4,969.293	4,967.309	4,967.105	4,966.886

8. Conclusion

From the Tables 1–9, it is concluded that Weibull distribution has more values for MTSF as compare to the exponential and Rayleigh distributions under stated conditions. Rayleigh distribution has more values for availability and profit of the system model than the exponential and Weibull distributions. If random variables associated with the system follow Rayleigh distribution then system can be made more available for use and profitable. It is also stated that when we give inspection to the degraded unit when it completely fail, by inspection we come to know that the unit is repairable or it should be replaced by a new unit which helps the system to attain more available to use and profitable.

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