



Received: 24 July 2016
Accepted: 10 September 2016
First Published: 26 September 2016

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Reviewing editor:
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PURE MATHEMATICS | RESEARCH ARTICLE

On q -series and continued fractions

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Abstract: In this paper we have established interesting results involving continued fraction. Special cases of the result established herein have also been discussed.

Subjects: Mathematics & Statistics; Physical Sciences; Science

Keywords: q -series; continued fraction identities

2010 Mathematics subject classification: 33D15

1. Introduction, notations and definitions

Continued fractions have been playing a very important role in number theory and classical analysis ever since the times of Euler and Gauss. Generalized hypergeometric series, both ordinary and basic, have been a very significant tool in the derivation of continued fraction representations. Bhargava and Adiga (1984), Bhargava, Adiga, and Somashekara (1987), Denis and Singh (2000, 2011), Srivastava, Singh, and Singh (2015) and many others have established a good number of results involving q -series and continued fractions. One is also referred to see the papers Cao and Srivastava (2013), Choi and Srivastava (2014), Luo and Srivastava (2011), Srivastava (2011), Srivastava and Choi (2012) and Srivastava and Choudhary (2015) for useful and interesting similar results. In what follows, we shall use the following usual notations and definitions. Some interesting applications in the direction of quantum calculus can be seen in Mishra, Khatri, Mishra, and Deepmala (2013), Mishra, Khan, Khatri, and Mishra (2013), Mishra, Khatri, and Mishra (2012, 2013a, 2013b), Mishra, Sharma, and Mishra (2016), Gairola, Deepmala, and Mishra (2016a, 2016b), and Singh, Gairola, and Deepmala (2016).

Let

$$[a; q]_n = (a; q)_n = \begin{cases} (1-a)(1-aq)(1-aq^2) \dots (1-aq^{n-1}) & \text{if } n \geq 1; \\ 1 & \text{if } n = 0. \end{cases}$$

Following (Srivastava and Karlsson, 1985, 272 p. 347) a basic hypergeometric series is defined as,



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Vishnu Narayan Mishra received the PhD in Mathematics from Indian Institute of Technology, Roorkee. His research interests are in the areas of pure and applied mathematics. He has published more than 110 research articles in reputed international journals of mathematical and engineering sciences. He is a referee and an editor of several international journals in frame of Mathematics. He guided many postgraduate and PhD students. Citations of his research contributions can be found in many books and monographs, PhD thesis and scientific journal articles.

PUBLIC INTEREST STATEMENT

This paper is interesting and useful from application point of view. Approximate values of many known functions can be evaluated with the help of continued fractions. Research scholars will get motivation through this paper.

$${}_r\Phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; q; z \\ b_1, b_2, \dots, b_s \end{matrix} \right] = \sum_{n=0}^{\infty} \frac{(a_1, a_2, \dots, a_r; q)_n z^n}{(q, b_1, b_2, \dots, b_s; q)_n} \left\{ (-)^n q^{n(n-1)/2} \right\}^{1+s-r},$$

where $|q| < 1$. When $1 + s > r$, series converges for $|z| < \infty$ and $1 + s = r$, it converges in the unit circle $|z| < 1$.

An expression of the form

$$\frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \dots} \frac{a_n}{b_n}}}$$

is said to be a finite continued fraction and as $n \rightarrow \infty$, it is called an infinite continued fraction. It is also represented by,

$$b_1 + \frac{a_1}{b_2 + \frac{a_2}{b_3 + \frac{a_3}{b_4 + \dots}}}$$

1.1. Main results

In this section we shall establish our main result:

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{(b; q)_n z^n}{(c; q)_n} &= \sum_{n=0}^{\infty} \frac{(b; q)_n (bzq/c; q)_n c^n z^n (1 - bzq^{2n}) q^{n^2 - n}}{(c; q)_n (z; q)_{n+1}} \\ &= \frac{1}{1 - (1 - c) -} \frac{z(1 - b)}{(1 - cq) -} \frac{z(1 - q)(b - c)}{(1 - cq^2) -} \frac{zq(1 - bq)(1 - c)}{(1 - cq^3) -} \frac{zq^2(1 - q^2)(b - cq)}{(1 - cq^4) -} \\ &\quad \frac{zq^2(1 - bq^2)(1 - cq)}{(1 - cq^5) -} \frac{zq^2(1 - q^3)(b - cq^2)}{(1 - cq^6) -} \frac{zq^3(1 - bq^3)(1 - cq^2)}{(1 - cq^7) -} \dots \end{aligned} \tag{1.1}$$

Proof of (1.1)

In order to prove (1.1), let us consider the ratio,

$$\begin{aligned} \frac{\sum_{n=0}^{\infty} \frac{(aq; q)_n (b; q)_n z^n}{(q; q)_n (c; q)_n}}{\sum_{n=0}^{\infty} \frac{(a; q)_n (b; q)_n z^n}{(c; q)_n (q; q)_n}} &= \frac{1}{1 + \frac{\sum_{n=0}^{\infty} \frac{(b; q)_n z^n}{(q; q)_n (c; q)_n} \{ (a; q)_n - (aq; q)_n \}}{\sum_{n=0}^{\infty} \frac{(aq; q)_n (c; q)_n z^n}{(q; q)_n (c; q)_n}} \\ &= \frac{1}{1 - \frac{\sum_{n=0}^{\infty} \frac{(aq; q)_{n-1} (b; q)_n z^n}{(q; q)_n (c; q)_n} (1 - q^n)}{\sum_{n=0}^{\infty} \frac{(aq; q)_n (b; q)_n z^n}{(q; q)_n (c; q)_n}} \\ &= \frac{1}{1 - \frac{az(1-b)/(1-c)}{1 + \frac{\sum_{n=0}^{\infty} \frac{(aq; q)_n z^n}{(q; q)_n} \left\{ \frac{(b; q)_n}{(c; q)_n} - \frac{(bq; q)_n}{(cq; q)_n} \right\}}{\sum_{n=0}^{\infty} \frac{(aq; q)_n (bq; q)_n z^n}{(q; q)_n (cq; q)_n}}} \end{aligned} \tag{1.2}$$

On simplifications (1.2) gives

$$= \frac{1}{1 - \frac{az(1-b)/(1-c)}{1 - \frac{z(b-c)(1-aq)/(1-c)(1-cq)}{1 + \frac{\sum_{n=0}^{\infty} \frac{(bq;q)_n z^n}{(q;q)_n} \left[\frac{(aq;q)_n}{(cq;q)_n} - \frac{(aq^2;q)_n}{(cq^2;q)_n} \right]}}}. \tag{1.3}$$

Iterating this process, we get

$$\frac{\sum_{n=0}^{\infty} \frac{(aq;q)_n (b;q)_n z^n}{(q;q)_n (c;q)_n}}{\sum_{n=0}^{\infty} \frac{(a;q)_n (b;q)_n z^n}{(c;q)_n (q;q)_n}} = \frac{1}{1 -} \frac{az(1-b)/(1-c)}{1 -} \frac{z(b-c)(1-aq)/(1-c)(1-cq)}{1 -} \frac{zq(a-c)(1-bq)/(1-cq)(1-cq^2)}{1 -} \frac{zq(b-cq)(1-aq^2)/(1-cq^2)(1-cq^3)}{1 -} \frac{zq^2(a-cq)(1-bq^2)/(1-cq^3)(1-cq^4)}{1 - \dots}. \tag{1.4}$$

Now, by an appeal of (Jones & Thoren, 1980, (2.3.14), p. 33) we have

$$\frac{\sum_{n=0}^{\infty} \frac{(aq;q)_n (b;q)_n z^n}{(q;q)_n (c;q)_n}}{\sum_{n=0}^{\infty} \frac{(a;q)_n (b;q)_n z^n}{(c;q)_n (q;q)_n}} = \frac{1}{1 -} \frac{az(1-b)}{(1-c) -} \frac{z(b-c)(1-aq)}{(1-cq) -} \frac{zq(a-c)(1-bq)}{(1-cq^2) -} \frac{zq(b-cq)(1-aq^2)}{(1-cq^3) -} \frac{zq^2(a-cq)(1-bq^2)}{(1-cq^4) -} \frac{zq^2(b-cq^2)(1-aq^3)}{(1-cq^5) -} \frac{zq^3(a-cq^2)(1-bq^3)}{(1-cq^6) -} \frac{zq^3(b-cq^3)(1-aq^4)}{(1-cq^7) - \dots}. \tag{1.5}$$

Finally, taking $a = 1$ in (1.5) and using Rogers-Fine identity (Andrews & Berndt, 2005, (9.1.1), p. 223) we get (1.1).

2. Special cases

In this section we shall discuss some special cases of the results (1.1) and (1.5).

(i) Putting z/b for z in (1.1) and then taking $b \rightarrow \infty$ and $c = 0$ in it we get,

$$\sum_{n=0}^{\infty} (-)^n q^{n(n-1)/2} z^n = \sum_{n=0}^{\infty} q^{2n^2-n} z^{2n} (1 - zq^{2n}) = \frac{1}{1+} \frac{z}{1-} \frac{z(1-q)}{1+} \frac{zq^2}{1-} \frac{zq(1-q^2)}{1+} \frac{zq^4}{1-} \dots. \tag{2.1}$$

(ii) If we take $z = aq$ in (2.1) we have

$$\sum_{n=0}^{\infty} (-)^n q^{n(n+1)/2} a^n = \sum_{n=0}^{\infty} q^{n(2n+1)} (1 - aq^{2n+1}) a^{2n} = \frac{1}{1+} \frac{aq}{1-} \frac{aq(1-q)}{1+} \frac{aq^3}{1-} \frac{aq^2(1-q^2)}{1+} \frac{aq^5}{1-} \dots, \tag{2.2}$$

which is a known result (Andrews & Berndt, 2005, (6.2.29), p. 152).

(iii) Taking $a = -1$ in (2.2) we get,

$$\sum_{n=0}^{\infty} q^{n(n+1)/2} = \sum_{n=0}^{\infty} q^{n(2n+1)}(1 + q^{2n+1}) \tag{2.3}$$

$$= \frac{1}{1-} \frac{q}{1+} \frac{q(1-q)}{1-} \frac{q^3}{1+} \frac{q^2(1-q^2)}{1-} \frac{q^5}{1+} \dots,$$

where $\psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2}$ is Ramanujan's theta function. Also

$$\psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2} = \frac{(q^2; q^2)_{\infty}}{(q; q^2)_{\infty}}.$$

Thus, from (2.3) we have,

$$\Psi(q) = \sum_{n=0}^{\infty} q^{n(n+1)/2} = \sum_{n=0}^{\infty} q^{n(2n+1)}(1 + q^{2n+1}) = \frac{(q^2; q^2)_{\infty}}{(q; q^2)_{\infty}} \tag{2.4}$$

$$= \frac{1}{1-} \frac{q}{1+} \frac{q(1-q)}{1-} \frac{q^3}{1+} \frac{q^2(1-q^2)}{1-} \frac{q^5}{1+} \dots.$$

(iv) Replacing q by q^2 in (2.1) and then taking $z = -aq$ in it we find,

$$\sum_{n=0}^{\infty} a^n q^{n^2} = \frac{1}{1-} \frac{aq}{1+} \frac{aq(1-q^2)}{1-} \frac{aq^5}{1+} \frac{aq^3(1-q^4)}{1-} \frac{aq^9}{1+} \dots \tag{2.5}$$

(v) Taking $c = bq$ in (1.1) we find,

$$(1-b) \sum_{n=0}^{\infty} \frac{z^n}{(1-bq^n)} = (1-b) \sum_{n=0}^{\infty} \frac{b^n z^n q^{n^2} (1-bzq^{2n})}{(1-bq^n)(1-zq^n)} \tag{2.6}$$

$$= \frac{1}{1-} \frac{z(1-b)}{(1-bq)-} \frac{bz(1-q^2)}{(1-bq^2)-} \frac{zq(1-bq)^2}{(1-bq^3)-} \frac{bzq(1-q^2)^2}{(1-bq^4)-} \frac{zq^2(1-bq^2)^2}{(1-bq^5)-} \dots$$

(vi) Replacing q by q^5 and then taking $z = q^j$ and $b = q^i$ in (2.6) we have,

$$(1-q^j) \sum_{n=0}^{\infty} \frac{q^{nj}}{(1-q^{5n+i})} = (1-q^j) \sum_{n=0}^{\infty} \frac{q^{5n^2+(i+j)n} (1-q^{i+j}q^{10n})}{(1-q^{5n+i})(1-q^{5n+j})} \tag{2.7}$$

$$= \frac{1}{1-} \frac{q^j(1-q^i)}{(1-q^{5+i})-} \frac{q^{i+j}(1-q^5)^2}{(1-q^{i+10})-} \frac{q^{5+i}(1-q^{5+i})^2}{(1-q^{15+i})-} \frac{q^{5+i+j}(1-q^{10})^2}{(1-q^{20+i})-} \frac{q^{10+j}(1-q^{10+i})^2}{(1-q^{25+i})-} \dots$$

(vii) For $j = i$, (2.7) yields,

$$(1-q^i) \sum_{n=0}^{\infty} \frac{q^{ni}}{(1-q^{5n+i})} = (1-q^i) \sum_{n=0}^{\infty} \frac{q^{5n^2+2ni} (1+q^{5n+i})}{(1-q^{5n+i})} \tag{2.8}$$

$$= \frac{1}{1-} \frac{q^i(1-q^i)}{(1-q^{5+i})-} \frac{q^{2i}(1-q^5)^2}{(1-q^{10+i})-} \frac{q^{5+i}(1-q^{5+i})}{(1-q^{15+i})-} \frac{q^{5+2i}(1-q^{10})^2}{(1-q^{20+i})-} \dots,$$

which gives the continued fraction representation of a Lambert series and is believed to be new.

(viii) Taking $c = q$ in (1.1) we get,

$$\frac{(bz; q)_{\infty}}{(z; q)_{\infty}} = \sum_{n=0}^{\infty} \frac{(b; q)_n (bz; q)_n z^n q^{n^2} (1-bzq^{2n})}{(q; q)_n (z; q)_{n+1}} \tag{2.9}$$

$$= \frac{1}{1-} \frac{z(1-b)}{(1-q)-} \frac{z(1-q)(b-q)}{(1-q^2)-} \frac{zq(1-bq)(1-q)}{(1-q^3)-} \frac{zq(1-q^2)(b-q^2)}{(1-q^4)-} \dots$$

(ix) Replacing q by q^3 and then taking $b = z = q$ in (2.9) we get,

$$\frac{(q^3; q^3)_\infty}{(q; q^3)_\infty} = \frac{1}{1-} \frac{q(1-q)}{(1-q^3)-} \frac{q^2(1-q^2)(1-q^3)}{(1-q^6)-} \frac{q^4(1-q^3)(1-q^4)}{(1-q^9)-} \frac{q^5(1-q^5)(1-q^6)}{(1-q^{12})-} \dots \quad (2.10)$$

(x) Replacing q by q^4 and then taking $z = q, b = q^2$ in (2.9) we have,

$$\begin{aligned} \frac{(q^3; q^4)_\infty}{(q; q^4)_\infty} &= \sum_{n=0}^{\infty} \frac{(q^2; q^4)_n (q^3; q^4)_n q^{4n^2+n} (1-q^{8n+3})}{(q^4; q^4)_n (q; q^4)_{n+1}} \\ &= \frac{1}{1-} \frac{q(1-q^2)}{(1-q^4)-} \frac{q^3(1-q^4)(1-q^2)}{(1-q^8)-} \frac{q^5(1-q^4)(1-q^6)}{(1-q^{12})-} \frac{q^7(1-q^6)(1-q^8)}{(1-q^{16})-} \dots \end{aligned}$$

(which by an appeal of (Jones & Thoren, 1980, (2.3.14), p. 33) gives)

$$= \frac{1}{1-} \frac{q}{(1+q^2)-} \frac{q^3}{(1+q^4)-} \frac{q^5}{(1+q^6)-} \frac{q^7}{(1+q^8)-} \dots, \quad (2.11)$$

which is known result (Andrews & Berndt, 2005, (7.1.2), p. 179).

(xi) Taking $b = 0$ and $z = q$ in (2.9) we have,

$$\begin{aligned} \frac{1}{(q; q)_\infty} &= \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q; q)_n (q; q)_{n+1}} \\ &= \frac{1}{1-} \frac{q}{(1-q)+} \frac{q^2(1-q)}{(1-q^2)-} \frac{q^2(1-q)}{(1-q^3)+} \frac{q^4(1-q^2)}{(1-q^4)-} \dots \end{aligned} \quad (2.12)$$

(xii) Again, putting z/b for z and then taking $b \rightarrow \infty$ in (2.9) we get,

$$\begin{aligned} (z; q)_\infty &= \sum_{n=0}^{\infty} \frac{(-)^n q^{n(n-1)/2} (z; q)_n q^{n^2} (1-zq^{2n}) z^n}{(q; q)_n} \\ &= \frac{1}{1+} \frac{z}{(1-q)-} \frac{z(1-q)}{(1-q^2)+} \frac{zq^2(1-q)}{(1-q^3)-} \frac{zq(1-q^2)}{(1-q^4)+} \dots \\ &= \frac{1}{1+} \frac{z}{(1-q)-} \frac{z}{(1+q)+} \frac{zq^2}{(1-q^3)-} \frac{zq}{(1+q^2)+} \dots \end{aligned} \quad (2.13)$$

(xiii) For $z = -q$, (2.13) yields

$$\begin{aligned} (-q; q)_\infty &= \sum_{n=0}^{\infty} \frac{q^{n(3n+1)/2} (-q; q)_n (1+q^{2n+1})}{(q; q)_n} \\ &= \frac{1}{1-} \frac{q}{(1-q)+} \frac{q}{(1+q)-} \frac{q^2}{(1+q^2)-} \frac{q^3}{(1-q^3)+} \frac{q^5}{(1-q^5)+} \dots \end{aligned} \quad (2.14)$$

In (2.12) and (2.14) we have established the continued fraction representations for partition generating functions.

(xiv) Replacing z by z/ab in (1.5) and then taking $a, b \rightarrow \infty$ we find,

$$\frac{\sum_{n=0}^{\infty} \frac{q^{n^2} z^n}{(q; q)_n (c; q)_n}}{\sum_{n=0}^{\infty} \frac{q^{n(n-1)} z^n}{(q; q)_n (c; q)_n}} = \frac{1}{1 + \frac{z}{(1-c) + \frac{zq}{(1-cq) + \frac{zq^2}{(1-cq^2) + \frac{zq^3}{(1-cq^3) + \frac{zq^4}{(1-cq^4) + \frac{zq^5}{(1-cq^5) + \dots}}}}}} \quad (2.15)$$

(xv) Taking $z = aq$ and $c = 0$ in (2.15) we find,

$$\frac{\sum_{n=0}^{\infty} \frac{q^{n(n+1)} a^n}{(q; q)_n}}{\sum_{n=0}^{\infty} \frac{q^{n^2} a^n}{(q; q)_n}} = \frac{1}{1 + \frac{aq}{1 + \frac{aq^2}{1 + \frac{aq^3}{1 + \frac{aq^4}{1 + \frac{aq^5}{1 + \dots}}}}}} \quad (2.16)$$

which is generalized Rogers–Ramanujan continued fraction.

(xvi) Taking $z = q$ and $c = q$ in (2.15) and using the special case of (Andrews & Berndt, 2005, (6.2.28), p. 152) we get,

$$\frac{\sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q; q)_n^2}}{\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q)_n^2}} = \sum_{n=0}^{\infty} (-1)^n q^{n(n+1)/2} = \frac{1}{1 + \frac{q}{(1-q) + \frac{q^2}{(1-q^2) + \frac{q^3}{(1-q^3) + \frac{q^4}{(1-q^4) + \frac{q^5}{(1-q^5) + \dots}}}}}} \quad (2.17)$$

(xvii) For $c = -q$ and $z = q$, (2.15) yields

$$\frac{\sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q^2; q^2)_n}}{\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^2; q^2)_n}} = \frac{1}{1 - \frac{q}{(1+q) - \frac{q^2}{(1+q^2) - \frac{q^3}{(1+q^3) - \frac{q^4}{(1+q^4) - \frac{q^5}{(1+q^5) - \dots}}}}}} \quad (2.18)$$

A number of other interesting results can also be scored.

Funding

The authors received no direct funding for this research.

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Citation information

Cite this article as: On q -series and continued fractions, Vishnu Narayan Mishra, S.N. Singh, S.P. Singh & Vijay Yadav, *Cogent Mathematics* (2016), 3: 1240414.

References

- Andrews, G. E., & Berndt, B. C. (2005). *Ramanujan's Lost Notebook, Part I*. New York, NY: Springer, Berlin, Heidelberg.
- Bhargava, S., & Adiga, C. (1984). On some continued fraction of Srinivasa Ramanujan. *Proceedings of the American Mathematical Society*, 92, 13–18.
- Bhargava, S., Adiga, C., & Somashekara, D. (1987). On some generalizations of Ramanujan's continued fraction identities. *Proceedings of the Indian Academy of Science*, 97, 31–43.
- Cao, J., & Srivastava, H. M. (2013). Some q -generating functions of the Carlitz and Srivastava–Agarwal types associated with the generalized Hahn polynomials and the generalized Rogers–Szegő polynomials. *Applied Mathematics and Computation*, 219, 8398–8406.
- Choi, J., & Srivastava, H. M. (2014). q -extensions of a multivariable and multiparameter generalization of the Gottlieb polynomials in several variables. *Tokyo Journal of Mathematics*, 37, 111–137.
- Denis, R. Y., & Singh, S. N. (2000). Hypergeometric functions and continued fractions. *Far East Journal of Mathematical Sciences*, 2, 385–400.
- Denis, R. Y., & Singh, S. P. (2011). Basic Hypergeometric series and continued fractions. *Indian Journal of Pure and Applied Mathematics*, 32, 1777–1787.

- Gairola, A. R., Deepmala, & Mishra, L. N. (2016a). Rate of approximation by finite iterates of q -Durrmeyer operators. *Proceedings of the Indian Academy Of Science, Section A*, 86, 229–234. doi:10.1007/s40010-016-0267-z
- Gairola, A. R., Deepmala, & Mishra, L. N. (2016b). On the q -derivatives of a certain linear positive operators. *Iranian Journal of Science & Technology*.
- Jones, W. B., & Thoren, W. J. (1980). *Continued fractions, Encyclopedia of Mathematics and its applications* (Vol. 11). Addison-Wesley Publishing Company.
- Luo, Q. M., & Srivastava, H. M. (2011). q -extensions of some relationships between the Bernoulli and Euler polynomials. *Taiwanese Journal of Mathematics*, 15, 241–257.
- Mishra, V. N., Khan, H. H., Khatri, K., & Mishra, L. N. (2013). Hypergeometric representation for Baskakov—Durrmeyer—Stancu type operators. *Bulletin of Mathematical Analysis and Applications*, 5, 18–26.
- Mishra, V. N., Khatri, K., & Mishra, L. N. (2012). On simultaneous approximation for Baskakov—Durrmeyer—Stancu type operators. *Journal of Ultra Scientist of Physical Sciences*, 24, 567–577.
- Mishra, V. N., Khatri, K., & Mishra, L. N. (2013a). Some approximation properties of q -Baskakov—Beta—Stancu type operators. *Journal of Calculus of Variations*, 2013, 814824, 8 p. doi:10.1155/2013/814824
- Mishra, V. N., Khatri, K., & Mishra, L. N. (2013b). Statistical approximation by Kantorovich type discrete q -beta operators. *Advances in Difference Equations*, 2013, 345. doi:10.1186/10.1186/1687-1847-2013-345
- Mishra, V. N., Khatri, K., Mishra, L. N., & Deepmala (2013). Inverse result in simultaneous approximation by Baskakov—Durrmeyer—Stancu operators. *Journal of Inequalities and Applications*, 2013, 586. doi:10.1186/1029-242X-2013-586
- Mishra, V.N., Sharma, P. & Mishra, L.N. (2016). On statistical approximation properties of q -Baskakov—Szász—Stancu operators. *Journal of Egyptian Mathematical Society*, 24, 396–401. doi:10.1016/j.joems.2015.07.005
- Singh, K. K., Gairola, A. R., & Deepmala. (2016). Approximation theorems for q - analogue of a linear positive operator by A. Lupas. *International Journal of Analysis and Applications*, 12, 30–37.
- Srivastava, H. M. (2011). Some generalizations and basic (or q) extensions of the Bernoulli. *Euler and Genocchi polynomials, Applied Mathematics & Information Sciences*, 5, 390–444.
- Srivastava, H. M., & Choi. (2012). *Zeta and q -Zeta Functions Associated Series and Integrals*. Austerdam, London and New York: Elsevier Science Publishers.
- Srivastava, H. M., & Choudhary, M. P. (2015). Some relationships between q -product identities, combinatorial partition identities and continued fraction identities. *Advanced Studies in Contemporary Mathematics*, 25, 265–272.
- Srivastava, H. M., & Karlsson, P. W. (1985). *Multiple Gaussian hypergeometric series*. New York, Chichester, Brisbane, and Toronto: A Halsted Press Book (Ellis Horwood Limited, Chichester), John Wiley and Sons. (ISBN: 0-470-20100-2 and 0-85312-602-X).
- Srivastava, H. M., Singh, S. N., & Singh, S. P. (2015). Some families of q -series identities and associated continued fractions. *Theory and Applications of Mathematics & Computer Science*, 5, 203–212.



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