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Some aspects on hesitant fuzzy soft set

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Abstract: In this paper, we introduce some operations on hesitant fuzzy soft sets and discuss some of their properties.

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1. Introduction

The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets, introduced by Zadeh in (1965). This theory brought a paradigmatic change in mathematics. But there exists difficulty, how to set the membership function in each particular case. The theory of intuitionistic fuzzy sets (see Atanassov, 1986) is a more generalized concept than the theory of fuzzy sets, but this theory has the same difficulties. All the above-mentioned theories are successful to some extent in dealing with problems arising due to vagueness present in the real world. But there are also cases where these theories failed to give satisfactory results, possibly due to inadequacy of the parameterization tool in them. As a necessary supplement to the existing mathematical tools for handling uncertainty, in Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling the problems in engineering, physics, computer science, economics, social sciences, and medical sciences. In Molodtsov, Leonov, and Kovkov (2006) successfully applied soft sets in directions such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, and theory of measurement. Maji, Biswas, and Roy (2002) gave the first practical application of soft sets in decision-making problems. Maji, Biswas, and Roy (2003) defined and studied several basic notions of the soft set theory. Also, Çağman and

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PUBLIC INTEREST STATEMENT

The hesitant fuzzy set, as one of the extension of fuzzy set, allows the membership degree that an element to a set presented by several possible values, and it can express the hesitant information more comprehensively than other extensions of fuzzy set. The hesitant fuzzy set is an effective tool used to express the decision-makers hesitant preferences in the process of decision-making, aggregation, distance, similarity and correlation measures, clustering analysis, and decision-making with hesitant fuzzy information.

Enginoğlu (2010) studied several basic notions of the soft set theory. Maji, Biswas, and Roy (2001) introduced the concepts of fuzzy soft set theory. The hesitant fuzzy set, as one of the extension of Zadeh (1965) fuzzy set, allows the membership degree that an element to a set presented by several possible values, and it can express the hesitant information more comprehensively than other extensions of fuzzy set. In Torra and Narukawa (2009) introduced the concept of hesitant fuzzy set. In Xu and Xia (2011) defined the concept of hesitant fuzzy element, which can be considered as the basic unit of a hesitant fuzzy set, and is a simple and effective tool used to express the decision-makers hesitant preferences in the process of decision-making. So many researchers (see Liao & Xu, 2014; Xia & Xu, 2011) has done lots of research work on aggregation, distance, similarity and correlation measures, clustering analysis, and decision-making with hesitant fuzzy information. In Babitha and John (2013) defined another important soft set hesitant fuzzy soft sets. They introduced basic operations such as intersection, union, compliment, and De Morgan's law was proved. Broumi and Smarandache (2014) introduced the operations over interval-valued intuitionistic hesitant fuzzy sets and proved some basic results. In Wang, Li, and Chen (2014) applied hesitant fuzzy soft sets in multicriteria group decision-making problems. Torra (2010), Torra and Narukawa (2009), and Verma and Sharma (2013) discussed the relationship between hesitant fuzzy set and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. A lot of work has been done about hesitant fuzzy sets, however, little has been done about the hesitant fuzzy soft sets.

In this paper, we study some operations on hesitant fuzzy soft set. We also establish some interesting properties of this notion.

2. Preliminary results

In this section, we recall some basic concepts and definitions regarding fuzzy soft sets, hesitant fuzzy set, and hesitant fuzzy soft set.

Definition 2.1 Maji et al. (2001) Let U be an initial universe and F be a set of parameters. Let $\tilde{P}(U)$ denote the power set of U and A be a non-empty subset of F . Then, F_A is called a fuzzy soft set over U , where $F:A \rightarrow \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$.

Definition 2.2 Molodstov (1999) F_E is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U .

In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\epsilon)$, $\epsilon \in E$, from this family may be considered as the set of ϵ -element of the soft set F_E or as the set of ϵ -approximate elements of the soft set.

Definition 2.3 Torra (2010) Given a fixed set X , then a hesitant fuzzy set (shortly HFS) in X is in terms of a function that when applied to X return a subset of $[0, 1]$. We express the HFS by a mathematical symbol:

$F = \{ \langle h, \mu_F(x) \rangle : h \in X \}$, where $\mu_F(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $h \in X$ to the set F . $\mu_F(x)$ is called a hesitant fuzzy element (HFE) and H is the set of all HFEs.

Definition 2.4 Torra (2010) Let $\mu_1, \mu_2 \in H$ and three operations are defined as follows:

- (1) $\mu_1^c = \cup_{\gamma_1 \in \mu_1} \{1 - \gamma_1\}$;
- (2) $\mu_1 \cup \mu_2 = \cup_{\gamma_1 \in \mu_1, \gamma_2 \in \mu_2} \max\{\gamma_1, \gamma_2\}$;
- (3) $\mu_1 \cap \mu_2 = \cap_{\gamma_1 \in \mu_1, \gamma_2 \in \mu_2} \min\{\gamma_1, \gamma_2\}$.

Definition 2.5 Wang, Li, and Chen (2014) Let U be an initial universe and E be a set of parameters. Let $\tilde{F}(U)$ be the set of all hesitant fuzzy subsets of U . Then, F_E is called a hesitant fuzzy soft set (HFSS) over U , where $\tilde{F} : E \rightarrow \tilde{F}(U)$.

A HFSS is a parameterized family of hesitant fuzzy subsets of U , i.e. $\tilde{F}(U)$. For all $\epsilon \in E$, $F(\epsilon)$ is referred to as the set of ϵ -approximate elements of the HFSS F_E . It can be written as $F(\tilde{\epsilon}) = \{ \langle h, \mu_{F(\tilde{\epsilon})}(x) \rangle : h \in U \}$.

Since HFE can represent the situation, in which different membership function are considered possible (see Torra, 2010), $\mu_{F(\tilde{\epsilon})}(x)$ is a set of several possible values, which is the hesitant fuzzy membership degree. In particular, if $F(\tilde{\epsilon})$ has only one element, $F(\tilde{\epsilon})$ can be called a hesitant fuzzy soft number. For convenience, a hesitant fuzzy soft number (HFSN) is denoted by $\{ \langle h, \mu_{F(\tilde{\epsilon})}(x) \rangle \}$.

Example 2.6 Suppose $U = \{a, b\}$ be an initial universe and $E = \{e_1, e_2, e_3, e_4\}$ be a set of parameters. Let $A = \{e_1, e_2\}$. Then, the hesitant fuzzy soft set F_A is given as $F_A = \{F(e_1) = \{ \langle a, \{0.6, 0.8\} \rangle, \langle b, \{0.8, 0.4, 0.9\} \rangle \}, F(e_2) = \{ \langle a, \{0.9, 0.1, 0.5\} \rangle, \langle b, \{0.2\} \rangle \}$.

Definition 2.7 Wang, Li, and Chen (2014) Let $F(e_i) = \{ \langle h_t, \mu_{it} \rangle \}$ be hesitant fuzzy soft number, where $t = 1, 2, \dots, m$. Then

$$(F(e_i))^c = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{1 - \gamma_{it}\} \rangle \}.$$

3. Aspect on hesitant fuzzy soft sets

Definition 3.1 Let $F(e_i) = \{ \langle h_t, \mu_{it} \rangle \}$ and $F(e_j) = \{ \langle h_t, \mu_{jt} \rangle \}$ be two hesitant fuzzy soft numbers with $\lambda > 0$ and $(t = 1, 2, \dots, m)$, then

- (i) $\lambda F(e_i) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{(1 - (1 - \gamma_{it})^\lambda)\} \rangle \}$.
- (ii) $F(e_i)^\lambda = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \gamma_{it}^\lambda \rangle \}$.
- (iii) $F(e_i) \oplus F(e_j) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma_{it} + \gamma_{jt} - \gamma_{it} \cdot \gamma_{jt}\} \rangle \}$.
- (iv) $F(e_i) \otimes F(e_j) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma_{it} \cdot \gamma_{jt}\} \rangle \}$.

Example 3.2 Let $F_A = \{F(e_1) = \{ \langle a, \{0.6, 0.8\} \rangle, \langle b, \{0.8, 0.4, 0.9\} \rangle \}, F(e_2) = \{ \langle a, \{0.9, 0.1, 0.5\} \rangle, \langle b, \{0.2\} \rangle \}$ and $G_B = \{G(e_1) = \{ \langle a, \{0.4, 0.2\} \rangle, \langle b, \{0.7, 0.1\} \rangle \}$.

Then, we have

- (i) $2F_A = \{e_1 = \{ \langle a, \{0.84, 0.96\} \rangle, \langle b, \{0.96, 0.64, 0.99\} \rangle \}, e_2 = \{ \langle a, \{0.99, 0.19, 0.75\} \rangle, \langle b, \{0.36\} \rangle \}$
- (ii) $(F_A)^2 = \{e_1 = \{ \langle a, \{0.36, 0.64\} \rangle, \langle b, \{0.64, 0.16, 0.81\} \rangle \}, e_2 = \{ \langle a, \{0.81, 0.01, 0.25\} \rangle, \langle b, \{0.4\} \rangle \}$
- (iii) $F_A \oplus G_B = \{e_1 = \{ \langle a, \{0.76, 0.68, 0.88, 0.84\} \rangle, \langle b, \{0.94, 0.82, 0.82, 0.46, 0.97, 0.91\} \rangle \}, e_2 = \{ \langle a, \{0.9, 0.1, 0.5\} \rangle, \langle b, \{0.2\} \rangle \}$
- (iv) $F_A \otimes G_B = \{e_1 = \{ \langle a, \{0.24, 0.12, 0.32, 0.16\} \rangle, \langle b, \{0.56, 0.08, 0.28, 0.04, 0.63, 0.09\} \rangle \}, e_2 = \{ \langle a, \{0.9, 0.1, 0.5\} \rangle, \langle b, \{0.2\} \rangle \}$

Proposition 3.3 Let $F(e_i) = \{ \langle h_t, \mu_{it} \rangle \}$ and $F(e_j) = \{ \langle h_t, \mu_{jt} \rangle \}$ be two hesitant fuzzy soft numbers with $\lambda > 0, \lambda_1 > 0, \lambda_2 > 0$ and $(t = 1, 2, \dots, m)$, then

- (i) $(\lambda F(e_i))^c = ((F(e_i))^c)^\lambda$
- (ii) $\lambda((F(e_i))^c) = (F(e_i)^\lambda)^c$
- (iii) $F(e_i) \oplus F(e_j) = F(e_j) \oplus F(e_i)$

- (iv) $F(e_i) \tilde{\otimes} F(e_j) = F(e_i) \tilde{\otimes} F(e_j)$
- (v) $(F(e_i) \tilde{\oplus} F(e_j))^C = (F(e_i))^C \tilde{\otimes} (F(e_j))^C$
- (vi) $(F(e_i) \tilde{\otimes} F(e_j))^C = (F(e_i))^C \tilde{\oplus} (F(e_j))^C$
- (vii) $\lambda(F(e_i) \tilde{\oplus} F(e_j)) = \lambda F(e_i) \tilde{\oplus} \lambda F(e_j)$
- (viii) $(F(e_i) \tilde{\otimes} F(e_j))^\lambda = F(e_i)^\lambda \tilde{\otimes} F(e_j)^\lambda$
- (ix) $\lambda_1 F(e_i) \tilde{\oplus} \lambda_2 F(e_i) = (\lambda_1 + \lambda_2) F(e_i)$
- (x) $F(e_i)^{\lambda_1} \tilde{\otimes} F(e_i)^{\lambda_2} = F(e_i)^{\lambda_1 + \lambda_2}$.

Proof For $t = 1, 2, \dots, m$

(i) By definition, $\lambda F(e_i) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - (1 - \gamma_{it})^\lambda) \} \rangle \}$.

Therefore,

$$\begin{aligned} (\lambda F(e_i))^C &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - (1 - (1 - \gamma_{it})^\lambda)) \} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - \gamma_{it}^\lambda) \} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - \gamma_{it}) \} \rangle \}^\lambda \\ &= \{ \langle h_t, \mu_{it}^C \rangle \}^\lambda \\ &= (F(e_i))^C. \end{aligned}$$

(ii) By definition, $(F(e_i))^C = \{ \langle h_t, \mu_{it}^C \rangle \} = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - \gamma_{it}) \} \rangle \}$.

Therefore,

$$\begin{aligned} \lambda(F(e_i))^C &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - (1 - (1 - \gamma_{it})^\lambda)) \} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (1 - \gamma_{it}^\lambda) \} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ (\gamma_{it}^\lambda) \} \rangle \}^C \\ &= \{ \langle h_t, \mu_{it}^\lambda \rangle \}^C \\ &= (F(e_i)^\lambda)^C. \end{aligned}$$

(iii) $F(e_i) \tilde{\oplus} F(e_j) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{ \gamma_{it} + \gamma_{jt} - \gamma_{it} \cdot \gamma_{jt} \} \rangle \}$
 $= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{ \gamma_{it} + \gamma_{jt} - \gamma_{it} \cdot \gamma_{jt} \} \rangle \}$
 $= F(e_i) \tilde{\oplus} F(e_j)$.

(iv) Similar as (iii).

(v) $(F(e_i) \tilde{\oplus} F(e_j))^C = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{ 1 - (\gamma_{it} + \gamma_{jt} - \gamma_{it} \cdot \gamma_{jt}) \} \rangle \}$
 $= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{ (1 - \gamma_{it})(1 - \gamma_{jt}) \} \rangle \}$
 $= \{ \langle h_t, \mu_{it}^C \cdot \mu_{jt}^C \rangle \}$
 $= (F(e_i))^C \tilde{\otimes} (F(e_j))^C$.

(vi) Similar to (v).

$$\begin{aligned} \text{(vii)} \quad \lambda(F(e_i) \oplus F(e_j)) &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{1 - (1 - \gamma_{it} - \gamma_{jt} + \gamma_{it} \cdot \gamma_{jt})^\lambda\} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{1 - (1 - \gamma_{it})^\lambda \cdot (1 - \gamma_{jt})^\lambda\} \rangle \} \end{aligned} \tag{A}$$

$$\text{Again } \lambda(F(e_i)) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{1 - (1 - \gamma_{it})^\lambda\} \rangle \}$$

and

$$\lambda(F(e_j)) = \{ \langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{1 - (1 - \gamma_{jt})^\lambda\} \rangle \}. \text{ Therefore,}$$

$$\begin{aligned} \lambda F(e_i) \oplus \lambda F(e_j) &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{1 - (1 - \gamma_{it})^\lambda + 1 - (1 - \gamma_{jt})^\lambda - (1 - (1 - \gamma_{it})^\lambda)(1 - (1 - \gamma_{jt})^\lambda)\} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{1 - (1 - \gamma_{it})^\lambda \cdot (1 - \gamma_{jt})^\lambda\} \rangle \}. \end{aligned} \tag{B}$$

From (A) and (B), we get the proved.

(viii) Since

$$(F(e_i) \otimes F(e_j))^\lambda = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma_{it}^\lambda \cdot \gamma_{jt}^\lambda\} \rangle \} \tag{C}$$

and

$$\begin{aligned} F(e_i)^\lambda \otimes F(e_j)^\lambda &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{\gamma_{it}^\lambda\} \rangle \} \otimes \{ \langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{\gamma_{jt}^\lambda\} \rangle \} \\ &= \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma_{it}^\lambda \cdot \gamma_{jt}^\lambda\} \rangle \}. \end{aligned} \tag{D}$$

From (C) and (D), we get the proved.

(ix) Similar as (viii).

(x) Similar as (viii). □

Definition 3.4 Let $F(e_i) = \{ \langle h_t, \mu_{it} \rangle \}$ and $F(e_j) = \{ \langle h_t, \mu_{jt} \rangle \}$ be two hesitant fuzzy soft numbers with $(t = 1, 2, \dots, m)$, then

$$\text{(i) } F(e_i) \oplus F(e_j) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma\} \rangle \}, \text{ where}$$

$$\gamma = \begin{cases} \frac{(\gamma_{it} - \gamma_{jt})}{(1 - \gamma_{jt})} & \text{if } \gamma_{it} > \gamma_{jt} \text{ and } \gamma_{jt} \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{(ii) } F(e_i) \otimes F(e_j) = \{ \langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}} \{\gamma\} \rangle \}, \text{ where}$$

$$\gamma = \begin{cases} \frac{\gamma_{it}}{\gamma_{jt}} & \text{if } \gamma_{it} \leq \gamma_{jt} \text{ and } \gamma_{jt} \neq 0 \\ 1 & \text{otherwise} \end{cases}$$

Proposition 3.5 Let $F(e_i) = \langle h_t, \mu_{it} \rangle$ be hesitant fuzzy soft number and $(t = 1, 2, \dots, m)$, then the following are true:

- (i) $F(e_i) \tilde{\ominus} F(e_i) = \tilde{\phi}$
- (ii) $F(e_i) \tilde{\ominus} \tilde{\phi} = F(e_i)$
- (iii) $F(e_i) \tilde{\ominus} \tilde{E} = \tilde{\phi}$
- (iv) $F(e_i) \tilde{\oslash} F(e_i) = \tilde{E}$
- (v) $F(e_i) \tilde{\oslash} \tilde{E} = F(e_i)$
- (vi) $F(e_i) \tilde{\oslash} \tilde{\phi} = \tilde{E}$
- (vii) $\tilde{E} \tilde{\ominus} \tilde{E} = \tilde{\phi}$
- (viii) $\tilde{\phi} \tilde{\ominus} \tilde{E} = \tilde{\phi}$
- (ix) $\tilde{E} \tilde{\ominus} \tilde{\phi} = \tilde{E}$
- (x) $\tilde{\phi} \tilde{\ominus} \tilde{\phi} = \tilde{\phi}$
- (xi) $\tilde{E} \tilde{\oslash} \tilde{E} = \tilde{E}$
- (xii) $\tilde{\phi} \tilde{\oslash} \tilde{E} = \tilde{\phi}$
- (xiii) $\tilde{E} \tilde{\oslash} \tilde{\phi} = \tilde{E}$
- (xiv) $\tilde{\phi} \tilde{\oslash} \tilde{\phi} = \tilde{E}$.

Proof Obvious. □

Proposition 3.6 Let $F(e_i) = \langle h_t, \mu_{it} \rangle$ and $F(e_j) = \langle h_t, \mu_{jt} \rangle$ be two hesitant fuzzy soft numbers with $(t = 1, 2, \dots, m)$, then

- (i) $(F(e_i) \tilde{\ominus} F(e_j)) \tilde{\oplus} F(e_j) = F(e_i)$ if $\gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1$;
- (ii) $(F(e_i) \tilde{\oslash} F(e_j)) \tilde{\otimes} F(e_j) = F(e_i)$ if $\gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0$.

Proof

$$\begin{aligned}
 \text{(i) } (F(e_i) \tilde{\ominus} F(e_j)) \tilde{\oplus} F(e_j) &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \right\} \right\rangle \tilde{\oplus} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{ \gamma_{jt} \} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} + \gamma_{jt} - \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \cdot \gamma_{jt} \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{\gamma_{it}(1 - \gamma_{jt})}{1 - \gamma_{jt}} \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ \gamma_{it} \} \right\rangle \\
 &= F(e_i).
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & (F(e_i) \tilde{\otimes} F(e_j)) \tilde{\otimes} F(e_j) = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\} \right\rangle \tilde{\otimes} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{ \gamma_{jt} \} \right\rangle \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \cdot \gamma_{jt} \right\} \right\rangle \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ \gamma_{it} \} \right\rangle \\
 & = F(e_i).
 \end{aligned}$$

□

Proposition 3.7 Let $F(e_i) = \langle h_t, \mu_{it} \rangle$ and $F(e_j) = \langle h_t, \mu_{jt} \rangle$ be two hesitant fuzzy soft numbers with $\lambda_1 \geq \lambda_2 > 0$ and $(t = 1, 2, \dots, m)$, then

- (i) $\lambda(F(e_i) \tilde{\otimes} F(e_j)) = \lambda F(e_i) \tilde{\otimes} \lambda F(e_j)$ if $\gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1$
- (ii) $(F(e_i) \tilde{\otimes} F(e_j))^\lambda = F(e_i)^\lambda \tilde{\otimes} F(e_j)^\lambda$ if $\gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0$
- (iii) $\lambda_1 F(e_i) \tilde{\otimes} \lambda_2 F(e_i) = (\lambda_1 - \lambda_2) F(e_i)$ if $\gamma_{it} \neq 1$
- (iv) $F(e_i)^{\lambda_1} \tilde{\otimes} F(e_i)^{\lambda_2} = F(e_i)^{\lambda_1 - \lambda_2}$ if $\gamma_{it} \neq 0$.

Proof

$$\begin{aligned}
 \text{(i)} \quad & \lambda(F(e_i) \tilde{\otimes} F(e_j)) = \lambda \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \right\} \right\rangle \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ 1 - \left(1 - \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \right)^\lambda \right\} \right\rangle \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{(1 - \gamma_{jt})^\lambda - (1 - \gamma_{it})^\lambda}{(1 - \gamma_{jt})^\lambda} \right\} \right\rangle \tag{E}
 \end{aligned}$$

Again

$$\begin{aligned}
 \lambda F(e_i) \tilde{\otimes} \lambda F(e_j) & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ 1 - (1 - \gamma_{it})^\lambda \} \right\rangle \tilde{\otimes} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{ 1 - (1 - \gamma_{jt})^\lambda \} \right\rangle \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{(1 - (1 - \gamma_{it})^\lambda) - (1 - (1 - \gamma_{jt})^\lambda)}{1 - (1 - (1 - \gamma_{jt})^\lambda)} \right\} \right\rangle \\
 & \quad (\text{since, } \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1, \text{ it follows that } (1 - (1 - \gamma_{it})^\lambda) \geq (1 - (1 - \gamma_{jt})^\lambda).) \\
 & = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{(1 - \gamma_{jt})^\lambda - (1 - \gamma_{it})^\lambda}{(1 - \gamma_{jt})^\lambda} \right\} \right\rangle \tag{F}
 \end{aligned}$$

From (E) and (F), we get the result.

$$\begin{aligned}
 \text{(ii)} \quad (F(e_i) \tilde{\otimes} F(e_j))^\lambda &= \left\langle \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\} \right\rangle \right\rangle^\lambda \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\}^\lambda \right\rangle.
 \end{aligned} \tag{G}$$

Again

$$\begin{aligned}
 F(e_i)^\lambda \tilde{\otimes} F(e_j)^\lambda &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \{ \gamma_{it}^\lambda \} \right\rangle \tilde{\otimes} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \{ \gamma_{jt}^\lambda \} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}^\lambda}{\gamma_{jt}^\lambda} \right\} \right\rangle \\
 &\quad (\text{since, } \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0, \text{ this implies that } \gamma_{it}^\lambda \leq \gamma_{jt}^\lambda) \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\}^\lambda \right\rangle
 \end{aligned} \tag{H}$$

From (G) and (H), we get the result.

(iii) Same as (i)

(iv) Same as (ii). □

Proposition 3.8 Let $F(e_i) = \langle h_t, \mu_{it} \rangle$, $F(e_j) = \langle h_t, \mu_{jt} \rangle$ and $F(e_k) = \langle h_t, \mu_{kt} \rangle$ be three hesitant fuzzy soft numbers with $(t = 1, 2, \dots, m)$, then

- (i) $F(e_i) \tilde{\otimes} F(e_j) \tilde{\otimes} F(e_k) = F(e_i) \tilde{\otimes} F(e_k) \tilde{\otimes} F(e_j)$,
if $\gamma_{it} \geq \gamma_{jt}, \gamma_{it} \geq \gamma_{kt}, \gamma_{jt} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt} \geq 0$,
- (ii) $F(e_i) \tilde{\otimes} F(e_j) \tilde{\otimes} F(e_k) = F(e_i) \tilde{\otimes} F(e_k) \tilde{\otimes} F(e_j)$,
if $\gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt}, \gamma_{jt} \neq 0, \gamma_{kt} \neq 0$
- (iii) $F(e_i) \tilde{\otimes} F(e_j) \tilde{\otimes} F(e_k) = F(e_i) \tilde{\otimes} (F(e_j) \tilde{\otimes} F(e_k))$,
if $\gamma_{it} \geq \gamma_{jt}, \gamma_{it} \geq \gamma_{kt}, \gamma_{jt} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt} \geq 0$,
- (iv) $F(e_i) \tilde{\otimes} F(e_j) \tilde{\otimes} F(e_k) = F(e_i) \tilde{\otimes} F(e_j) \tilde{\otimes} F(e_k)$,
if $\gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt}, \gamma_{jt} \neq 0, \gamma_{kt} \neq 0$.

Proof

$$\begin{aligned}
 \text{(i) } F(e_i) \tilde{\ominus} F(e_j) \tilde{\ominus} F(e_k) &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{it} \neq 1} \left\{ \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{kt} \in \mu_{kt}, \gamma_{kt} \neq 1} \{ \gamma_{kt} \} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \geq \gamma_{jt}, \gamma_{it} \geq \gamma_{kt}, \gamma_{it} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt} \geq 0} \left\{ \frac{\frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} - \gamma_{kt}}{1 - \gamma_{kt}} \right\} \right\rangle \\
 &\quad \text{(since, } \gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt} \geq 0, \text{ and } \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \geq \gamma_{kt} \text{)} \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \geq \gamma_{jt}, \gamma_{it} \geq \gamma_{kt}, \gamma_{it} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt} \geq 0} \left\{ \frac{\gamma_{it} - \gamma_{jt} - \gamma_{kt} + \gamma_{jt} \cdot \gamma_{kt}}{(1 - \gamma_{jt})(1 - \gamma_{kt})} \right\} \right\rangle. \tag{I}
 \end{aligned}$$

Again

$$\begin{aligned}
 F(e_i) \tilde{\ominus} F(e_k) \tilde{\ominus} F(e_j) &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{kt} \in \mu_{kt}, \gamma_{it} \geq \gamma_{kt}, \gamma_{it} \neq 1} \left\{ \frac{\gamma_{it} - \gamma_{kt}}{1 - \gamma_{kt}} \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}, \gamma_{jt} \neq 1} \{ \gamma_{jt} \} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \geq \gamma_{kt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{it} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{kt} - \gamma_{jt} + \gamma_{kt} \cdot \gamma_{jt} \geq 0} \left\{ \frac{\frac{\gamma_{it} - \gamma_{kt}}{1 - \gamma_{kt}} - \gamma_{jt}}{1 - \gamma_{jt}} \right\} \right\rangle \\
 &\quad \text{(since, } \gamma_{it} - \gamma_{kt} - \gamma_{jt} + \gamma_{kt} \cdot \gamma_{jt} \geq 0, \text{ and } \frac{\gamma_{it} - \gamma_{kt}}{1 - \gamma_{kt}} \geq \gamma_{jt} \text{)} \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \geq \gamma_{kt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{it} \neq 1, \gamma_{kt} \neq 1, \gamma_{it} - \gamma_{kt} - \gamma_{jt} + \gamma_{kt} \cdot \gamma_{jt} \geq 0} \left\{ \frac{\gamma_{it} - \gamma_{kt} - \gamma_{jt} + \gamma_{kt} \cdot \gamma_{jt}}{(1 - \gamma_{kt})(1 - \gamma_{jt})} \right\} \right\rangle. \tag{J}
 \end{aligned}$$

From (I) and (J), we get the result.

$$\begin{aligned}
 \text{(ii) } F(e_i) \tilde{\ominus} F(e_j) \tilde{\ominus} F(e_k) &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{kt} \in \mu_{kt}, \gamma_{kt} \neq 0} \{ \gamma_{kt} \} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{kt}} \right\} \right\rangle \\
 &\quad \text{(since, } \gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt}, \gamma_{jt} \neq 0, \text{ this implies that } \gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt} \leq \gamma_{jt}, \text{ and } \frac{\gamma_{it}}{\gamma_{jt}} \leq \gamma_{kt} \text{)} \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{it} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt} \cdot \gamma_{kt}} \right\} \right\rangle. \tag{K}
 \end{aligned}$$

Again

$$\begin{aligned}
 & F(e_i) \tilde{\ominus} F(e_k) \tilde{\ominus} F(e_j) \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{kt} \in \mu_{kt}, \gamma_{it} \leq \gamma_{kt}, \gamma_{kt} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{kt}} \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}, \gamma_{jt} \neq 0} \left\{ \gamma_{jt} \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt}} \right\} \right\rangle \\
 & \text{(since, } \gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt}, \gamma_{kt} \neq 0, \text{ this implies that } \gamma_{it} \leq \gamma_{jt} \cdot \gamma_{kt} \leq \gamma_{kt}, \text{ and } \frac{\gamma_{it}}{\gamma_{kt}} \leq \gamma_{jt}\text{)} \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ \frac{\gamma_{it}}{\gamma_{jt} \cdot \gamma_{kt}} \right\} \right\rangle \tag{L}
 \end{aligned}$$

From (K) and (L), we get the result.

(iii) Obvious.

(iv) Obvious. □

Proposition 3.9 Let $F(e_i) = \langle h_t, \mu_{it} \rangle$ and $F(e_j) = \langle h_t, \mu_{jt} \rangle$ be two hesitant fuzzy soft numbers with $(t = 1, 2, \dots, m)$, then

- (i) $(F(e_i))^c \tilde{\ominus} (F(e_j))^c = (F(e_i) \tilde{\ominus} F(e_j))^c$
- (ii) $(F(e_i))^c \tilde{\ominus} (F(e_j))^c = (F(e_i) \tilde{\ominus} F(e_j))^c$.

Proof

$$\begin{aligned}
 \text{(i)} \quad & (F(e_i))^c \tilde{\ominus} (F(e_j))^c = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \left\{ (1 - \gamma_{it}) \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \left\{ (1 - \gamma_{jt}) \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ \frac{(1 - \gamma_{it}) - (1 - \gamma_{jt})}{1 - (1 - \gamma_{jt})} \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \leq \gamma_{jt}, \gamma_{jt} \neq 0} \left\{ 1 - \frac{\gamma_{it}}{\gamma_{jt}} \right\} \right\rangle \\
 &= (F(e_i) \tilde{\ominus} F(e_j))^c. \\
 \text{(ii)} \quad & (F(e_i))^c \tilde{\ominus} (F(e_j))^c = \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}} \left\{ (1 - \gamma_{it}) \right\} \right\rangle \tilde{\ominus} \left\langle h_t, \bigcup_{\gamma_{jt} \in \mu_{jt}} \left\{ (1 - \gamma_{jt}) \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ \frac{1 - \gamma_{it}}{1 - \gamma_{jt}} \right\} \right\rangle \\
 &= \left\langle h_t, \bigcup_{\gamma_{it} \in \mu_{it}, \gamma_{jt} \in \mu_{jt}, \gamma_{it} \geq \gamma_{jt}, \gamma_{jt} \neq 1} \left\{ 1 - \frac{\gamma_{it} - \gamma_{jt}}{1 - \gamma_{jt}} \right\} \right\rangle \\
 &= (F(e_i) \tilde{\ominus} F(e_j))^c. \tag{□}
 \end{aligned}$$

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