



Received: 15 March 2016
Accepted: 09 May 2016
First Published: 26 May 2016

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Reviewing editor:
Song Wang, Curtin University, Australia

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COMPUTATIONAL SCIENCE | RESEARCH ARTICLE

New exact solutions for the Khokhlov-Zabolotskaya-Kuznetsov, the Newell-Whitehead-Segel and the Rabinovich wave equations by using a new modification of the tanh-coth method

Şamil Akçağıl^{1*} and Tuğba Aydemir²

Abstract: The family of the tangent hyperbolic function methods is one of the most powerful method to find the solutions of the nonlinear partial differential equations. In the mathematical literature, there are a great deal of tanh-methods completing each other. In this article, the unified tanh-function method as a unification of the family of tangent hyperbolic function methods is introduced and implemented to find traveling wave solutions for three important physical models, namely the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation, the Newell-Whitehead-Segel (NWS) equation, and the Rabinovich wave equation with nonlinear damping. Various exact traveling wave solutions of these physical structures are formally derived.

Subjects: Applied Mathematics; Mathematics & Statistics; Physical Sciences; Science

Keywords: the unified tanh-function method; the Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation; the Newell-Whitehead-Segel (NWS) equation; the Rabinovich wave equation with nonlinear damping; traveling wave solution

1. Introduction

The (3+1) dimensional Zabolotskaya-Khokhlov (ZK) equation

$$(u_t + uu_x)_x + \gamma \Delta_y u + \theta \Delta_z u = 0 \quad (1)$$

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PUBLIC INTEREST STATEMENT

Over the past two decades, several expansion methods for finding solutions of nonlinear differential equations (NPDEs) have been proposed, developed, and extended. In the recent years, direct searching for exact solutions of NPDEs has become more and more attractive partly due to symbolic computation. One of the most effective direct methods to construct wave solutions of NPDEs is the family of the tanh-function methods. The first member of this family was introduced by Malfliet firstly and developed and used many researchers. The most known members of this family are the tanh-function method, the extended tanh-function method, the modified extended tanh-function method, and the complex tanh-function method. In this article, the authors have given a unification between these different tanh-function methods. Therefore, it can be obtained the solutions of different tanh-function methods using merely one method called the unified tanh-function method.

was proposed by Zabolotskaya and Khokhlov to describe the propagation of sound beam in a slightly nonlinear medium without dispersion or absorption (Zabolotskaya & Khokhlov, 1969). This equation enables to analyze the beam deformation associated with the nonlinear properties of the medium.

The ZK equation with a dissipative term may be written as

$$(u_t + uu_x + \lambda u_{xx})_x + \gamma \Delta_y u + \theta \Delta_z u = 0, \quad (2)$$

where λ, γ and θ are some constants. This equation known as the (3+1) dimensional Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation and derived by Kuznetsov which took into account of the thermoviscous term of adsorption (Kuznetsov, 1971).

The ZK and KZK equations have been investigated by many authors. Vinogradov and Vorob'ev (1976) investigated exact solutions of ZK equation using symmetries. Chowdhury and Nasker (1986) obtained the explicit structure for the generating function of Lie symmetries for the 3+1 dimensional Khokhlov–Zabolotskaya equation. Taniuti (1990) showed that systems of nonlinear evolutionary equations are reducible to the Kadomtsev and Petviashvili equation and the Zabolotskaya–Khokhlov equation in the weakly dispersive and dissipative cases respectively, by means of an extension of the reductive perturbation method to quasi-one-dimensional propagation. Murakami (1990) presented N-traveling-wave solutions to this equation using the bilinear transformation method. Tajiri (1995) investigated similarity reductions of the Zabolotskaya–Khokhlov equation with a dissipative term to one-dimensional partial differential equations including the Burgers equation by means of Lie's method of infinitesimal transformation and obtained some similarity solutions of the ZK equation. Using the theory of nonclassical symmetry reductions, some traveling wave solutions of the dissipative Zabolotskaya–Khokhlov equation are obtained (Bruzon, Gandarias, Torrisi, & Tracinà, 2009).

The Newell–Whitehead–Segel (NWS) equation is a nonlinear parabolic partial differential equation and written as

$$u_t - ku_{xx} - au + bu^q = 0, \quad (3)$$

where a, b, k are real numbers, $k > 0$ is the coefficient of diffusion and q is a positive integer. The NWS equation models the interaction of the effect of the diffusion term with the nonlinear effect of the reaction term. This equation can be viewed as a generalization of the NWS equation which appeared in the investigation of fluid mechanics (Newell & Whitehead, 1969; Segel, 1969). The function u may be thought of as the distribution of temperature in an infinitely thin and long rod or as the flow velocity of a fluid in an infinitely long pipe with small diameter (Macías-Díaz & Ruiz-Ramírez, 2011). Besides, u is a function of the spatial variable x and the temporal variable t , with $x \in \mathbb{R}$ and $t \geq 0$.

The applications of the NWS equation may be seen widely in mechanical and chemical engineering, ecology, biology, and bio-engineering. For more details, we refer the reader to Fisher (1937), FitzHugh (1955), Kastenber and Chambré (1968), Nagumo, Arimoto, and Yoshizawa (1962) and references therein.

The NWS equation has been considered by many authors. For instance, Macías-Díaz and Ruiz-Ramírez have proposed a finite-difference scheme to approximate the solutions of a generalization of the classical, one-dimensional Newell–Whitehead–Segel equation which is in the form

$$u_t - ku_{xx} - u + u^{2p+1} = 0, \quad (4)$$

where p is a positive integer (Macías-Díaz & Ruiz-Ramírez, 2011). Nourazar, Soori and Nazari-Golshan have obtained solutions of special cases of Equation (3) using the homotopy perturbation method.

They have solved this equation with $(a, b, k, q) = (2, 3, 1, 2), (1, 1, 1, 2), (1, 1, 1, 4), (3, 4, 1, 3)$ (Nourazar, Soori, Nazari-Golshan, 2011).

Rabinovich has considered how the establishment of self-oscillations takes place for explosion instability (Rabinovich, 1974). He has investigated such a mechanism using the example of medium described by the equation

$$-\beta u_{xxtt} - u_{tt} + \left(-\gamma u + u^2 - \alpha u^3\right)_t + \left(V + \delta u^2\right)u_{xx} = 0. \tag{5}$$

This equation describes electric signals in telegraph lines on the basis of the tunnel diode. In Korpusov (2011), setting $\beta = -1, \gamma = \alpha = \delta = 0$ and $V = 1$ in (Equation 5), Korpusov has considered the equation

$$u_{xxtt} - u_{tt} + u_t - \left(u^2\right)_t + u_{xx} = 0 \tag{6}$$

and named as “Rabinovich wave equation with nonlinear damping”. Also he has obtained sufficient conditions of the blow-up for Equation (6).

Over the last 20 years, several different hyperbolic tangent function method has been proposed for searching traveling wave solutions of nonlinear evolution equations. This technique was used by Huibin and Kelin (1990) first and then developed by Malfliet, Hereman, Fan, Senthilvelan, Wazwaz, and others since 1990. It has been used extensively in the literature to the present. The most common of these methods were the tanh-function method, the extended tanh-function method, the modified extended tanh-function method, and the complex tanh-function method (Wazwaz, 2006, 2007; Khuri, 2004).

In this article, a unification of the family of tangent hyperbolic function methods called the unified tanh-function method has been proposed. The other members of the family of hyperbolic function methods could overlook some type solutions or sometimes it can need any other member of this family to find all of the solutions. The advantage of this method is to give all type solutions within one method in a straightforward, concise and elegant manner without reproducing a lot of different forms of the same solution.

The rest of the paper is organized as follows: The unified tanh-function method is introduced in Section 2. The traveling wave solutions of (3+1) dimensional Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation are obtained in Section 3. Equation (3) is considered and exact solutions of the generalized Newell–Whitehead–Segel (NWS) equation are obtained in Section 4. The new traveling wave, trigonometric, and solitary wave solutions of the Rabinovich wave equation are obtained in Section 5. Finally, our conclusions are summarized in Section 6.

2. The unified tanh-function method

The authors describe the unified tanh method for finding traveling wave solutions of nonlinear partial differential equations. Suppose that a nonlinear partial differential equation (NPDE), say in two independent variables x and t , is given by

$$P(u, u_t, u_x, u_{xt}, u_{tt}, u_{xx}, \dots) = 0 \tag{7}$$

where $u(x, t)$ is an unknown function, P is a polynomial in $u = u(x, t)$, and its various partial derivatives, in which highest order derivatives and nonlinear terms are involved.

The summary of the unified tanh method can be presented in the following six steps:

Step 1: To find the traveling wave solutions of Equation (7), one uses the wave variable

$$u(x, t) = U(\xi), \xi = x - ct, \tag{8}$$

where the constant c is generally termed the wave velocity. Substituting Equation (8) into Equation (7), one obtains the following ordinary differential equation (ODE) in ξ (which illustrates a principal advantage of a traveling wave solution, i.e. a PDE is reduced to an ODE).

$$P(U, cU', U', c^2U'', U'', \dots) = 0 \tag{9}$$

Step 2: If necessary one integrates Equation (9) as many times as possible and set the constants of integration to be zero for simplicity.

Step 3: Suppose the solution of nonlinear partial differential equation can be expressed by a polynomial in Y as follows:

$$u(\xi) = a_0 + \sum_{i=1}^M (a_i Y^i(\xi) + b_i Y^{-i}(\xi)) \tag{10}$$

where $Y = Y(\xi)$ satisfies the Riccati differential equation

$$Y'(\xi) = k^2 - Y^2(\xi), \tag{11}$$

where $Y' = \frac{dY}{d\xi}$, and a_i, b_i and k are constants. The general solution of Equation (11) as follows:

$$Y(\xi) = \begin{cases} \begin{cases} (a + ib) \tanh((a + ib)(\xi + \xi_0)) \\ (a + ib) \coth((a + ib)(\xi + \xi_0)) \end{cases}, k = a + ib \\ \frac{1}{\xi + \xi_0}, k = 0 \end{cases} . \tag{12}$$

If one takes $k = a$, then the first solutions group in (12) are $a \tanh(a(\xi + \xi_0))$ and $a \coth(a(\xi + \xi_0))$. On the other hand, if one takes $k = ib$, then the first solutions group in (12) are $-b \tan(b(\xi + \xi_0))$ and $b \cot(b(\xi + \xi_0))$. If the general solutions of Equation (11) are summed up, then one has

$$Y(\xi) = \begin{cases} \begin{cases} (a + ib) \tanh((a + ib)(\xi + \xi_0)) \\ (a + ib) \coth((a + ib)(\xi + \xi_0)) \end{cases}, k = a + ib \\ \begin{cases} a \tanh(a(\xi + \xi_0)) \\ a \coth(a(\xi + \xi_0)) \end{cases}, k = a \\ \begin{cases} -b \tan(b(\xi + \xi_0)) \\ b \cot(b(\xi + \xi_0)) \end{cases}, k = ib \\ \frac{1}{\xi + \xi_0}, k = 0 \end{cases} . \tag{13}$$

Step 4: The positive integer M can be accomplished by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in Equation (3.3) as follows:

If one defines the degree of $u(\xi)$ as $D[u(\xi)] = M$, then the degree of other expressions is defined by

$$D\left[\frac{d^q u}{d\xi^q}\right] = M + q,$$

$$D\left[u^r \left(\frac{d^q u}{d\xi^q}\right)^s\right] = Mr + s(q + M).$$

Therefore, one gets the value of M in Equation (3.4).

Step 5: Substituting Equation (10) and (11) into Eq.(9) and collecting all terms with the same order of Y together, then setting each coefficient of this polynomial to zero yield a set of algebraic equations for a_i, b_i, c and k .

Step 6: Substituting a_i, b_i, c and k obtained in Step 5 into (10) and using the general solutions of Equation (11) in (12) or (13), one can obtain the explicit solutions of Equation (7) immediately depending on the value of k .

3. The (3+1) dimensional Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation

The (3 + 1)-dimensional KZK equation reads:

$$u_{xt} + u_x^2 + uu_{xx} + \lambda u_{xxx} + \gamma u_{yy} + \theta u_{zz} = 0, \tag{14}$$

where $\lambda, \gamma,$ and θ are real constants and $\lambda \neq 0$. We first substitute the wave variable $\xi = x + y + z - Vt$, V as the wave speed, with the wave transformation $u(x, t) = U(\mu\xi)$ into (14) and integrating once to obtain

$$(\theta + \gamma - V)U + \frac{1}{2}U^2 + \lambda U' = 0. \tag{15}$$

Using the balance process leads to $M = 1$. The tanh-coth method allows us to use the substitution

$$U(\mu\xi) = S(Y) = \sum_{k=0}^1 a_k Y^k + \sum_{k=1}^1 b_k Y^{-k} \tag{16}$$

Substituting (16) into (15), collecting the coefficients of each power of Y , setting each coefficient to zero we find a system of algebraic equations for a_0, a_1, b_1 and V in the following form:

$$\begin{aligned} Y^2: & \quad a_1^2 - 2a_1\lambda\mu = 0 \\ Y^1: & \quad 2a_1\theta + 2a_1\gamma - 2Va_1 + 2a_0a_1 = 0 \\ Y^0: & \quad 2a_0\theta + 2a_0\gamma - 2Va_0 + 2a_1b_1 + a_0^2 + 2a_1\lambda\mu + 2b_1\lambda\mu = 0 \\ Y^{-1}: & \quad 2b_1\theta + 2b_1\gamma - 2Vb_1 + 2a_0b_1 = 0 \\ Y^{-2}: & \quad b_1^2 - 2b_1\lambda\mu = 0. \end{aligned} \tag{17}$$

Solving the resulting system of algebraic equations, we find the following sets of solutions:

$$a_0 = -2\lambda\mu, \quad a_1 = 0, \quad b_1 = 2\lambda\mu, \quad V = \theta - 2\lambda\mu + \gamma \tag{18}$$

$$a_0 = 2\lambda\mu, \quad a_1 = 0, \quad b_1 = 2\lambda\mu, \quad V = \theta + 2\lambda\mu + \gamma \tag{19}$$

$$a_0 = -4\lambda\mu, \quad a_1 = 2\lambda\mu, \quad b_1 = 2\lambda\mu, \quad V = \theta - 4\lambda\mu + \gamma \tag{20}$$

$$a_0 = 2\lambda\mu, \quad a_1 = 2\lambda\mu, \quad b_1 = 0, \quad V = \theta + 2\lambda\mu + \gamma \tag{21}$$

$$a_0 = -2\lambda\mu, \quad a_1 = 2\lambda\mu, \quad b_1 = 0, \quad V = \theta - 2\lambda\mu + \gamma \tag{22}$$

$$a_0 = 4\lambda\mu, \quad a_1 = 2\lambda\mu, \quad b_1 = 2\lambda\mu, \quad V = \theta + 4\lambda\mu + \gamma \quad (23)$$

where μ is left as a free parameter. Consequently, using these values, we obtain following hyperbolic solutions, respectively:

$$u_1(x, t) = -2\lambda a + 2\lambda a \coth a(x + y + z - (\theta - 2\lambda a + \gamma)t) \quad (24)$$

$$u_2(x, t) = 2\lambda a + 2\lambda a \coth a(x + y + z - (\theta + 2\lambda a + \gamma)t) \quad (25)$$

$$u_3(x, t) = -4\lambda a + 2\lambda a \tanh a(x + y + z - (\theta - 4\lambda a + \gamma)t) + 2\lambda a \coth a(x + y + z - (\theta - 4\lambda a + \gamma)t) \quad (26)$$

$$u_4(x, t) = 2\lambda a + 2\lambda a \tanh a(x + y + z - (\theta + 2\lambda a + \gamma)t) \quad (27)$$

$$u_5(x, t) = -2\lambda a + 2\lambda a \tanh a(x + y + z - (\theta - 2\lambda a + \gamma)t) \quad (28)$$

$$u_6(x, t) = 4\lambda a + 2\lambda a \tanh a(x + y + z - (\theta + 4\lambda a + \gamma)t) + 2\lambda a \coth a(x + y + z - (\theta + 4\lambda a + \gamma)t) \quad (29)$$

where $\mu = a$, a is a real constant. Using the hyperbolic identities $i \tanh(ix) = -\tan x$ and $i \coth(ix) = \cot x$, trigonometric solutions from the tanh-coth can be obtained method as follows:

$$u_7(x, t) = -2\lambda b i + 2\lambda b \cot b(x + y + z - (\theta - 2\lambda b i + \gamma)t) \quad (30)$$

$$u_8(x, t) = 2\lambda b i + 2\lambda b \cot b(x + y + z - (\theta + 2\lambda b i + \gamma)t) \quad (31)$$

$$u_9(x, t) = -4\lambda b i - 2\lambda b \tan b(x + y + z - (\theta - 4\lambda b i + \gamma)t) + 2\lambda b \cot b(x + y + z - (\theta - 4\lambda b i + \gamma)t) \quad (32)$$

$$u_{10}(x, t) = 2\lambda b i - 2\lambda b \tan b(x + y + z - (\theta + 2\lambda b i + \gamma)t) \quad (33)$$

$$u_{11}(x, t) = -2\lambda b i - 2\lambda b \tan b(x + y + z - (\theta - 2\lambda b i + \gamma)t) \quad (34)$$

$$u_{12}(x, t) = 4\lambda b i - 2\lambda b \tan b(x + y + z - (\theta + 4\lambda b i + \gamma)t) + 2\lambda b \cot b(x + y + z - (\theta + 4\lambda b i + \gamma)t) \quad (35)$$

where $\mu = ib$, b is a real constant and $i = \sqrt{-1}$.

4. The Newell-Whitehead-Segel (NWS) equation

The Newell-Whitehead-Segel (NWS) equation is of the form

$$u_t - k u_{xx} - a u + b u^q = 0, \quad (36)$$

where a, b, k are real numbers, $k > 0$ is the coefficient of diffusion, and q is a positive integer. Also, we assume that $q > 1$. Using the wave transformation $u(x, t) = U(\mu\xi)$ with wave variable $\xi = x - Vt$, Equation (36) will be converted to the ODE

$$-VU' - kU'' - aU + bU^q = 0. \quad (37)$$

Balancing the second term with the last term, we find $M + 2 = qM$ so that $M = \frac{2}{q-1}$. To get analytic closed solution, M should be an integer, hence we use the transformation

$$U = W^{\frac{1}{q-1}}. \quad (38)$$

Using (38) into (37) gives

$$-V(q-1)WW' - k(2-q)(W')^2 - k(q-1)WW'' - (q-1)^2 aW^2 + (q-1)^2 bW^3 = 0. \quad (39)$$

Balancing WW'' with W^3 gives $M = 2$. The tanh-coth method admits the use of the finite expansion

$$W(\mu\xi) = S(Y) = \sum_{i=0}^2 a_i Y^i + \sum_{i=1}^2 b_i Y^{-i} \tag{40}$$

Substituting Equation (40) into Equation (39), we obtain a system of algebraic equations for $a_0, a_1, a_2, b_1, b_2, \mu$ and V . Solving this system of equation, we obtain the following sets of solutions:

$$a_0 = a_2 = \frac{a}{4b}, a_1 = -\frac{2\mu V(q+1)}{b(q^2+2q-3)}, b_1 = b_2 = 0, V = \mp 2\sqrt{ak+k^2\mu^2}, \mu = \mp \frac{\sqrt{2ak(q+1)(q-1)}}{4k(q+1)} \tag{41}$$

$$a_0 = b_2 = \frac{a}{4b}, a_1 = a_2 = 0, b_1 = -\frac{2\mu V(q+1)}{b(q^2+2q-3)}, V = \mp 2\sqrt{ak+k^2\mu^2}, \mu = \mp \frac{\sqrt{2ak(q+1)(q-1)}}{4k(q+1)} \tag{42}$$

Consequently, using these values, we obtain following hyperbolic solutions, respectively:

$$w_1(x, t) = \frac{a}{4b} - \frac{2pV(q+1)}{b(q^2+2q-3)} \tanh p(x-Vt) + \frac{a}{4b} \tanh^2 p(x-Vt), \tag{43}$$

$$w_2(x, t) = \frac{a}{4b} - \frac{2pV(q+1)}{b(q^2+2q-3)} \coth p(x-Vt) + \frac{a}{4b} \coth^2 p(x-Vt), \tag{44}$$

where $\mu = p, p$ is a real constant. Using the hyperbolic identities $i \tanh(ix) = -\tan x$ and $i \coth(ix) = \cot x$, the trigonometric solutions from tanh-coth method can be obtained as follows:

$$w_3(x, t) = \frac{a}{4b} + \frac{2rV(q+1)}{b(q^2+2q-3)} \tan r(x-Vt) - \frac{a}{4b} \tan^2 r(x-Vt), \tag{45}$$

$$w_4(x, t) = \frac{a}{4b} - \frac{2rV(q+1)}{b(q^2+2q-3)} \cot r(x-Vt) - \frac{a}{4b} \cot^2 r(x-Vt), \tag{46}$$

where $\mu = ir, r$ is a real constant and $i = \sqrt{-1}$.

Recalling that $u = w^{\frac{1}{q-1}}$ we find the traveling wave solutions for the NWS equation in the following form:

$$u_1(x, t) = \left\{ \frac{a}{4b} - \frac{2pV(q+1)}{b(q^2+2q-3)} \tanh p(x-Vt) + \frac{a}{4b} \tanh^2 p(x-Vt) \right\}^{\frac{1}{q-1}}, \tag{47}$$

$$u_2(x, t) = \left\{ \frac{a}{4b} - \frac{2pV(q+1)}{b(q^2+2q-3)} \coth p(x-Vt) + \frac{a}{4b} \coth^2 p(x-Vt) \right\}^{\frac{1}{q-1}}, \tag{48}$$

$$u_3(x, t) = \left\{ \frac{a}{4b} + \frac{2rV(q+1)}{b(q^2+2q-3)} \tan r(x-Vt) - \frac{a}{4b} \tan^2 r(x-Vt) \right\}^{\frac{1}{q-1}}, \tag{49}$$

$$u_4(x, t) = \left\{ \frac{a}{4b} - \frac{2rV(q+1)}{b(q^2+2q-3)} \cot r(x-Vt) - \frac{a}{4b} \cot^2 r(x-Vt) \right\}^{\frac{1}{q-1}}. \quad (50)$$

5. The Rabinovich wave equation

The Rabinovich wave equation with nonlinear damping is given by

$$u_{xxtt} - u_{tt} + u_t - (u^2)_t + u_{xx} = 0. \quad (51)$$

Using the wave variable $\xi = x - Vt$ in Equation (51), then integrating this equation and considering the integration constant to not be zero, we obtain

$$V^2 U''' + (1 - V^2)U' - VU + VU^2 = 0 \quad (52)$$

Balancing U^2 and U''' gives $M = 3$. Therefore, the solutions of (52) can be written in the form

$$U(\mu\xi) = S(Y) = \sum_{i=0}^3 a_i Y^i + \sum_{i=1}^3 b_i Y^{-i}. \quad (53)$$

Substituting (53) into (52), collecting the coefficients of each power of Y , setting each coefficient to zero, and solving the system of algebraic equations, we find sets of solutions in the following form:

Set 1.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{19V^2 - 19}}{38V}, b_3 = b_2 = b_1 = 0, a_3 = \frac{15\mu(V^2 - 1)}{19V}, \\ a_2 &= 0, a_1 = -\frac{45\mu(V^2 - 1)}{19V}, a_0 = \frac{1}{2}; \end{aligned} \quad (54)$$

Set 2.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{19V^2 - 19}}{38V}, a_3 = a_2 = a_1 = 0, b_3 = \frac{15\mu(V^2 - 1)}{19V}, \\ b_2 &= 0, b_1 = -\frac{45\mu(V^2 - 1)}{19V}, a_0 = \frac{1}{2}; \end{aligned} \quad (55)$$

Set 3.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{209 - 209V^2}}{38V}, b_3 = b_2 = b_1 = 0, a_3 = -\frac{165\mu(V^2 - 1)}{19V}, \\ a_2 &= 0, a_1 = \frac{135\mu(V^2 - 1)}{19V}, a_0 = \frac{1}{2}; \end{aligned} \quad (56)$$

Set 4.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{209 - 209V^2}}{38V}, a_3 = a_2 = a_1 = 0, b_3 = -\frac{165\mu(V^2 - 1)}{19V}, \\ b_2 &= 0, b_1 = \frac{135\mu(V^2 - 1)}{19V}, a_0 = \frac{1}{2}; \end{aligned} \quad (57)$$

Set 5.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{209 - 209V^2}}{76V}, a_2 = b_2 = 0, a_0 = \frac{1}{2}, b_3 = a_3 = -\frac{165\mu(V^2 - 1)}{76V}, \\ a_1 &= -\frac{495(V^2 - 1)^2}{23104\mu V^3}, b_1 = \frac{45\mu(V^2 - 1)}{76V}; \end{aligned} \tag{58}$$

Set 6.

$$\begin{aligned} \mu &= \mp \frac{\sqrt{19c^2 - 19}}{76V}, a_2 = b_2 = 0, a_0 = \frac{1}{2}, b_3 = a_3 = \frac{15\mu(V^2 - 1)}{76V}, \\ a_1 &= -\frac{135(V^2 - 1)^2}{23104\mu V^3}, b_1 = -\frac{135\mu(V^2 - 1)}{76V}. \end{aligned} \tag{59}$$

Using these values, we obtain following hyperbolic solutions respectively:

$$\begin{aligned} u_1(x, t) &= \frac{1}{2} + \frac{15(1 - V^2)\sqrt{19V^2 - 19}}{722V^2} \left(3 \tanh \left(\frac{\sqrt{19V^2 - 19}}{38V}(x - Vt) \right) \right. \\ &\quad \left. - \tanh^3 \left(\frac{\sqrt{19V^2 - 19}}{38V}(x - Vt) \right) \right), \end{aligned} \tag{60}$$

$$\begin{aligned} u_2(x, t) &= \frac{1}{2} + \frac{15(1 - V^2)\sqrt{19V^2 - 19}}{722V^2} \left(3 \coth \left(\frac{\sqrt{19V^2 - 19}}{38V}(x - Vt) \right) \right. \\ &\quad \left. - \coth^3 \left(\frac{\sqrt{19V^2 - 19}}{38V}(x - Vt) \right) \right), \end{aligned} \tag{61}$$

where $V^2 > 1$;

$$\begin{aligned} u_3(x, t) &= \frac{1}{2} - \frac{(1 - V^2)\sqrt{209 - 209V^2}}{722V^2} \left(135 \tanh \left(\frac{\sqrt{209 - 209V^2}}{38V}(x - Vt) \right) \right. \\ &\quad \left. - 165 \tanh^3 \left(\frac{\sqrt{209 - 209V^2}}{38V}(x - Vt) \right) \right), \end{aligned} \tag{62}$$

$$\begin{aligned} u_4(x, t) &= \frac{1}{2} - \frac{(1 - V^2)\sqrt{209 - 209V^2}}{722V^2} \left(135 \coth \left(\frac{\sqrt{209 - 209V^2}}{38V}(x - Vt) \right) \right. \\ &\quad \left. - 165 \coth^3 \left(\frac{\sqrt{209 - 209V^2}}{38V}(x - Vt) \right) \right), \end{aligned} \tag{63}$$

where $V^2 < 1$;

$$\begin{aligned}
 u_5(x, t) = & \frac{1}{2} - \frac{495(V^2 - 1)^2}{304V^2 \sqrt{209 - 209V^2}} \tanh\left(\frac{\sqrt{209 - 209V^2}}{76c}(x - Vt)\right) \\
 & + \frac{(V^2 - 1) \sqrt{209 - 209V^2}}{5776V^2} (-165 \tanh^3\left(\frac{\sqrt{209 - 209V^2}}{76V}(x - Vt)\right)) \\
 & + 45 \coth\left(\frac{\sqrt{209 - 209V^2}}{76V}(x - Vt)\right) - 165 \coth^3\left(\frac{\sqrt{209 - 209V^2}}{76V}(x - Vt)\right),
 \end{aligned} \tag{64}$$

where $V^2 < 1$;

$$\begin{aligned}
 u_6(x, t) = & \frac{1}{2} - \frac{135(V^2 - 1)^2}{304V^2 \sqrt{19V^2 - 19}} \tanh\left(\frac{\sqrt{19V^2 - 19}}{76V}(x - Vt)\right) \\
 & + \frac{(V^2 - 1) \sqrt{19V^2 - 19}}{5776V^2} (15 \tanh^3\left(\frac{\sqrt{19V^2 - 19}}{76V}(x - Vt)\right)) \\
 & - 135 \coth\left(\frac{\sqrt{19V^2 - 19}}{76V}(x - Vt)\right) + 15 \coth^3\left(\frac{\sqrt{19V^2 - 19}}{76V}(x - Vt)\right),
 \end{aligned} \tag{65}$$

where $V^2 > 1$;

Using the hyperbolic identities $i \tanh(ix) = -\tan x$ and $i \coth(ix) = \cot x$, the trigonometric solutions from tanh-method can be obtained as follows:

$$\begin{aligned}
 u_7(x, t) = & \frac{1}{2} + \frac{15(1 - V^2) \sqrt{19 - 19V^2}}{722V^2} \left(-3 \tan\left(\frac{\sqrt{19 - 19V^2}}{38V}(x - Vt)\right)\right. \\
 & \left. - \tan^3\left(\frac{\sqrt{19 - 19V^2}}{38V}(x - Vt)\right)\right),
 \end{aligned} \tag{66}$$

$$\begin{aligned}
 u_8(x, t) = & \frac{1}{2} + \frac{15(1 - V^2) \sqrt{19 - 19V^2}}{722V^2} \left(3 \cot\left(\frac{\sqrt{19 - 19V^2}}{38V}(x - Vt)\right)\right. \\
 & \left. + \cot^3\left(\frac{\sqrt{19 - 19V^2}}{38V}(x - Vt)\right)\right),
 \end{aligned} \tag{67}$$

where $V^2 < 1$;

$$\begin{aligned}
 u_9(x, t) = & \frac{1}{2} - \frac{(1 - V^2) \sqrt{209V^2 - 209}}{722V^2} \left(-135 \tan\left(\frac{\sqrt{209V^2 - 209}}{38V}(x - Vt)\right)\right. \\
 & \left. - 165 \tan^3\left(\frac{\sqrt{209V^2 - 209}}{38V}(x - Vt)\right)\right),
 \end{aligned} \tag{68}$$

$$\begin{aligned}
 u_{10}(x, t) = & \frac{1}{2} - \frac{(1 - V^2) \sqrt{209V^2 - 209}}{722V^2} \left(135 \cot\left(\frac{\sqrt{209V^2 - 209}}{38c}(x - Vt)\right)\right. \\
 & \left. + 165 \cot^3\left(\frac{\sqrt{209V^2 - 209}}{38V}(x - Vt)\right)\right),
 \end{aligned} \tag{69}$$

where $V^2 > 1$;

$$\begin{aligned}
 u_{11}(x, t) = & \frac{1}{2} - \frac{495(V^2 - 1)^2}{304V^2 \sqrt{209V^2 - 209}} \tan\left(\frac{\sqrt{209V^2 - 209}}{76V}(x - Vt)\right) \\
 & + \frac{(V^2 - 1) \sqrt{209V^2 - 209}}{5776V^2} \left(-165 \tan^3\left(\frac{\sqrt{209V^2 - 209}}{76V}(x - Vt)\right)\right. \\
 & \left. + 45 \cot\left(\frac{\sqrt{209V^2 - 209}}{76V}(x - Vt)\right) + 165 \cot^3\left(\frac{\sqrt{209c^2 - 209}}{76V}(x - Vt)\right)\right),
 \end{aligned} \tag{70}$$

where $V^2 > 1$;

$$\begin{aligned}
 u_{12}(x, t) = & \frac{1}{2} - \frac{135(V^2 - 1)^2}{304V^2 \sqrt{19 - 19V^2}} \tan\left(\frac{\sqrt{19 - 19V^2}}{76V}(x - Vt)\right) \\
 & + \frac{(V^2 - 1) \sqrt{19 - 19V^2}}{5776V^2} \left(15 \tanh^3\left(\frac{\sqrt{19 - 19c^2}}{76V}(x - Vt)\right)\right. \\
 & \left. - 135 \coth\left(\frac{\sqrt{19 - 19V^2}}{76V}(x - Vt)\right) + 15 \coth^3\left(\frac{\sqrt{19 - 19c^2}}{76V}(x - Vt)\right)\right),
 \end{aligned} \tag{71}$$

where $V^2 < 1$.

6. Conclusion

In this paper, the (3+1) dimensional Khokhlov-Zabolotskaya-Kuznetsov (KZK), the Newell-Whitehead-Segel (NWS), and the Rabinovich wave equations were investigated using the unified tanh-function produced more general solutions in a straightforward, concise, and elegant manner. The reason why it is needed to give the unified tanh method is to give a unification for the tanh-function methods in the literature without reproducing a lot of different forms of the same solutions. Thus, the unified tanh method gives the solutions in a straightforward and brief way without requiring more effort. On the other hand, the obtained results clearly show the efficiency of the method used in this work. Throughout the entire study, Maple facilitates the tedious algebraic calculations.

Funding

The authors received no direct funding for this research.

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Citation information

Cite this article as: New exact solutions for the Khokhlov-Zabolotskaya-Kuznetsov, the Newell-Whitehead-Segel and the Rabinovich wave equations by using a new modification of the tanh-coth method, Şamil Akçağıl & Tuğba Aydemir, *Cogent Mathematics* (2016), 3: 1193104.

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