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## PURE MATHEMATICS | RESEARCH ARTICLE

# Some new fixed point results on intuitionistic fuzzy metric spaces

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**Abstract:** Fuzzy set theory originated from the fact that reasoning is not crisp and admits degree. This theory plays a leading role in ample field of Science and Technology. In this paper, by utilizing the concept of E.A. and common (E.A) property, we prove new common fixed point theorems on intuitionistic fuzzy metric spaces. Moreover, we extend the main result for finite number of mappings and integral-type contractive condition on intuitionistic fuzzy metric spaces.

**Subjects:** Foundations & Theorems; Mathematics & Statistics; Science

**Keywords:** fuzzy set; fixed point; intuitionistic fuzzy metric spaces; E.A. property; common property (E.A.); weakly compatible mapping; complete subspace

**Subject classification codes:** 47H10; 54H25

### 1. Introduction

The Banach fixed point theorem in Banach (1922) is an significant tool in the theory of metric spaces. The idea of fuzzy logic was invented by professor Zadeh (1965) of the University of California, Berkeley. Fuzzy set introduces vagueness by eliminating the sharp boundary which divides members of the class from non-member. There has been an extensive research on fuzzy sets. In the literature, there are several notions of fuzzy metric space. The first one was introduced by Kramosil and Michalek (1975), its motivation derives from statistical metric space. Later, the notion of fuzzy metric space was modified by George and Veeramani (1994). This work forms a pertinent basis for the construction of fixed point theory in fuzzy metric spaces.

### ABOUT THE AUTHOR



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Vishal Gupta, having more than 12 years of teaching experience, is working as Associate Professor in the Department of Mathematics, Maharishi Markandeshwar University, Haryana (India). He has published one research book with international publisher and his immense contribution to journals of national and international repute is more than 60. He has presented more than 35 research papers in national and international conferences. He is also reviewer of many prestigious professional bodies like Mathematical Reviews, etc. His research interests focus on fixed point theory, fuzzy set theory and fuzzy mappings, topology, differential and integral equations.

### PUBLIC INTEREST STATEMENT

The idea of fixed point theory on fuzzy metric spaces is developed with swift tempo. The notion of defining intuitionistic fuzzy set as generalized fuzzy set is fairly interesting and constructive in many application areas like sale analysis, new product marketing, financial services, negotiation process, psychological investigations, etc. This work seeks to highlight the use of the concept of E.A property for proving some fixed point results on intuitionistic fuzzy metric space. In view of their interesting applications, we have given fixed point theorems for finite number of mappings and also for integral-type contractive condition on intuitionistic fuzzy metric space. In the last result, we replace E.A property by common E.A property on intuitionistic fuzzy metric space. We hope these new results will further assist to comprehend the concept of fixed point theory on intuitionistic fuzzy metric space.

Sessa (1982) initiated the tradition of improving commutativity in fixed point theorem. He introduced the notion of weakly commuting maps in metric spaces. The first step to extend the commutativity to generalized commutativity, known as compatible maps is done by Jungck (1986). Jungck and Rhoades (1998) derived a significant result in which notion of weak compatible map is given. Aamri and El Moutawakil (2002) generalized the concept of non-compatibility by defining E.A. property for self-mappings. It contained the class of non-compatible mappings in metric space. Many interesting and valuable results on fuzzy metric space were given by various authors as Gupta and Kanwar (2012), Gupta, Kanwar, and Gulati (2016), Vijayaraju and Sajath (2009), Gupta, Saini, Mani, and Tripathi (2015), Kang, Gupta, Singh, and Kumar (2013), and Saini, Gupta, and Singh (2007). Branciari (2002) presented the idea of Banach contraction principle with the help of Lebesgue-integrable function and proved a fixed point theorem satisfying contractive conditions of integral type. Gupta and Mani (2013) proved fixed point result for contractive mapping of integral type.

The significance of fixed point theory is evident from the fact that it has its applications in diverse disciplines of Science, Engineering, and Economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. It is commonly accepted that fuzzy logic emerged from the theory of fuzzy set. Today, fuzzy logic is very relevant concept in the field of Science and Technology.

In fuzzy set theory, the membership of an element to a fuzzy set is a single value between zero and one. The concept of intuitionistic fuzzy sets is a generalization of fuzzy sets which incorporate the degree of hesitation. The idea of an intuitionistic fuzzy set is initiated by Atanassov (1986). With help of continuous t-norm and continuous t-conorm, Alaca, Turkoglu, and Yildiz (2006) defined the notion of intuitionistic fuzzy metric space and introduced the notion of Cauchy sequence in intuitionistic fuzzy metric space. For more fixed point results on intuitionistic fuzzy metric space, we refer to Beg, Vetro, Gopal, and Imdad (2014), Turkoglu, Alaca, Cho, and Yildiz (2006), Turkoglu, Alaca, and Yildiz (2006), and Sharma and Deshpande (2009). For the extraction of information by reflecting and modeling the hesitancy present in real-life situation, intuitionistic fuzzy set theory has been playing an important role. The application of intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into set description. Intuitionistic fuzzy fixed point theory has become a subject of great interest for specialist in fixed point theory because this branch of mathematics has covered new possibilities for fixed point theorists.

## 2. Preliminaries

*Definition 2.1* (Schweizer & Sklar, 1960): A binary operation  $*$ : $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is called continuous t-norm if  $*$  satisfies the following conditions:

- (i)  $*$  is commutative and associative,
- (ii)  $*$  is continuous,
- (iii)  $a * 1 = a, \quad \forall a \in [0, 1]$ ,
- (iv)  $a * b \leq c * d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

*Definition 2.2* (Schweizer & Sklar, 1960): A binary operation  $\diamond$ : $[0, 1] \times [0, 1] \rightarrow [0, 1]$  is called continuous t-conorm if  $\diamond$  satisfies the following conditions:

- (i)  $\diamond$  is commutative and associative,
- (ii)  $\diamond$  is continuous,
- (iii)  $a \diamond 0 = a, \quad \forall a \in [0, 1]$ ,

(iv)  $a \diamond b \leq c \diamond d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a, b, c, d \in [0, 1]$ .

**Definition 2.3** (Alaca et al., 2006): The 5-tuple  $(X, M, N, *, \diamond)$  is said to be an intuitionistic fuzzy metric space (Shortly, IFM-space) if  $X$  is an arbitrary set,  $*$  is a continuous t-norm,  $\diamond$  is a continuous t-co-norm.  $M$  and  $N$  are fuzzy sets in  $X^2 \times [0, \infty)$  satisfying following conditions for all  $x, y, z \in X$  and  $s, t > 0$ ,

$$\text{IFM}_1. M(x, y, t) + N(x, y, t) \leq 1,$$

$$\text{IFM}_2. M(x, y, 0) = 0,$$

$$\text{IFM}_3. M(x, y, t) = 1 \text{ for all } t > 0 \text{ if and only if } x = y,$$

$$\text{IFM}_4. M(x, y, t) = M(y, x, t),$$

$$\text{IFM}_5. M(x, y, t) = 1, \text{ as } t \rightarrow \infty,$$

$$\text{IFM}_6. M(x, y, t) * M(y, z, s) \leq M(x, z, t + s),$$

$$\text{IFM}_7. M(x, y, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is left continuous,}$$

$$\text{IFM}_8. N(x, y, 0) = 1,$$

$$\text{IFM}_9. N(x, y, t) = 0 \text{ for all } t > 0 \text{ if and only if } x = y,$$

$$\text{IFM}_{10}. N(x, y, t) = N(y, x, t),$$

$$\text{IFM}_{11}. N(x, y, t) = 0, \text{ as } t \rightarrow \infty,$$

$$\text{IFM}_{12}. N(x, y, t) * N(y, z, s) \geq N(x, z, t + s),$$

$$\text{IFM}_{13}. N(x, y, \cdot): [0, \infty) \rightarrow [0, 1] \text{ is right continuous.}$$

Here,  $M(x, y, t)$  and  $N(x, y, t)$  denote the degree of nearness and the degree of non-nearness between  $x$  and  $y$  with respect to  $t$ , respectively.

**Remark** In intuitionistic fuzzy metric spaces,  $M(x, y, t)$  is non-decreasing and  $N(x, y, t)$  is non-increasing.

**Definition 2.4** (Alaca et al., 2006): Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space in which every Cauchy sequence is convergent, then it is said to be a complete fuzzy metric space.

**Definition 2.5** (Jungck & Rhoades, 1998): Two self-maps  $P$  and  $Q$  on set  $X$  are said to be weakly compatible if they commute at their coincident point.

**Definition 2.6** (Aamri & El Moutawakil, 2002): Two self-maps  $P$  and  $Q$  from an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  into itself are said to satisfy E.A. property if there exists a sequence  $\{x_n\}$  in  $X$  such that  $\lim_{n \rightarrow \infty} Px_n = \lim_{n \rightarrow \infty} Qx_n = x$  where  $x \in X$ .

**Definition 2.7** (Abbas, Altun, & Gopal, 2009): Two pairs  $(A, T)$  and  $(B, S)$  of self-mappings of an intuitionistic fuzzy metric space  $(X, M, N, *, \diamond)$  share the common property (E.A.) if there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  such that

$$\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} Ty_n = m \text{ for some } m \in X.$$

LEMMA 2.8 (Alaca, Altun, & Turkoglu, 2008): Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space and if for a number  $k \in (0, 1)$ ,

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t) \text{ for all } x, y \in X, t > 0, \text{ then } x = y.$$

LEMMA 2.9 (Branciari, 2002): Branciari-Integral contractive-type condition: For a given  $\epsilon > 0$ , there exists a real number  $c \in (0, 1)$  and a locally Lebesgue integrable function  $g: [0, \infty) \rightarrow [0, \infty)$  such that

$$\int_0^{d(x,y)} g(t)dt \geq c \int_0^{d(x,y)} g(t)dt \text{ and } \int_0^\epsilon g(t)dt > 0 \text{ for all } x, y \in X, \epsilon > 0.$$

Then  $f$  has a unique fixed point  $a \in X$  such that  $x \in X, \lim_{n \rightarrow \infty} f^n x = a$ .

Also, Branciari-Integral contractive-type condition is a generalization of Banach contraction map if  $g(t) = 1, \forall t > 0$ .

### 3. Main results

In this section, we are proving new fixed point theorems with contractive condition on intuitionistic fuzzy metric spaces.

THEOREM 3.1 Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Let  $A, B, S$  and  $T$  be self-mappings such that for all  $x, y \in X, t > 0$  and constant  $k \in (0, 1)$ ,

$$\left. \begin{aligned} M(Ax, By, kt) * [M(Tx, Ax, kt) \times M(Sy, By, kt)] &\geq \{M(Tx, Ax, t) \\ &* M(Tx, Sy, t) * M(Tx, By, t)\}, \\ N(Ax, By, kt) \diamond [N(Tx, Ax, kt) \times N(Sy, By, kt)] &\leq \{N(Tx, Ax, t) \\ \diamond N(Tx, SGy, t) \diamond N(Tx, By, t)\} \end{aligned} \right\} \quad (1)$$

Also satisfying the following condition

- i.(1) the pairs  $(A, T)$  and  $(B, S)$  share the common property (E.A.),
- ii.(2)  $T(X)$  and  $S(X)$  are closed subsets of  $X$ .

Then the pairs  $(A, T)$  and  $(B, S)$  have a coincident point. Further, if both the pairs  $(A, T)$  and  $(B, S)$  are weakly compatible, then  $A, B, S$  and  $T$  have a unique common fixed point in  $X$ .

*Proof* Since the pairs  $(A, T)$  and  $(B, S)$  share the common property (E.A.), therefore there exist two sequences  $\{x_n\}$  and  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} Ty_n = m \text{ for some } m \in X. \quad (2)$$

Since  $S(X)$  is a closed subset of  $X$ , therefore  $\lim_{n \rightarrow \infty} Sx_n = m \in S(X)$ .

Hence, there exists a point  $u \in X$  such that

$$Su = m. \quad (3)$$

With the help of (1) we have

$$M(Ay_n, Bu, kt) * [M(Ty_n, Ay_n, kt) \times M(Su, Bu, kt)] \geq \{M(Ty_n, Ay_n, t) * M(Ty_n, Su, t) * M(Ty_n, Bu, t)\},$$

and

$$N(Ay_n, Bu, kt) \diamond [N(Ty_n, Ay_n, kt) \times N(Su, Bx_n, kt)] \leq \{N(Ty_n, Ay_n, t) \diamond N(Ty_n, Su, t) \diamond N(Ty_n, Bu, t)\}$$

Taking limit as  $n \rightarrow \infty$  in (20), (21), we get  $Bu = m$  or  $Bu = m = Su$ .

It shows that the pair  $(B, S)$  has a coincident point. (4)

In a similar way,  $T(X)$  is also a closed subset of  $X$ , hence we have

$$\lim_{n \rightarrow \infty} Ty_n = m \in T(X).$$

Hence, there exists a point  $v \in X$  such that  $Tv = m$ . (5)

From condition (1), we get

$$M(Av, Bx_n, kt) * [M(Tv, Av, kt) \times M(Sx_n, Bx_n, kt)] \geq \{M(Tv, Av, t) * M(Tv, Sx_n, t) * M(Tv, Bx_n, t)\}$$

and

$$N(Av, Bx_n, kt) \diamond [N(Tv, Av, kt) \times N(Sx_n, Bx_n, kt)] \leq \{N(Tv, Av, t) \diamond N(Tv, Sx_n, t) \diamond N(Tv, Bx_n, t)\}.$$

Letting  $n \rightarrow \infty$  and using (2), (5), we get  $Av = m$  or  $Av = m = Tv$ . It shows that the pair  $(A, T)$  has a coincident point. (6)

Since, the pair  $(B, S)$  is weakly compatible, therefore

$$Bm = BSu = SBu = Sm. \tag{7}$$

By putting  $x_n = v, y_n = m$  in (1), we obtain

$$M(Av, Bm, kt) * [M(Tv, Av, kt) \times M(Sm, Bm, kt)] \geq \{M(Tv, Av, t) * M(Tv, Sm, t) * M(Tv, Bm, t)\},$$

and

$$N(Av, Bm, kt) \diamond [N(Tv, Av, kt) \times N(Sm, Bm, kt)] \leq \{N(Tv, Av, t) \diamond N(Tv, Sm, t) \diamond N(Tv, Bm, t)\}.$$

So, by (6) and (7), we have  $Bm = m = Sm$ , which shows that  $m$  is a common fixed point of the pair  $(B, S)$ .

As  $Av = Tv$  and pair  $(A, T)$  is weakly compatible, therefore

$$Am = ATv = TAv = Tm. \tag{8}$$

From condition (1), we obtain

$$M(Am, Bu, kt) * [M(Tm, Am, kt) \times M(Su, Bu, kt)] \geq \{M(Tm, Am, t) * M(Tm, Su, t) * M(Tm, Bu, t)\},$$

and

$$N(Am, Bu, kt) \diamond [N(Tm, Am, kt) \times N(Su, Bu, kt)] \leq \{N(Tm, Am, t) \diamond N(Tm, Su, t) \diamond N(Tm, Bu, t)\}.$$

Using (4) and (8), we get  $Am = m = Tm$ , which shows that  $m$  is a common fixed point of the pair  $(A, T)$ .

Hence,  $m$  is a common fixed point of  $A, B, S,$  and  $T$ . Uniqueness of common fixed point can be easily proved by using condition (1) of this theorem. This implies that  $m$  is a unique common fixed point of  $A, B, S,$  and  $T$ .

Our next theorem is a common fixed point result via E.A property on intuitionistic fuzzy metric space.

**THEOREM 3.2** *Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space with  $t$ -norm  $a * b = \min \{a, b\}$  and  $t$ -conorm  $a \diamond b = \max \{a, b\}$ .*

Let  $A, B, G, H, S$  and  $T$  be self-mappings such that for  $k \in (0, 1)$  and every  $x, y \in X, t > 0,$

$$\left. \begin{aligned} M(Ax, By, kt) * [M(THx, Ax, kt) \times M(SGy, By, kt)] &\geq \{M(THx, Ax, t) * M(THx, SGy, t) * M(THx, By, t)\} \\ N(Ax, By, kt) \diamond [N(THx, Ax, kt) \times N(SGy, By, kt)] &\leq \{N(THx, Ax, t) \diamond N(THx, SGy, t) \diamond N(THx, By, t)\} \end{aligned} \right\} \quad (9)$$

Also, satisfying the following conditions:

- (a)  $A(X) \subset SG(X), B(X) \subset TH(X),$
- (b) the pair  $(A, TH)$  or  $(B, SG)$  satisfies E.A property.
- (c) If one of  $A(X), B(X), SG(X)$  or  $TH(X)$  is a complete subspace of  $X,$  then  $(A, TH)$  and  $(B, SG)$  have a coincident point.

Moreover, if  $(A, TH)$  and  $(B, SG)$  are weakly compatible then,

$A, B, TH$  and  $SG$  have a unique common fixed point in  $X.$

**Proof:** By considering  $(B, SG)$  satisfies E.A. property, there exists a sequence  $\{x_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} Bx_n = m = \lim_{n \rightarrow \infty} SGx_n \text{ where } m \in X. \quad (10)$$

From condition (a), then there exists  $\{y_n\}$  in  $X$  such that

$$\lim_{n \rightarrow \infty} THy_n = m. \quad (11)$$

$Bx_n = THy_n.$  We have,

Using (9), we get

$$M(Ay_n, Bx_n, kt) * [M(THy_n, Ay_n, kt) \times M(SGx_n, Bx_n, kt)] \geq \{M(THy_n, Ay_n, t) * M(THy_n, SGx_n, t) * M(THy_n, Bx_n, t)\},$$

and

$$N(Ay_n, Bx_n, kt) \diamond [N(THy_n, Ay_n, kt) \times N(SGx_n, Bx_n, kt)] \leq \{N(THy_n, Ay_n, t) \diamond N(THy_n, SGx_n, t) \diamond N(THy_n, Bx_n, t)\}.$$

Taking limit as  $n \rightarrow \infty,$  we obtain

$$\lim_{n \rightarrow \infty} Ay_n = m \text{ and } \lim_{n \rightarrow \infty} Ay_n = m = \lim_{n \rightarrow \infty} SGy_n. \quad (12)$$

The property of complete subspace  $SG(X)$  of  $X$  implies that  $m = SG(l)$  for some  $l \in X.$

So, we get

$$\lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} THy_n = \lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} SGx_n = m = TH(l). \quad (13)$$

The followings conditions are obtained from

$$M(AI, Bx_n, kt) * [M(THI, AI, kt) \times M(SGx_n, Bx_n, kt)] \geq \{M(THI, AI, t) * M(THI, SGx_n, t) * M(THI, Bx_n, t)\}$$

and

$$N(AI, Bx_n, kt) \diamond [N(THI, AI, kt) \times N(SGx_n, Bx_n, kt)] \leq \{N(THI, AI, t) \diamond N(THI, SGx_n, t) \diamond N(THI, Bx_n, t)\}.$$

From (13) as  $n \rightarrow \infty$ , we get,  $A(l) = TH(l)$  which show that the pair  $(A, TH)$  has a coincident point  $m \in X$ . (14)

Since the pair  $(A, TH)$  is weakly compatible, therefore we have

$$A(TH)l = (TH)Al.$$

Thus

$$AA(l) = ATH(l) = THA(l) = THTH(l). \quad (15)$$

With help of (a), there exists  $q \in X$  such that  $A(l) = SG(q)$ . (16)

From (9), (14), and (15), we obtain  $Al = Bq$  which gives

$$Al = THI = SGq = Bq. \quad (17)$$

The weak compatibility of  $(B, SG)$  implies that  $BSGq = SGBq$ .

This implies,  $BSGq = SGBq = BBq = SGSGq$ .

By putting  $x = Al, y = q$  in (9), we get

$$M(AAl, Bq, kt) * [M(THAl, AAl, kt) \times M(SGq, Bq, kt)] \geq \{M(THAl, AAl, t) * M(THAl, SGq, t) * M(THAl, Bq, t)\},$$

and

$$N(AAl, Bq, kt) \diamond [N(THAl, AAl, kt) \times N(SGq, Bq, kt)] \leq \{N(THAl, AAl, t) \diamond N(THAl, SGq, t) \diamond N(THAl, Bq, t)\}.$$

Thus,  $Al = AAl = THAl$  is a common fixed point of  $A$  and  $TH$ . (18)

In same way as discussed above, we can prove that  $Bq$  is the common fixed point of  $SG$  and  $B$ .

Since  $Al = Bq$ , so  $Al$  is the common fixed point of  $A, B, TH$  and  $SG$ .

Uniqueness: If possible, let  $x$  and  $x'$  be two fixed points of  $A, B, TH$  and  $SG$ . Consider  $x = x, y = x'$  in (9), we obtain

$$M(Ax, Bx', kt) * [M(THx, Ax, kt) \times M(SGx', Bx', kt)] \geq \{M(THx, Ax, t) * M(THx, SGx', t) * M(THx, Bx', t)\},$$

and

$$N(Ax, Bx', kt) \diamond [N(THx, Ax, kt) \times N(SGx', Bx', kt)] \leq \{N(THx, Ax, t) \diamond N(THx, SGx', t) \diamond N(THx, Bx', t)\}.$$

We get  $x = x'$  using the concept of fixed point and intuitionistic fuzzy metric space. Therefore, the mappings  $A$ ,  $B$ ,  $TH$  and  $SG$  have a unique common fixed point.

As an application of the previously proved result, Integral-type contractive condition is employed for proving the next theorem on intuitionistic fuzzy metric space.

**THEOREM 3.3** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space with  $t$ -norm  $a * b = \min\{a, b\}$  and  $t$ -conorm  $a \diamond b = \max\{a, b\}$ .

Let  $A, B, G, H, S$  and  $T$  be self-mappings such that for  $k \in (0, 1)$  and every  $x, y \in X, t > 0$ ,

$$\int_0^{M(Ax, By, kt) * [M(THx, Ax, kt) \times M(SGy, By, kt)]} \psi(t) dt \geq \int_0^{U(x, y, t)} \psi(t) dt,$$

and

$$\int_0^{N(Ax, By, kt) \diamond [N(THx, Ax, kt) \times N(SGy, By, kt)]} \psi(t) dt \leq \int_0^{V(x, y, t)} \psi(t) dt$$

where  $\psi: \mathbb{R}^+ \rightarrow \mathbb{R}$  is Lebesgue integrable mapping which is summable, non-negative and

$$\begin{aligned} U(x, y, t) &= \{M(THx, Ax, t) * M(THx, SGy, t) * M(THx, By, t)\}, V(x, y, t) \\ &= \{N(THx, Ax, t) \diamond N(THx, SGy, t) \diamond N(THx, By, t)\}. \end{aligned}$$

Also, satisfying the following conditions:

- (a)  $A(X) \subset SG(X), B(X) \subset TH(X)$ ,
- (b) the pair  $(A, TH)$  or  $(B, SG)$  satisfies E.A property.
- (c) If one of  $A(X), B(X), SG(X)$  or  $TH(X)$  is a complete subspace of  $X$ ,

Then,  $(A, TH)$  and  $(B, SG)$  have a coincident point.

Moreover, if  $(A, TH)$  and  $(B, SG)$  are weakly compatible then,

$A, B, TH$  and  $SG$  have a unique common fixed point in  $X$ .

*Proof* Since  $(B, SG)$  satisfies E.A. property, then there exists a sequence  $\{x_n\}$  in  $X$ , such that

$$\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} SGx_n = m \text{ where } m \in X. \quad (19)$$

The condition (a) gives a sequence  $y_n \in X$  such that

$$Bx_n = THy_n. \text{ This implies}$$

$$\lim_{n \rightarrow \infty} THy_n = m. \quad (20)$$

Using (19), (20), Lemma (2.9–2.10), we get

$$\int_0^{M(Ay_n, Bx_n, kt) * [M(THy_n, Ay_n, kt) \times M(SGx_n, Bx_n, kt)]} \psi(t) dt \geq \int_0^{U(y_n, x_n, t)} \psi(t) dt,$$

and

$$\int_0^{N(Ay_n, Bx_n, kt) \diamond [N(THy_n, Ay_n, kt) \times N(SGx_n, Bx_n, kt)]} \psi(t) dt \leq \int_0^{V(y_n, x_n, t)} \psi(t) dt.$$

where

$$U(y_n, x_n, t) = M(THy_n, Ay_n, t) * M(THy_n, SGx_n, t) * M(THy_n, Bx_n, t), \quad V(y_n, x_n, t) \\ = N(THy_n, Ay_n, t) \diamond N(THy_n, SGx_n, t) \diamond N(THy_n, Bx_n, t).$$

$$\text{This implies } \lim_{n \rightarrow \infty} Ay_n = m \text{ and } \lim_{n \rightarrow \infty} Bx_n = m = \lim_{n \rightarrow \infty} SGy_n. \quad (21)$$

The concept of complete subspace  $SG(X)$  of  $X$  gives  $m = SG(l)$  for some  $l \in X$ .

This gives

$$\lim_{n \rightarrow \infty} Ay_n = \lim_{n \rightarrow \infty} THy_n = \lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} SGx_n = m = TH(l). \quad (22)$$

Taking  $x = l, y = x_n$  in contractive condition of Theorem (3.3), we have

$$\int_0^{M(AI, Bx_n, kt) * [M(THI, AI, kt) \times M(SGx_n, Bx_n, kt)]} \psi(t) dt \geq \int_0^{U(l, x_n, t)} \psi(t) dt,$$

and

$$\int_0^{N(AI, Bx_n, kt) \diamond [N(THI, AI, kt) \times N(SGx_n, Bx_n, kt)]} \psi(t) dt \leq \int_0^{V(l, x_n, t)} \psi(t) dt,$$

where

$$U(l, x_n, t) = M(THI, Ay_n, t) * M(THI, SGx_n, t) * M(THI, Bx_n, t), \quad V(l, x_n, t) \\ = N(THI, Ay_n, t) \diamond N(THI, SGx_n, t) \diamond N(THI, Bx_n, t).$$

$$\text{By considering } n \rightarrow \infty, \text{ we get } A(l) = TH(l). \quad (23)$$

This implies  $(A, TH)$  have a coincident point  $m \in X$ .

The weak compatibility of  $(A, TH)$  implies that  $A(TH)l = (TH)Al$ .

$$\text{Thus, } AA(l) = ATH(l) = THA(l) = THTH(l). \quad (24)$$

As  $A(X) \subset SG(X)$ , there exists  $q \in X$  such that  $A(l) = SG(q)$ . (25)

From (24), (25), we obtain

$$\int_0^{M(AI, Bq, kt) * [M(THI, AI, kt) \times M(SGq, Bq, kt)]} \psi(t) dt \geq \int_0^{U(l, q, t)} \psi(t) dt,$$

and

$$\int_0^{N(AI, Bq, kt) \diamond [N(THI, AI, kt) \times N(SGq, Bq, kt)]} \psi(t) dt \leq \int_0^{V(l, q, t)} \psi(t) dt,$$

where

$$U(l, q, t) = M(THI, Aq, t) * M(THI, SGq, t) * M(THI, Bq, t), \quad V(l, q, t) = N(THI, Aq, t) \diamond N(THI, SGq, t) \diamond N(THI, Bq, t).$$

Hence, we obtain  $Al = Bq$ .

Thus we have  $Al = THl = SGq = Bq$ . (26)

The weak compatibility of  $(B, SG)$  implies that  $BSGq = SGBq$ .

This implies,  $BSGq = SGBq = BBq = SGSGq$ .

Again taking  $x = Al, y = q$  in contractive condition of this theorem, we get  $Al = AAl = THAl$  is a common fixed point of  $A$  and  $TH$ . In the same way, We can prove that  $Bq$  is the common fixed point of  $SG$  and  $B$ . Since  $Al = Bq$ . So,  $Al$  is the common fixed point of  $A, B, TH$  and  $SG$ .

In next theorem, we generalize Theorem (3.2) for finite number of mappings:

**THEOREM 3.4** Let  $(X, M, N, *, \diamond)$  be an intuitionistic fuzzy metric space. Let  $T_1, T_2, T_3, \dots, T_z, S_1, S_2, S_3, \dots, S_z, A$  and  $B$  be mappings from  $X$  into itself such that

- (i)  $A(X) \subset S_1 S_2 S_3 \dots S_z(X), B(X) \subset T_1 T_2 T_3 \dots T_z(X)$ ,
- (ii) the pair  $(A, T_1 T_2 T_3 \dots T_z)$  or  $(B, S_1 S_2 S_3 \dots S_z)$  satisfies E.A property,
- (iii) there exists  $k \in (0, 1)$  such that every  $x, y \in X, t > 0$ ,

$$\left. \begin{aligned} &M(Ax, By, kt) * [M(T_1 T_2 T_3 \dots T_z x, Ax, kt) \times M(S_1 S_2 S_3 \dots S_z y, By, kt)] \\ &\geq \{M(T_1 T_2 T_3 \dots T_z x, Ax, t) * M(T_1 T_2 T_3 \dots T_z x, S_1 S_2 S_3 \dots S_z y, t) * M(T_1 T_2 T_3 \dots T_z x, By, t)\}, \\ &N(Ax, By, kt) \diamond [N(T_1 T_2 T_3 \dots T_z x, Ax, kt) \times N(S_1 S_2 S_3 \dots S_z y, By, kt)] \\ &\leq \{N(T_1 T_2 T_3 \dots T_z x, Ax, t) \diamond N(T_1 T_2 T_3 \dots T_z x, S_1 S_2 S_3 \dots S_z y, t) \diamond N(T_1 T_2 T_3 \dots T_z x, By, t)\}. \end{aligned} \right\}$$

If one of  $A(X), B(X), T_1 T_2 T_3 \dots T_z(X), S_1 S_2 S_3 \dots S_z(X)$  is complete subspace of  $X$  then  $(A, T_1 T_2 T_3 \dots T_z)$  and  $(B, S_1 S_2 S_3 \dots S_z)$  have a coincident point. Further, if  $(A, T_1 T_2 T_3 \dots T_z)$  and  $(B, S_1 S_2 S_3 \dots S_z)$  are weakly compatible, then  $A(X), B(X), T_1 T_2 T_3 \dots T_z(X)$  and  $S_1 S_2 S_3 \dots S_z(X)$  have unique fixed point in  $X$ .

*Proof* Since  $(B, S_1 S_2 S_3 \dots S_z)$  satisfies E.A. property, then there exists a sequence  $\{x_n\} \in X$ , such that  $\lim_{n \rightarrow \infty} Bx_n = \lim_{n \rightarrow \infty} S_1 S_2 S_3 \dots S_z x_n = m$ , where  $m \in X$ .

Also,  $B(X) \subset T_1 T_2 T_3 \dots T_z(X)$ , then there exists a sequence  $\{y_n\}$  in  $X$ , such that  $Bx_n = T_1 T_2 T_3 \dots T_z y_n$ .

Using the method of proof of Theorem (3.2), we can see that this result holds.

**COROLLARY 3.5** Let  $A, B, S$  and  $T$  be self-mappings on intuitionistic fuzzy metric spaces  $(X, M, N, *, \diamond)$  with t-norm  $a * b = \min \{a, b\}$  and t-conorm  $a \diamond b = \max \{a, b\}$  such that

- (i)  $A(X) \subset S(X), B(X) \subset T(X)$ ,
- (ii) the pair  $(A, T)$  or  $(B, S)$  satisfies E.A property,
- (iii) there exists  $k \in (0, 1)$  such that for every  $x, y \in X, t > 0$ ,

$$M(Ax, By, kt) * [M(Tx, Ax, kt) \times M(Sy, By, kt)] \geq \{M(Tx, Ax, t) * M(Tx, Sy, t) * M(Tx, By, t)\}$$

and

$$N(Ax, By, kt) \diamond [N(Tx, Ax, kt) \times N(Sy, By, kt)] \leq \{N(Tx, Ax, t) \diamond N(Tx, SGy, t) \diamond N(Tx, By, t)\}.$$

If one of  $A(X)$ ,  $B(X)$ ,  $S(X)$  or  $T(X)$  is a complete subspace of  $X$ , the pairs  $(A, T)$  and  $(B, S)$  have a coincident point. Further, if  $(A, T)$  and  $(B, S)$  are weakly compatible, then  $A$ ,  $B$ ,  $S$  and  $T$  have a unique common fixed point in  $X$ .

**Proof:** If we put  $G = H = I_x$  (the identity map on  $X$ ) in Theorem (3.2), then we have above result.

#### 4. Conclusions

Intuitionistic fuzzy set includes the degree of belongingness, degree of non-belongingness, and the hesitation margin. Many applications of intuitionistic fuzzy sets are carried out using distance measures approach. Distance measure between intuitionistic fuzzy sets is an indispensable concept in fuzzy mathematics.

In the present paper, the work has been proved using different contractive conditions on intuitionistic fuzzy metric space. Some assumptions are required for proving results. In this continuation, we extend the main result for finite number of mappings in which pairs of mappings satisfy contractive conditions, E.A property, common E.A. property, and weak compatibility. We have also incorporated the concept of Branciari-Integral contractive-type condition on intuitionistic fuzzy metric space with E.A property.

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