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*Corresponding author: Ashish Kumar, Department of Mathematics, Manipal University Jaipur, Jaipur 303007, Rajasthan, India
E-mail: ashishbarak2020@gmail.com

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STATISTICS | RESEARCH ARTICLE

Analysis of a redundant system with priority and Weibull distribution for failure and repair

Ashish Kumar[1*], Monika Saini[1] and Kuntal Devi[1]

Abstract: The main objective of the present study is to analyse a redundant system under the concept of priority and Weibull distribution for all random variables with different scale parameter and common shape parameter. For this purpose, by using semi-Markovian approach and regenerative point technique a reliability model is developed for a two non-identical unit system. A single repair facility is provided to the system to perform all repair activities. After a pre-specified time to enhance the performance and efficiency of the system the unit goes for preventive maintenance (PM). Priority to the repair of original unit is given over the PM of duplicate unit. Numerical results for the mean time of system failure, availability and profit function have been derived for a particular case to highlight the importance of the study.

Subjects: Mathematics & Statistics; Science; Statistics & Probability

Keywords: redundant system; non-identical units; Weibull failure and repair laws; priority; preventive maintenance and maximum operation time

1. Introduction

In the current scenario, the configuration and design of industrial systems such as air crafts, textile manufacturing systems, computer systems, communication systems, carbon recovery systems in fertiliser plants and satellite systems become more and more complex. The reliability, availability and efficiency of any system depend more or less on the design of the system. So, system designers continuously make efforts to design high-reliability systems using various techniques. Redundancy, i.e. the provision of spare unit is always considered as an effective technique to enhance the reliability of the system. Redundancy is classified into three categories: cold standby, hot standby and warm standby. In cold standby redundancy, the probability of failure of the standby unit is zero. Many researchers, such as Gopalan and Nagarwalla (1985), Goel and Sharma (1989), Cao and Wu (1989) and Gopalan and Bhanu (1995) analysed standby systems using the concept of cold standby redundancy. Chandrasekhar, Natarajan, and Yadavalli (2004) analysed a two-unit cold standby

ABOUT THE AUTHOR

Ashish Kumar, PhD, works as an assistant professor in Department of Mathematics & Statistics, School of Basic Sciences, Manipal University Jaipur, Jaipur, Rajasthan, India. He published more than 50 research papers in the field of reliability modelling and sampling theory. He participated in various workshops/seminar/conferences and presented his research findings within India and abroad.

PUBLIC INTEREST STATEMENT

Reliability modelling and analysis play a very important role in the lifecycle management of a product. Reliability describes the ability of a system or component to function under stated conditions for a specified period of time. Reliability may also describe the ability to function at a specified moment or interval of time. Reliability plays a key role in the cost-effectiveness of systems. A lot of research work has already been carried out in the direction of cold standby redundant systems of identical units. But, for non-identical systems not much work has been carried out. So, this work helps in the analysis of non-identical units systems.

system with Erlangian repair time. Wu and Wu (2011) developed a reliability model with two cold standby units, one repairman and a switch under Poisson shocks. Moghaddass, Zuo, and Qu (2011) analysed the reliability and availability of repairable system with repairman subject to shut-off rules.

A lot of research papers such as Malik and Nandal (2010), Mahmoud and Moshref (2010) and Kumar, Malik, and Barak (2012) appear in the literature to strengthen the idea that by conducting preventive maintenance (PM) after a pre-specified time period, the system can be restored to a younger state. Kumar and Malik (2012) developed many stochastic models for a computer system using the concept of PM after maximum operation time and independent h/w and s/w failure. Priority in repair disciplines is also an effective technique for reliability improvement. Zhang and Wang (2009) suggested a geometric model for a repairable cold standby system with priority in use and repair. Chhillar, Barak, and Malik (2014) developed a reliability model for a cold standby system with priority to repair over corrective maintenance subject to random shocks.

Many researchers such as Malik and Barak (2013) and Kumar, Baweja, and Barak (2015) developed reliability models under the assumption of constant failure and repair rates. But, many mechanical and electrical system's failure and repair rates behave arbitrarily. Osaki and Asakura (1970), Kapur and Kapoor (1974), Gupta, Kumar, and Gupta (2013) and Kishan and Jain (2014) suggested some reliability models for cold standby redundant systems in which all random variables are arbitrarily distributed.

In all studies discussed above, stochastic models for cold standby systems having identical units under different set of assumptions are developed. But, it is not always possible to keep an identical unit in standby due to economic reasons. However, a duplicate unit can be kept in standby to improve the reliability and availability of the system. In most of the studies, all the researchers made the assumption that all the random variables related to failure time of the unit distributed exponentially and repair times are either arbitrary or constantly distributed. But, the performance of most of the mechanical, industrial and electrical systems varies with respect to passes of time. So, their repair and failure are not necessarily constantly distributed but may behave as any arbitrary distribution. There are many distributions such as Weibull, normal and lognormal distributions that are useful in analysing failure processes of standby systems. These distributions have hazard rate functions that are not constant over time, thus providing a necessary alternative to the exponential failure law. The most important probability distribution in reliability modelling is the Weibull distribution. The Weibull failure distribution may be used to model both increasing and decreasing failure rates. Suppose random variable V denotes the maximum operation time of an item/device having Weibull distribution, then its PDF is denoted by $f_1(t) = \theta\eta t^{\eta-1}\exp(-\theta t^{\eta})\ t \geq 0$ and $\theta,\ \eta > 0$. It is characterised by a hazard rate function of the form $h(t) = \theta\eta t^{\eta-1},\ t \geq 0$ and $\theta, \eta > 0$ which is a power function. The function $\lambda(t)$ is increasing for $\eta > 0, \theta > 0$ and is decreasing for $\eta < 0, \theta < 0$. The reliability function is given by $R(t) = \exp(-\theta t^{\eta})$. Thus the failure-free operating time of the system has a Weibull distribution with parameters θ and η. Here η is referred to as the shape parameter. Its effect on the distribution can be seen for several different values. For $\eta < 1$, the PDF is similar in shape to the exponential, and for large values of η ($\eta \geq 3$), the PDF is somewhat symmetrical, like the normal distribution. For $1 < \eta < 3$, the density is skewed. If we put $\eta = 1$ in PDF, Weibull distribution reduces to Exponential distribution and if $\eta = 2$, it reduces to Rayleigh distribution. Kumar and Saini (2014) analysed the cost-benefit of a single-unit system under PM and Weibull distribution for random variables.

In the present paper, we develop a reliability model for a non-identical cold standby system for the evaluation of various reliability measures of the system by considering all time random variables as Weibull distributed. Priority to repair of the original unit is given over the PM of the duplicate unit. The possible states of the proposed problem have been given in section entitled system description. A single repair facility has been provided to do repair and maintenance activities of original and duplicate units. After a pre-specified time, the unit goes for PM. All random variables are statistically independent. Switch devices and repairs are perfect. Semi-Markov process and regenerative point techniques are used to draw recurrence relations for various reliability characteristics. All time random variables are Weibull distributed. The probability density function of maximum operation time of original and duplicate units are denoted by $g(t) = \alpha\eta t^{\eta-1}\exp(-\alpha t^{\eta})$. The PDF of failure times of the original and duplicate unit are denoted by $f(t) = \beta\eta t^{\eta-1}\exp(-\beta t^{\eta})$ and $f_2(t) = h\eta t^{\eta-1}\exp(-ht^{\eta})$, respectively. The PM rate of the original and duplicate units is denoted by the probability density function $g_1(t) = \gamma\eta t^{\eta-1}\exp(-\gamma t^{\eta})$. The random variables corresponding to repair rate of the original and duplicate units have the probability density function $f_1(t) = k\eta t^{\eta-1}\exp(-kt^{\eta})$ and $f_3(t) = l\eta t^{\eta-1}\exp(-lt^{\eta})$, respectively with $t \geq 0$ and θ, η, α, β, h, k, $l > 0$. The probability/cumulative density functions of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in $(0, t]$ have been denoted by $q_{ij.kr}(t)/Q_{ij.kr}(t)$. To improve the importance of the study, graphs are drawn for a particular case for mean time to system failure (MTSF), availability and profit function.

2. System description

In this section, a reliability model for two non-identical unit's system using the concept of priority and arbitrary distribution is developed. The system may be any of the following states described as follows:

State 0 Original unit is operative, duplicate unit in cold standby and system is in upstate. The service facility at S_0 remains idle.

State 1 Original unit is under PM after completion of maximum operation time, duplicate unit is operative and system is in upstate. The service facility at S_1 is busy under PM of the original unit.

State 2 Original unit is under repair after failure, duplicate unit is operative and system is in upstate. The service facility at S_2 is busy in repair activity of the failed original unit.

State 3 Original unit is operative, duplicate unit is under repair after failure and system is in upstate. The service facility at S_3 is busy in repair activity of the failed duplicate unit.

State 4 Original unit is operative, duplicate unit is under PM after completion of maximum operation time and system is in upstate. The service facility at S_4 is busy in PM of the duplicate unit.

State 5 It is the priority state. Here priority is given to repair of failed original unit over PM of duplicate unit. The service facility at S_5 is busy in repair of the original unit.

State 6 Original unit has failed and continuously under repair from past state, duplicate unit has failed and waiting for repair and system is in downstate. The service facility at S_6 is busy in repair of the original unit.

State 7 Original unit has failed and continuously under repair from past state, duplicate unit is under waiting for PM and system is in downstate. The service facility at S_7 is busy in repair of the original unit.

State 8 Original unit is waiting for PM after completion of maximum operation time, duplicate unit is under PM continuously after completion of maximum operation time from previous state and system is in downstate. The service facility at S_8 is busy in PM of the duplicate unit.

State 9 Original unit is continuously under PM after completion of maximum operation time from previous state, duplicate unit is waiting for PM after completion of maximum operation time and system is in downstate. The service facility at S_9 is busy in PM of the original unit.

State 10 Original unit is continuously under PM after completion of maximum operation time from previous state, duplicate failed unit is waiting for repair and system is in downstate. The service facility at S_{10} is busy in PM of the original unit.

State 11 Original unit is waiting for repair, duplicate failed unit is continuously under repair from previous state and system is in downstate. The service facility at S_{11} is busy in repair of the failed duplicate unit.

State 12 Original unit is waiting for PM after completion of maximum operation time, duplicate failed unit is continuously under repair from previous state and system is in downstate. The service facility at S_{12} is busy in repair of the failed duplicate unit.

Out of these, states S_0, S_1, S_2, S_3, S_4 and $S_5$are regenerative states, while all other are non-regenerative and failed states.

3. Transition probabilities and mean sojourn times

Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{ij}(\infty) = \int q_{ij}(t)dt \text{ as} \tag{1}$$

$$\begin{aligned} p_{01} &= \frac{\alpha}{\alpha+\beta}, p_{02} = \frac{\beta}{\alpha+\beta}, p_{10} = \frac{\gamma}{\alpha+h+\gamma}, p_{1.10} = \frac{h}{\alpha+\gamma+h} = p_{13.10}, p_{19} = \frac{\alpha}{\alpha+\gamma+h} \\ &= p_{14.9}, p_{20} = \frac{k}{\alpha+k+h}, p_{26} = \frac{h}{\alpha+h+k} = p_{23.6}, p_{27} = \frac{\alpha}{\alpha+h+k} = p_{24.7}, p_{30} \\ &= \frac{l}{l+\alpha+\beta}, p_{3.12} = \frac{\alpha}{\alpha+\beta+l} = p_{31.12}, p_{3.11} = \frac{\beta}{\alpha+\beta+l} = p_{32.11}, p_{40} = \frac{\gamma}{\alpha+\beta+\gamma}, p_{45} \\ &= \frac{\beta}{\alpha+\beta+\gamma} = p_{42.5}, p_{48} = \frac{\alpha}{\alpha+\beta+\gamma} = p_{41.8}, p_{52} = p_{63} = p_{74} = p_{81} = p_{94} = p_{10.3} = p_{11.2} \\ &= p_{12.1} = 1 \end{aligned} \tag{2}$$

It can be easily verified that $p_{01} + p_{02} = p_{10} + p_{19} + p_{1.10} = p_{10} + p_{14.9} + p_{10} = p_{20} + p_{27} + p_{26}$

$$\begin{aligned} &= p_{20} + p_{23.6} + p_{24.7} = p_{30} + p_{3.12} + p_{3.11} = p_{30} + p_{31.12} + p_{32.11} = p_{40} + p_{45} + p_{48} = p_{40} + p_{418} \\ &\quad + p_{425} = p_{52} = p_{63} = p_{74} = p_{81} = p_{94} = p_{10.3} = p_{11.2} = p_{12.1} = 1 \end{aligned} \tag{3}$$

The mean sojourn times (μ_i) in the state S_i are

$$\begin{aligned} &\mu_0 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta)^{1/\eta}}, \quad \mu_3 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta+l)^{1/\eta}}, && \mu_4 = \frac{\Gamma(1+1/\eta)}{(\alpha+\beta+\gamma)^{1/\eta}}, \\ &\mu_1' = \Gamma(1/\eta+1)\left[\frac{1}{(\alpha+h+\gamma)^{\frac{1}{\eta}}} + \frac{1}{(\alpha+\gamma+h)(\gamma)^{\frac{1}{\eta}}}(\alpha+h)\right], && \\ &\mu_2' = \Gamma(1/\eta+1)\left[\frac{1}{(\alpha+h+k)^{\frac{1}{\eta}}} + \frac{1}{(\alpha+k+h)(k)^{\frac{1}{\eta}}}(\alpha+h)\right], && \mu_2 = \frac{\Gamma(1+1/\eta)}{(\alpha+k+h)^{1/\eta}} \\ &\mu_3' = \Gamma(1/\eta+1)\left[\frac{1}{(\alpha+\beta+l)^{\frac{1}{\eta}}} + \frac{1}{(\alpha+\beta+l)(l)^{\frac{1}{\eta}}}(\alpha+\beta)\right], && \mu_1 = \frac{\Gamma(1+1/\eta)}{(\alpha+\gamma+h)^{1/\eta}} \\ &\mu_4^1 = \Gamma(1/\eta+1)\left[\frac{1}{(\alpha+\beta+\gamma)^{\frac{1}{\eta}}} + \frac{\alpha}{(\alpha+\gamma+\beta)(\gamma)^{\frac{1}{\eta}}}\right], && \mu_5 = \frac{\Gamma(1+\frac{1}{\eta})}{(k)^{1/\eta}}, \end{aligned} \tag{4}$$

4. Reliability and MTSF

Let $\varphi_i(t)$ be the CDF of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$:

$$\varphi_i(t) = \sum_j Q_{i,j}(t)®\varphi_j(t) + \sum_k Q_{i,k}(t) \tag{5}$$

where j is an unfailed regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly. Taking LST of above relation (5) and solving for $\tilde{\varphi}_0(s)$, we have

$$R^*(s) = \frac{1-\tilde{\varphi}_0(s)}{s} \tag{6}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (6).

The MTSF is given by

$$\begin{aligned} &\text{MTSF} = \lim_{s\to 0} \frac{1-\tilde{\varphi}_0(s)}{s} = \frac{N}{D} \quad \text{where} \\ &N = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 \quad \text{and } D = 1 - p_{01}p_{10} - p_{02}p_{20} \end{aligned} \tag{7}$$

5. Steady state availability

Let $A_i(t)$ be the probability that the system is in upstate at instant "t" given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \tag{8}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $M_i(t)$ is the probability that the system is initially in state $S_i \in E$ up at time t without visiting any other regenerative state, we have

$$M_0(t) = e^{-(\alpha+\beta)t^{\eta}},\ M_1(t) = e^{-(\alpha+h+\gamma)t^{\eta}},\ M_2(t) = e^{-(\alpha+k+h)t^{\eta}},\ M_3(t) = e^{-(\alpha+\beta+l)t^{\eta}},\ M_4(t) = e^{-(\alpha+\beta+\gamma)t^{\eta}} \tag{9}$$

Taking LT of above relations (8) and solving for $A_0^*(s)$, the steady-state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \text{where} \tag{10}$$

$$\begin{aligned} N_2 = {} & (M_0(t)((-p_{41.8})(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + (1 - p_{45}p_{54})((1 - p_{31.12}p_{13.10}) \\ & (1 - p_{32.11}p_{23.6}) - p_{32.11}p_{13.10}p_{31.12}p_{23.6})) + (M_1(t) + M_3(t)p_{13.10})(p_{01}(1 - p_{32.11}p_{23.6}) \\ & (1 - p_{54}p_{45}) + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54}) + p_{24.7}p_{41.8})) + (M_2(t) + M_3(t)p_{23.6})(p_{01}(p_{32.11}p_{13.10} \\ & (1 - p_{45}p_{54})) + p_{02}((1 - p_{31.12}p_{13.10})(1 - p_{45}p_{54}) - p_{14.9}p_{41.8})) - M_4(t)(-p_{01}(p_{24.7}p_{32.11}p_{13.10} \\ & + p_{14.9}(1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10}) - p_{14.9}p_{31.12}p_{23.6})) \end{aligned}$$

$$\begin{aligned} D_2 = {} & \mu_0((-p_{41.8})(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + (1 - p_{45}p_{54})((1 - p_{31.12}p_{13.10}) \\ & (1 - p_{32.11}p_{23.6}) - p_{32.11}p_{13.10}p_{31.12}p_{23.6})) + (\mu_1' + \mu_3' p_{13.10})(p_{01}(1 - p_{32.11}p_{23.6})(1 - p_{54}p_{45}) \\ & + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54}) + p_{24.7}p_{41.8})) + (\mu_2' + \mu_3' p_{23.6})(p_{01}(p_{32.11}p_{13.10}(1 - p_{45}p_{54})) \\ & + p_{02}((1 - p_{31.12}p_{13.10})(1 - p_{45}p_{54}) - p_{14.9}p_{41.8})) - \mu_4'(-p_{01}(p_{24.7}p_{32.11}p_{13.10} + p_{14.9} \\ & (1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10}) - p_{14.9}p_{31.12}p_{23.6})) \end{aligned}$$

6. Busy period analysis for server

6.1. Due to repair

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit at an instant "t" given that the system entered state i at $t = 0$. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \tag{11}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so $W_2(t) = e^{-(\alpha+k+h)t^{\eta}}, W_3(t) = e^{-(\alpha+\beta+l)t^{\eta}}$

By taking LT of (11) and solving for $B_0^{*R}(s)$, the busy period of the server due to repair is given by $B_0^R = \lim_{s\to 0}\ sB_0^{*R}(s) = \frac{N_3^R}{D_2}$

$$\begin{aligned} N_3^r = {} & (W_3(t)p_{13.10})(p_{01}(1 - p_{32.11}p_{23.6})(1 - p_{54}p_{45}) + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54}) + p_{24.7}p_{41.8})) \\ & +(W_2(t) + W_3(t)p_{23.6})(p_{01}(p_{32.11}p_{13.10}(1 - p_{45}p_{54})) + p_{02}((1 - p_{31.12}p_{13.10})(1 - p_{45}p_{54}) - p_{14.9}p_{41.8})) \\ & -(W_5(t)p_{45})(-p_{01}(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10}) \\ & -p_{14.9}p_{31.12}p_{23.6})) \end{aligned}$$

And D_2 is already mentioned in the previous section.

6.2. Due to PM

Let $B_i^{Pm}(t)$ be the probability that the server is busy in preventive maintenance of the system (unit) at an instant "t" given that the system entered state i at $t = 0$. The recursive relations for $B_i^{Pm}(t)$ are as follows:

$$B_i^{pm}(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^{pm}(t) \tag{12}$$

where j is any successive regenerative state to which the regenerative state i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to PM up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so $W_1(t) = e^{-(\alpha+k+\gamma)t''}, W_4(t) = e^{-(\alpha+\beta+\gamma)t''}$

By taking LT of (12) and solving for $B_0^{*Pm}(s)$, the busy period of the server due to PM is given by

$$B_0^{Pm} = \lim_{s\to 0} sB_0^{*Pm}(s) = \frac{N_4^{Pm}}{D_2} \tag{13}$$

$$\begin{aligned}N_4^{Pm} &= (W_1(0))(p_{01}(1 - p_{32.11}p_{23.6})(1 - p_{54}p_{45}) + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54}) + p_{24.7}p_{41.8}))\\ &-W_4(0)(-p_{01}(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10})\\ &-p_{14.9}p_{31.12}p_{23.6}))\end{aligned}$$

And D_2 is already mentioned in previous section.

7. Expected number of repairs

Let $E_i^R(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state i at $t = 0$. The recursive relations for $E_i^R(t)$ are given as

$$E_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) ® \left[\delta_j + E_j^R(t)\right] \tag{14}$$

where j is any regenerative state to which the given regenerative state i transits and $\delta j = 1$ if j is the regenerative state where the server does the job afresh, otherwise $\delta j = 0$. Taking LST of relations (14) and solving for $\tilde{E}_0^{R}(s)$, the expected number of repairs per unit time is given by

$$E_0^R(\infty) = \lim_{s\to 0} s\tilde{E}_0^{R}(s) = \frac{N_5^R}{D_2} \tag{15}$$

$$\begin{aligned}N_5^R &= ((p_{30} + p_{31.12} + p_{32.11})p_{13.10})(p_{01}(1 - p_{32.11}p_{23.6})(1 - p_{54}p_{45}) + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54})\\ &\quad + p_{24.7}p_{41.8})) + ((p_{20} + p_{23.6} + p_{24.7}) + (p_{30} + p_{31.12} + p_{32.11})p_{23.6})(p_{01}(p_{32.11}p_{13.10}(1 - p_{45}p_{54}))\\ &\quad + p_{02}((1 - p_{31.12}p_{13.10})(1 - p_{45}p_{54}) - p_{14.9}p_{41.8})) - (p_{45}p_{54})(-p_{01}(p_{24.7}p_{32.11}p_{13.10}\\ &\quad + p_{14.9}(1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10}) - p_{14.9}p_{31.12}p_{23.6}))\end{aligned}$$

And D_2 is already mentioned in the previous section.

8. Expected number of PM (PM)

Let $E_i^{Pm}(t)$ be the expected number of PM by the server in (0, t] given that the system entered the regenerative state i at $t = 0$. The recursive relations for $E_i^{Pm}(t)$ are given as

$$E_i^{Pm}(t) = \sum_j Q_{i,j}^{(n)}(t) ® \left[\delta_j + E_j^{Pm}(t)\right] \tag{16}$$

where j is any regenerative state to which the given regenerative state i transits and $\delta j = 1$ if j is the regenerative state where the server does the job afresh, otherwise $\delta j = 0$. Taking LST of relations (16) and solving for $\tilde{E}_0^{Pm}(s)$, the expected number of PM per unit time is given by

$$E_0^{Pm}(\infty) = \lim_{s\to 0} s\tilde{E_0}^{Pm}(s) = \frac{N_6^{Pm}}{D_2} \tag{17}$$

$$N_6^{Pm} = (p_{10} + p_{13.10} + p_{14.9})(p_{01}(1 - p_{32.11}p_{23.6})(1 - p_{54}p_{45}) + p_{02}(p_{31.12}p_{23.6}(1 - p_{45}p_{54}) + p_{24.7}p_{41.8})) - (p_{40} + p_{41.8} + p_{45})(-p_{01}(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + p_{02}(-p_{24.7}(1 - p_{31.12}p_{13.10}) - p_{14.9}p_{31.12}p_{23.6}))$$

And D_2 is already mentioned in the previous section.

9. Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i(t)$ are given as

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t)® \left[\delta_j + N_j(t)\right] \tag{18}$$

where j is any regenerative state to which the given regenerative state i transits and $\delta j = 1$ if j is the regenerative state where the server does the job afresh, otherwise $\delta j = 0$. Taking LST of relation (18) and solving for $\tilde{N}_0(s)$, the expected number of visits per unit time by the server is given by

$$N_0'(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_7}{D_2} \tag{19}$$

$$N_7 = (p_{01} + p_{02})((-p_{41.8})(p_{24.7}p_{32.11}p_{13.10} + p_{14.9}(1 - p_{32.11}p_{23.6})) + (1 - p_{45}p_{54})((1 - p_{31.12}p_{13.10})(1 - p_{32.11}p_{23.6}) - p_{32.11}p_{13.10}p_{31.12}p_{23.6}))$$

And D_2 is already mentioned in the previous section.

10. Profit analysis

The profit incurred by the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^{Pm} - K_2B_0^R - K_3E_0^{Pm} - K_4E_0^R - K_5N_0 \tag{20}$$

K_0 is the revenue per unit up-time of the system; K_1 is the cost per unit time for which the server is busy due PM; K_2 is the cost per unit time for which the server is busy due to repair; K_3 is the cost per unit time for which the server is busy due to expected number of PM; K_4 is the cost per unit time due to expected number of repairs; K_5 is the cost per unit time visit by the server.

10.1. Case studies with discussions

(1) When shape parameter $\eta = 0.5$ then maximum operation/failure of original unit/failure of duplicate unit/PM/repair of original/repair of duplicate unit time distributions reduce to:

$$g(t) = \frac{\alpha}{2\sqrt{t}}e^{-\alpha\sqrt{t}},\ f(t) = \frac{\beta}{2\sqrt{t}}e^{-\beta\sqrt{t}},\ g_1(t) = \frac{\gamma_1}{2\sqrt{t}}e^{-\gamma_1\sqrt{t}},\ f_1(t) = \frac{k}{2\sqrt{t}}e^{-k\sqrt{t}}$$
$$f_3(t) = \frac{l}{2\sqrt{t}}e^{-l\sqrt{t}},\ f_2(t) = \frac{h}{2\sqrt{t}}e^{-h\sqrt{t}};\ \text{where } t \ge 0 \text{ and } \eta,\ \alpha\,\beta,,\gamma_1,\ h, k, l > 0 \tag{21}$$

(2) When shape parameter $\eta = 1.0$, then failure/PM/arrival time of the server/replacement/transition rate/repair time distributions reduce to exponentials, then

$$g(t) = \alpha e^{-\alpha t},\ f(t) = \beta e^{-\beta t},\ g_1(t) = \gamma_1 e^{-\gamma_1 t},\ f_1(t) = ke^{-kt},\ f_3(t) = le^{-lt},\ f_2(t) = he^{-ht};$$
$$\text{where } t \ge 0\ \ \textit{and}\ \ \eta,\ \alpha\,\beta,\ \gamma_1,\ h, k, l > 0 \tag{22}$$

(3) When shape parameter $\eta = 2.0$, then failure/PM/arrival time of the server/replacement/transition rate/repair time distributions reduce to Rayleigh having the pdf-

$$g(t) = 2\alpha e^{-\alpha t^2},\ f(t) = 2\beta e^{-\beta t^2},\ g_1(t) = 2\gamma_1 e^{-\gamma_1 t^2},\ f_1(t) = 2ke^{-kt^2},\ f_3(t) = 2le^{-lt^2},$$
$$f_2(t) = 2he^{-ht^2};\quad \text{where } t \geq 0 \ \textit{and} \ \eta,\ \alpha\, \beta, \gamma_1,\ h, k, l > 0 \tag{23}$$

Table 1. Values of MTSF for different values of α and η with respect to β

β	$\alpha = 2$, $\eta = 0.5$, $\gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 0.5$, $\gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2$, $\eta = 1, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 1, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2$, $\eta = 2, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 2, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$
0.01	4.9918	2.1095	5.9647	3.0460	8.9395	4.9639
0.02	4.7950	2.0642	5.7599	2.9933	8.6486	4.8851
0.03	4.6123	2.0207	5.5696	2.9426	8.3781	4.8092
0.04	4.4423	1.9788	5.3921	2.8937	8.1260	4.7362
0.05	4.2836	1.9385	5.2264	2.8466	7.8904	4.6657
0.06	4.1351	1.8998	5.0712	2.8012	7.6698	4.5978
0.07	3.9960	1.8624	4.9255	2.7573	7.4627	4.5323
0.08	3.8654	1.8264	4.7886	2.7150	7.2680	4.4689
0.09	3.7426	1.7916	4.6596	2.6741	7.0846	4.4078
0.1	3.6268	1.7581	4.5379	2.6346	6.9116	4.3486

Table 2. Values of availability for different values of α and η with respect to β

β	$\alpha = 2$, $\eta = 0.5$, $\gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 0.5$, $\gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2$, $\eta = 1, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 1, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2$, $\eta = 2, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$	$\alpha = 2.4$, $\eta = 2, \gamma = 5$, $h = 0.009$, $k = 1.5$, $l = 1.4$
0.01	0.9405	0.9066	0.8947	0.8627	0.8719	0.8457
0.02	0.9368	0.9026	0.8929	0.8611	0.8710	0.8449
0.03	0.9332	0.8986	0.8912	0.8596	0.8701	0.8442
0.04	0.9296	0.8946	0.8895	0.8580	0.8692	0.8435
0.05	0.9260	0.8907	0.8878	0.8565	0.8683	0.8427
0.06	0.9225	0.8869	0.8861	0.8550	0.8674	0.8420
0.07	0.9189	0.8831	0.8844	0.8535	0.8666	0.8413
0.08	0.9155	0.8793	0.8828	0.8521	0.8657	0.8407
0.09	0.9120	0.8756	0.8812	0.8506	0.8649	0.8400
0.1	0.9086	0.8719	0.8796	0.8492	0.8641	0.8393

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Table 3. Values of profit for different values of α and η with respect to β

β	α = 2, η = 0.5, γ = 5, h = 0.009, k = 1.5, l = 1.4	α = 2.4, η = 0.5, γ = 5, h = 0.009, k = 1.5, l = 1.4	α = 2, η = 1, γ = 5, h = 0.009, k = 1.5, l = 1.4	α = 2.4, η = 1, γ = 5, h = 0.009, k = 1.5, l = 1.4	α = 2, η = 2, γ = 5, h = 0.009, k = 1.5, l = 1.4	α = 2.4, η = 2, γ = 5, h = 0.009, k = 1.5, l = 1.4
0.01	4362.9	4064.9	4181.3	3982.3	4136.5	3993.4
0.02	4341.7	4042.3	4171.3	3973.4	4131.2	3989.1
0.03	4320.7	4020.0	4161.3	3964.6	4126.0	3984.9
0.04	4299.8	3998.0	4151.6	3956.0	4120.9	3980.7
0.05	4279.2	3976.2	4141.9	3947.4	4115.8	3976.6
0.06	4258.7	3954.7	4132.4	3939.0	4110.9	3972.6
0.07	4238.3	3933.4	4123.0	3930.7	4106.0	3968.6
0.08	4218.2	3912.4	4113.7	3922.5	4101.2	3964.7
0.09	4198.2	3891.6	4104.6	3914.4	4096.5	3960.9
0.1	4178.4	3871.1	4095.6	3906.4	4091.9	3957.1

11. Conclusion

For a particular case, having values $\alpha = 2, \gamma = 5, h = 0.009, k = 1.5, l = 1.4$ the behaviour of various reliability measures such as MTSF, availability and net expected steady-state profit of the system discussed here for a two-unit cold standby system under priority and Weibull distribution. The values of K_i for profit function are assumed as $K_1 = 200$, $K_0 = 5000$, $K_2 = 150$, $K_3 = 100$, $K_4 = 75$, $K_5 = 80$. From the numerical results depicted in Tables 1–3 show that the MTSF, availability and profit of the system decline with the increase of failure rate (β). While with respect to shape parameter (η) the value of MTSF increases, the availability and profit decrease. With the increment of the repair rate and PM rate, availability and profit show increasing behaviour. Finally, we conclude that by increasing the repair rate of the original and duplicate units, the system can be made more profitable.

Nomenclature

O	operative unit
DCs	duplicate cold standby unit
Do	duplicate unit is operative
~/*	symbol for Laplace–Steiltjes transform (LST)/Laplace transform (LT)
Ⓢ/©	symbol for Laplace–Stieltjes convolution/Laplace convolution
Fur/FUR	failed original unit under repair/continuously under repair
DFur/DFUR	failed duplicate unit under repair/continuously under repair
DPm/DPM	duplicate unit under preventive maintenance/continuously under preventive maintenance
Pm/PM	original unit under preventive maintenance/continuously under preventive maintenance
WPm/WPM	original unit waiting for preventive maintenance/continuously waiting for preventive maintenance
DWPm/DWPM	duplicate unit waiting for preventive maintenance/continuously waiting for preventive maintenance
Fwr/FWR	original unit after failure waiting for repair/continuously waiting for repair
DFwr/DFWR	duplicate unit after failure waiting for repair/continuously waiting for repair
MTSF	mean time to system failure

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Author details
Ashish Kumar[1]
E-mail: ashishbarak2020@gmail.com
Monika Saini[1]
E-mail: drmnksaini4@gmail.com
Kuntal Devi[1]
E-mail: akbrk@rediffmail.com
[1] Department of Mathematics, Manipal University Jaipur, Jaipur 303007, Rajasthan, India.

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