Ailawalia et al., *Cogent Mathematics* (2015), 2: 1086293
http://dx.doi.org/10.1080/23311835.2015.1086293

cogent mathematics

APPLIED & INTERDISCIPLINARY MATHEMATICS | RESEARCH ARTICLE

Two dimensional deformation in microstretch thermoelastic half space with microtemperatures and internal heat source

Praveen Ailawalia[1]*, Sunil Kumar Sachdeva[2,3] and Devinder Pathania[4]

Received: 17 April 2015
Accepted: 19 August 2015
Published: 14 September 2015

*Corresponding author: Praveen Ailawalia, Department of Applied Sciences and Humanities, M.M. University, Sadopur, Ambala City, Haryana, India
E-mail: praveen_2117@rediffmail.com

Reviewing editor:
Timothy Marchant, University of Wollongong, Australia

Additional information is available at the end of the article

Abstract: The purpose of this paper is to study the two dimensional deformation due to internal heat source in a microstretch thermoelastic solid with microtemperatures (MTSM). A mechanical force is applied along the interface of fluid half space and microstretch thermoelastic half space. The normal mode analysis has been applied to obtain the exact expressions for component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress for MTSM. The effect of internal heat source, micropolarity, and microstretch on the above components has been depicted graphically.

Subjects: Applied Mathematics; Mathematics & Statistics; Science

Keywords: thermoelasticity; microstretch; microtemperature; heat source; fluid half space

1. Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronautics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. Eringen and Suhubi (1964a, 1964b) developed a

Praveen Ailawalia

ABOUT THE AUTHOR

The author, Praveen Ailawalia working as a professor at M M University, Sadopur, Ambala, Haryana (India) is actively involved in the field of thermoelasticity, micropolar elasticity. He has 17 years of teaching experience in different Universities and institutions. The author has more than 70 research publications in international journals of repute. He has guided four PhD students and four students are currently working with him. The author has discussed deformation in thermoelastic medium and micropolar elastic medium in many of his research papers. The research problem discussed in the paper helps in analyzing the behavior of a medium with temperature changes if the medium undergoes deformation due to an internal heat source. The results obtained in the paper may be applied to various geological problems which involves sources acting in the medium. The problem can be further discussed in case of mechanical sources applied on the free surface of the medium or along the interface of two different mediums.

PUBLIC INTEREST STATEMENT

Studying the two dimensional deformation due to internal heat source in a microstretch thermoelastic solid with microtemperatures is very useful in the study of earthquake engineering, seismology, and volcanic eruptions. It helps us to study the effect of a heat source in the medium and the deformation caused in the medium due to the heat source.

nonlinear theory of microelastic solids. Later Eringen (1965, 1966a, 1996b) developed a theory for the special class of microelastic materials and called it the “linear theory of micropolar elasticity”. Under this theory, solids can undergo macro-deformations and microrotations. Eringen (1990) developed a theory of thermo-microstretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level and a two-phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibers, porous media whose pores are filled with gas or inviscid liquid, other elastic inclusions and “solid-liquid” crystals, etc., should be characterizable by microstretch solids. Eringen (1968) developed a theory of microstretch elastic solid in which he included microstructural expansions and contractions, Singh and Kumar (1998) studied wave propagation in a generalized thermo-microstretch elastic solid, Kumar and Rupender (2008) studied the reflection at free surface of magneto-thermo-microstretch elastic solid, Tomar and Khurana (2009) discussed reflection and transmission of elastic waves from a plane interface between two thermo-microstretch solid half-spaces. Marin (2010) discussed Lagrange identity method for microstretch thermoelastic materials, Othman and Lotfy (2010) studied the plane waves of generalized thermo-microstretch elastic half space under three theories, Kumar and Partap (2009) presented the analysis of free vibrations for Rayleigh–Lamb waves in a microstretch thermoelastic plate with two relaxation times, Othman, Lotfy, and Farouk (2010) studied generalized thermo-microstretch elastic medium with temperature-dependent properties for different theories, Kumar and Kansal (2011) studied fundamental solution in the theory of thermo-microstretch elastic diffusive solids, Othman and Lotfy (2011) studied the effect of rotation on plane waves in generalized thermo-microstretch elastic solid with one relaxation time, Kumar, Sharma, and Sharma (2011) discussed the generalized thermoelastic waves in microstretch plates loaded with fluid of varying temperature. Abbas and Othman (2012) studied the plane waves in generalized thermo-microstretch elastic solid with thermal relaxation using finite element method, Kumar and Rupender (2009) discussed the propagation of plane waves at imperfect boundary of elastic and electro-microstretch generalized thermoelastic solids.

Grot (1969) discussed a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. Říha (1976) studied heat conduction in materials with microtemperatures. Iesan and Quintanilla (2000) studied a theory of thermoelasticity with microtemperatures. Iesan (2001) proposed the theory of micromorphic elastic solids with microtemperatures. Exponential stability in thermoelasticity with microtemperatures was studied by Casas and Quintanilla (2005). Scalia and Svanadze (2006) gave the solutions of the theory of thermoelasticity with microtemperatures. Magaña and Quintanilla (2006) discussed the time decay of solutions in one-dimensional theories of porous materials. Aouadi (2008) discussed some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. Ieşan and Quintanilla (2009) discussed thermoelastic bodies with inner structure and microtemperatures. Scalia, Svanadze, and Tracinà (2010) studied basic theorems in the equilibrium theory of thermoelasticity with microtemperatures. Quintanilla (2011) discussed the growth and continuous dependence in thermoelasticity with microtemperatures. Steeb, Singh, and Tomar (2013) studied time harmonic waves in thermoelastic material with microtemperatures. Chiriţă, Ciarletta, and D’Apice (2013) studied the theory of thermoelasticity with microtemperatures. Singh, Kumar, and Kumar (2014) discussed a problem in microstretch thermoelastic diffusive medium. Kumar and Kaur (2014) studied the reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures (MTSM).

In the present problem, the authors have discussed deformation due to internal heat source and a mechanical force which is applied along the interface of fluid half space and microstretch thermoelastic half space with microtemperatures. The normal mode analysis has been applied to obtain the exact expressions for component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress for MTSM. The effect of internal heat source, micropolarity, and microstretch on the above components has been depicted graphically.

The behavior of a thermo-microstretch isotropic material with microtemperatures without body forces, body couples, stretch force, heat sources, and first heat source moment is governed by the following equations given by Eringen (1990) and Ieşan (2007) as,

$$t_{ji,j} = \rho \ddot{u}_i \tag{1}$$

$$m_{ij,i} + \varepsilon_{ijk} t_{jk} - \mu_1 \varepsilon_{ijr} w_{r,j} = \rho J \ddot{\varphi}_j \tag{2}$$

$$h_{i,i} - s = \frac{\rho j_0}{2} \ddot{\phi}^* \tag{3}$$

The constitutive relations are,

$$t_{ij} = \lambda u_{r,r} \delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr} \varphi_r) - \nu T \delta_{ij} + \lambda_0 \phi^* \delta_{ij} \tag{4}$$

$$m_{ij} = \alpha \varphi_{r,r} \delta_{ij} + \beta \varphi_{i,j} + \gamma \varphi_{j,i} + b_0 \varepsilon_{mji} \phi^*_{,m} \tag{5}$$

$$\lambda^*_i = \alpha_0 \phi^*_{,i} + b_0 \varepsilon_{ijm} \varphi_{j,m} \tag{6}$$

$$q_{ij} = -k_4 w_{r,r} \delta_{ij} - k_5 w_{i,j} - k_6 w_{j,i} \tag{7}$$

$$h_i = \alpha_0 \phi^*_{,i} - \mu_2 w_i \tag{8}$$

$$s = \lambda_0 e_{rr} - \nu_1 T + \lambda_1 \varphi^*; \quad i, j, m = 1, 2, 3 \tag{9}$$

using Equations 4–9 in Equations 1–3, we get the equations,

$$(\mu + K) u_{i,ii} + (\lambda + \mu) u_{i,ij} - K \varepsilon_{ijr} \varphi_r + \lambda_0 \phi^*_{,i} - \nu T_{,i} = \rho \ddot{u}_i \tag{10}$$

$$\gamma \varphi_{i,ii} + K \varepsilon_{ijr} u_r - 2K \varphi_i - \mu_1 \varepsilon_{ijr} w_r = \rho J \ddot{\varphi}_i \tag{11}$$

$$\alpha_0 \phi^*_{,ii} + \nu_1 T - \lambda_1 \phi^* - \lambda_0 u_{i,i} - \mu_2 w_{i,i} = \rho \frac{j_0}{2} \ddot{\phi}^* \tag{12}$$

$$K^* T_{,ii} - \rho c^* \dot{T} - \nu_1 T_0 \dot{\phi}^* - \nu T_0 u_{i,i} + k_1 w_{i,i} = Q_1 \tag{13}$$

$$k_6 w_{i,ii} + (k_4 + k_5) w_{i,ij} + \mu_1 \varepsilon_{ijr} \dot{\varphi}_r - \mu_2 \dot{\phi}^*_{,i} - b \dot{w}_i - k_2 w_i - k_3 T_{,i} = 0 \tag{14}$$

where

$\nu = (3\lambda + 2\mu + K)\alpha_{t_1}$, $\nu_1 = (3\lambda + 2\mu + K)\alpha_{t_2}$, α_{t_1}, α_{t_2} are the coefficents of linear thermal expansion, λ and μ are Lame's constants, K, α, β, γ are the micropolar constants of the solid, $\alpha_0, \lambda_0, \lambda_1$ are the stretch constants, and $j_0, \mu_1, \mu_2, k_1, k_2, k_3, k_4, k_5, k_6$ are the constitutive coefficients. t_{ij} is the component of stress tensor, m_{ij} is the coupled stress tensor, λ^*_i is the microstress tensor, q_{ij} is the first heat flux moment tensor, $\vec{u} = (u_i)$ is the displacement vector, $\vec{\varphi} = (\varphi_i)$ is the microrotation vector, $\vec{w} = (w_i)$ is the microtemperature vector and ϕ^* is the scalar microstretch, ρ is the density, J is the microinertia, c^* is the specific heat at constant strain, Q_1 is the internal heat source, K^* is the thermal conductivity, and T is the thermodynamic temperature above reference temperature T_0.

The equations of motion and stress components in fluid (Ewing, Jardetzky, & Press, 1957) are:

$$\lambda^f u^f_{i,ij} = \rho^f \ddot{u}^f_i, \tag{15}$$

$$t_{ij}^f = \lambda^f u_{r,r}^f \delta_{ij} \tag{16}$$

where $\vec{u}^f = (u_i^f)$ is the displacement vector, λ^f is the fluid constant, and ρ^f is the density of fluid.

We consider a normal force of magnitude F_1 acting along the interface of microstretch thermoelastic medium with microtemperatures (medium I) occupying the region $0 \le z \le \infty$ and a non-viscous fluid (medium II) in the region $-\infty \le z \le 0$ is shown in Figure 1.

A homogeneous isotropic, microstretch thermoelastic solid half space with microtemperatures is considered. We have restricted our analysis to the plane strain parallel to xz plane with displacement vector $u_i = (u_1, 0, u_3)$, microtemperature vector $w_i = (w_1, 0, w_3)$, and microrotation vector $\varphi_i = (0, \varphi_2, 0)$.

For convenience, the following non-dimensional variables are used:

$$x' = \tfrac{1}{L}x,\ z' = \tfrac{1}{L}z,\ u_i' = \tfrac{1}{L}u_i,\ u_i^{f'} = \tfrac{1}{L}u_i^f,\ w_i' = Lw_i,\ t' = \tfrac{c_1}{L}t,\ t_{ij}' = \tfrac{t_{ij}}{\nu T_0},\ t_{ij}^{f'} = \tfrac{t_{ij}^f}{\nu T_0},\ \varphi_i' = \varphi_i,\ \phi^{*\prime} = \phi^*,$$
$$m_{ij}' = \tfrac{m_{ij}}{L\nu T_0}, q_{ij}' = \tfrac{q_{ij}}{Lc_1\nu T_0}, \lambda_i^{*'} = \tfrac{\lambda_i^*}{L\nu T_0}, T' = \tfrac{T}{T_0}, F_1' = \tfrac{F_1}{\nu T_0}, Q_1' = \tfrac{Q_1}{Q_0}.$$

where $L = \left(\frac{b}{\rho c^* T_0}\right)^{\frac{1}{2}}, c_1^2 = \frac{\lambda+2\mu+K}{\rho}$.

Assuming the scalar potential functions $\psi_1(x, z, t)$, $\psi_2(x, z, t)$, $\psi_3(x, z, t)$, and $\psi_4(x, z, t)$ defined by the relation in non-dimensional form as,

$$u_1 = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z};\quad u_3 = \frac{\partial \psi_1}{\partial z} + \frac{\partial \psi_2}{\partial x};\quad w_1 = \frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_4}{\partial z};\quad w_3 = \frac{\partial \psi_3}{\partial z} + \frac{\partial \psi_4}{\partial x}. \tag{17}$$

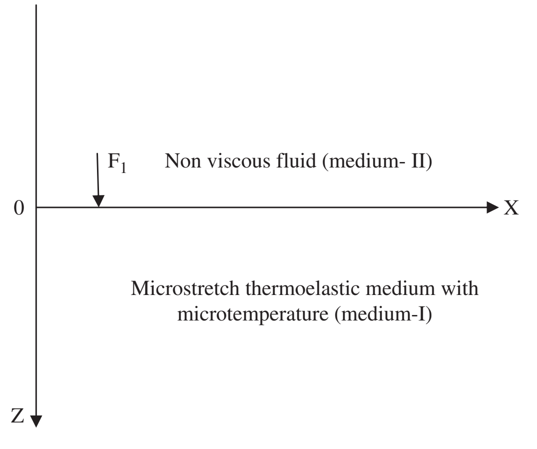
using above non-dimensional variables and relation given by Equation 17, Equations 10–14 reduce to (after dropping superscripts),

$$\left\{ (A_1 + 1)\nabla^2 - A_2 \frac{\partial^2}{\partial t^2} \right\} \psi_1 + A_3 \phi^* - A_4 T = 0 \tag{18}$$

$$\left(\nabla^2 - A_2 \frac{\partial^2}{\partial t^2} \right) \psi_2 + A_5 \varphi_2 = 0 \tag{19}$$

$$\left(\nabla^2 - 2A_6 - A_7 \frac{\partial^2}{\partial t^2} \right) \varphi_2 - A_6 \nabla^2 \psi_2 + A_8 \nabla^2 \psi_4 = 0 \tag{20}$$

Figure 1. Geometry of the problem.



$$\left(\nabla^2 - A_9 - A_{10}\frac{\partial^2}{\partial t^2}\right)\phi^* - A_{11}\nabla^2\psi_1 - A_{12}\nabla^2\psi_3 + A_{13}T = 0 \tag{21}$$

$$\left(\nabla^2 - A_{14}\frac{\partial}{\partial t}\right)T - A_{15}\frac{\partial\phi^*}{\partial t} - A_{16}\nabla^2\psi_1 + A_{17}\nabla^2\psi_3 = YQ_1 \tag{22}$$

$$\left(\nabla^2(1 + A_{18}) - A_{19} - A_{20}\frac{\partial}{\partial t}\right)\psi_3 - A_{21}\frac{\partial\phi^*}{\partial t} - A_{22}T = 0 \tag{23}$$

$$\left(\nabla^2 - A_{19} - A_{20}\frac{\partial}{\partial t}\right)\psi_4 + A_{23}\frac{\partial\varphi_2}{\partial t} = 0 \tag{24}$$

where

$A_1 = \frac{\lambda+\mu}{\mu+K}$, $A_2 = \frac{\rho c_1^2}{\mu+K}$, $A_3 = \frac{\lambda_0}{\mu+K}$, $A_4 = \frac{\nu T_0}{\mu+K}$, $A_5 = \frac{K}{\mu+K}$, $A_6 = \frac{KL^2}{\gamma}$, $A_7 = \frac{\rho J c_1^2}{\gamma}$, $A_8 = \frac{\mu_1}{\gamma}$, $A_9 = \frac{\lambda_1 L^2}{\alpha_0}$, $A_{10} = \frac{\rho j_0 c_1^2}{2\alpha_0}$, $A_{11} = \frac{\lambda_0 L^2}{\alpha_0}$, $A_{12} = \frac{\mu_2}{\alpha_0}$, $A_{13} = \frac{\nu_1 T_0 L^2}{\alpha_0}$, $A_{14} = \frac{\rho c^* c_1 L}{K^*}$, $A_{15} = \frac{\nu_1 c_1 L}{K^*}$, $A_{16} = \frac{\nu c_1 L}{K^*}$, $A_{17} = \frac{k_1}{K^* T_0}$, $A_{18} = \frac{k_4+k_5}{k_6}$, $A_{19} = \frac{k_2 L^2}{k_6}$, $A_{20} = \frac{bc_1 L}{k_6}$, $A_{21} = \frac{\mu_2 c_1 L}{k_6}$, $A_{22} = \frac{k_3 T_0 L^2}{k_6}$, $A_{23} = \frac{\mu_1 c_1 L}{k_6}$, $Y = \frac{L^2}{K^*}Q_0$.

2. Analytic solution

The solution of the considered physical variable can be decomposed in terms of normal mode and can be considered in the following form,

$$(\psi_i, \phi^*, T, \varphi_2, t_{ij}, q_{ij}, u_i^f, t_{ij}^f, m_{ij}, \lambda_i^*, Q_1)(x, z, t) = (\bar{\psi}_i, \bar{\phi}^*, \bar{T}, \bar{\varphi}_2, \bar{t}_{ij}, \bar{q}_{ij}, \bar{u}_i^f, \bar{t}_{ij}^f, \bar{m}_{ij}, \bar{\lambda}_i^*, \bar{Q}_1)(z)e^{\omega t + iax}$$

where ω is the complex frequency, a is the wave number in x-direction, and $\bar{\psi}_i(z), \bar{\phi}^*(z), \bar{T}(z), \bar{\varphi}_2(z), \bar{\sigma}_{ij}(z), \bar{q}_{ij}(z), \bar{u}_i^f, \bar{t}_{ij}^f, \bar{m}_{ij}(z), \bar{\lambda}_i^*(z), \bar{Q}_1(z)$ are the amplitudes of field quantities.

Using normal mode in Equations 18–24, we get,

$$(D^2 - B_8)\bar{\psi}_1 + B_2\bar{\phi}^* - B_3\bar{T} = 0 \tag{25}$$

$$(D^2 - B_9)\bar{\psi}_2 + A_5\bar{\varphi}_2 = 0 \tag{26}$$

$$(D^2 - B_{10})\bar{\varphi}_2 - A_6(D^2 - a^2)\bar{\psi}_2 + A_8(D^2 - a^2)\bar{\psi}_4 = 0 \tag{27}$$

$$(D^2 - B_{11})\bar{\phi}^* - A_{11}(D^2 - a^2)\bar{\psi}_1 - A_{12}(D^2 - a^2)\bar{\psi}_3 + A_{13}\bar{T} = 0 \tag{28}$$

$$(D^2 - B_{12})\bar{T} - A_{15}\omega\bar{\phi}^* - A_{16}(D^2 - a^2)\bar{\psi}_1 + A_{17}(D^2 - a^2)\bar{\psi}_3 = Y\bar{Q}_1 \tag{29}$$

$$(D^2 - B_{13})\bar{\psi}_3 - B_6\bar{\phi}^* - B_7\bar{T} = 0 \tag{30}$$

$$(D^2 - B_{14})\bar{\psi}_4 + A_{23}\omega\bar{\varphi}_2 = 0 \tag{31}$$

where

$B_1 = \frac{A_2}{A_1+1}$, $B_2 = \frac{A_3}{A_1+1}$, $B_3 = \frac{A_4}{A_1+1}$, $B_4 = \frac{A_{19}}{A_{18}+1}$, $B_5 = \frac{A_{20}}{A_{18}+1}$, $B_6 = \frac{A_{21}\omega}{A_{18}+1}$, $B_7 = \frac{A_{22}}{A_{18}+1}$, $B_8 = a^2 + B_1\omega^2$, $B_9 = a^2 + A_2\omega^2$, $B_{10} = a^2 + 2A_6 + A_7\omega^2$, $B_{11} = a^2 + A_9 + A_{10}\omega^2$, $B_{12} = a^2 + A_{14}\omega$, $B_{13} = a^2 + B_4 + B_5\omega$, $B_{14} = a^2 + A_{19} + A_{20}\omega$.

and constitutive relations (4–7) become,

$$\bar{t}_{xx} = \left(A_{25}D^2 - a^2 A_{24}\right)\bar{\psi}_1 + ia\left(A_{25} - A_{24}\right)D\bar{\psi}_2 - \bar{T} + A_{26}\bar{\phi}^* \tag{32}$$

$$\bar{t}_{xz} = ia\left(A_{27} + A_{28}\right)D\bar{\psi}_1 - \left(A_{27}D^2 + a^2A_{28}\right)\bar{\psi}_2 - K\bar{\varphi}_2 \tag{33}$$

$$\bar{t}_{zz} = \left(A_{24}D^2 - a^2A_{25}\right)\bar{\psi}_1 + ia\left(A_{24} - A_{25}\right)D\bar{\psi}_2 - \bar{T} + A_{26}\bar{\phi}^* \tag{34}$$

$$\bar{q}_{xx} = \left(A_{29}a^2 - A_{30}D^2\right)\bar{\psi}_3 + ia\left(A_{29} - A_{30}\right)D\bar{\psi}_4 \tag{35}$$

$$\bar{q}_{xz} = -ia\left(A_{31} + A_{32}\right)D\bar{\psi}_3 + \left(A_{31}D^2 + a^2A_{32}\right)\bar{\psi}_4 \tag{36}$$

$$\bar{q}_{zz} = \left(A_{30}a^2 - A_{29}D^2\right)\bar{\psi}_3 + ia\left(A_{30} - A_{29}\right)D\bar{\psi}_4 \tag{37}$$

$$\bar{m}_{yz} = A_{33}D\bar{\phi}_2 - iaA_{34}\bar{\phi}^* \tag{38}$$

$$\bar{\lambda}_3^* = A_{35}D\bar{\phi}^* - iaA_{34}\bar{\varphi}_2 \tag{39}$$

where

$A_{24} = \frac{\lambda+2\mu+K}{\nu T_0}$, $A_{25} = \frac{\lambda}{\nu T_0}$, $A_{26} = \frac{\lambda_0}{\nu T_0}$, $A_{27} = \frac{\mu}{\nu T_0}$, $A_{28} = \frac{\mu+K}{\nu T_0}$, $A_{29} = \frac{k_4+k_5+k_6}{L^3 c_1 \nu T_0}$, $A_{30} = \frac{k_4}{L^3 c_1 \nu T_0}$, $A_{31} = \frac{k_5}{L^3 c_1 \nu T_0}$, $A_{32} = \frac{k_6}{L^3 c_1 \nu T_0}$, $A_{33} = \frac{\beta}{L^2 \nu T_0}$, $A_{34} = \frac{b_0}{L^2 \nu T_0}$, $A_{35} = \frac{\alpha_0}{L^2 \nu T_0}$.

Eliminating $\bar{\phi}^*(z)$, $\bar{\psi}_3(z)$, $\bar{T}(z)$ from Equations 25, 28–30, we get the following eight-order differential equation for $\bar{\psi}_1(z)$ as,

$$(D^8 + PD^6 + QD^4 + RD^2 + S)\bar{\psi}_1(z) = B_{15}\bar{Q}_1 \tag{40}$$

Eliminating $\bar{\psi}_4(z)$ and $\bar{\varphi}_2(z)$ from Equations 26–27 and 31, we get the following sixth-order differential equation for $\bar{\psi}_2(z)$ as,

$$(D^6 + ED^4 + FD^2 + G)\bar{\psi}_2(z) = 0 \tag{41}$$

where

$$P = \left[-(B_{12} + B_{13}) + B_7A_{17} - B_8 - B_{11} - A_{12}B_6 + B_2A_{11} - B_3A_{16}\right]$$

$$\begin{aligned} Q = \Big[& B_{12}B_{13} - B_7A_{17}a^2 + (B_8 + B_{11})(B_{12} + B_{13}) - B_7A_{17}(B_8 + B_{11}) + B_8B_{11} + A_{13}A_{15}\omega - A_{13}A_{17}B_6 \\ & - A_{12}A_{15}B_7\omega + A_{12}B_{12}B_6 + A_{12}(a^2 + B_8)B_6 + B_2B_7(A_{11}A_{17} + A_{12}A_{16}) \\ & - B_2A_{11}(B_{12} + B_{13}) - B_2A_{13}A_{16} - a^2B_2A_{11} + (A_{11}A_{17} + A_{12}A_{16})B_3B_6 - B_3A_{11}A_{15}\omega + B_3B_{11} \\ & + A_{16}(a^2 + B_{13})B_3\Big] \end{aligned}$$

$$\begin{aligned} R = \Big[& -B_{12}B_{13}(B_8 + B_{11}) - B_7A_{17}a^2(B_8 + B_{11}) - B_8B_{11}(B_{12} + B_{13}) + B_7A_{17}B_8B_{11} - A_{13}A_{15}B_{13}\omega \\ & + a^2A_{13}A_{17}B_6 - B_8A_{13}A_{15}\omega + B_8B_6A_{13}A_{17} + A_{12}(a^2 + B_8)A_{15}B_7\omega - A_{12}(a^2 + B_8)B_6B_{12} \\ & - A_{12}B_8B_6a^2 - 2a^2B_2B_7(A_{11}A_{17} + A_{12}A_{16}) + B_2A_{11}B_{12}B_{13} + B_2A_{13}A_{16}B_{13} + a^2B_2A_{11}(B_{12} + B_{13}) \\ & + a^2A_{13}A_{16}B_2 - 2a^2B_3B_6(A_{11}A_{17} + A_{12}A_{16}) + B_3(B_{13} + a^2)A_{11}A_{15}\omega - B_3(B_{13} + a^2)A_{16}B_{11} \\ & - a^2B_3A_{16}B_{13}\Big] \end{aligned}$$

$$S = \Big[B_{12}B_{13}B_8B_{11} - B_7A_{17}a^2B_8B_{11} + B_8B_{13}A_{13}A_{15}\omega - B_6B_8A_{13}A_{17}a^2 - B_7B_8A_{12}A_{15}a^2\omega + B_6B_8B_{12}A_{12}a^2 + a^4B_2B_7(A_{11}A_{17} + A_{12}A_{16}) - a^2B_2A_{11}B_{12}B_{13} - a^2B_2A_{13}A_{16}B_{13} + a^4B_3B_6(A_{11}A_{17} + A_{12}A_{16}) - B_3B_{13}A_{11}A_{15}a^2\omega + B_3B_{13}B_{11}A_{16}a^2\Big]$$

$$E = \left[-(B_{10} + B_{14}) - A_8A_{23}\omega - B_9 + A_5A_6\right]$$

$$F = \left[B_{10}B_{14} + A_8A_{23}\omega a^2 + B_9(B_{10} + B_{14}) + B_9A_8A_{23}\omega - A_5A_6(B_{14} + a^2)\right]$$

$$G = \left[-B_9B_{10}B_{14} - B_9A_8A_{23}\omega a^2 + a^2A_5A_6B_{14}\right]$$

$$B_{15} = YA_{12}B_3\left[-A_3B_{13}B_2 + B_7A_{12}B_2a^2 - a^2A_{12}B_6B_3 - B_3B_{13}B_{11}\right]$$

In a similar manner, we can show that $\bar{\phi}^*(z)$, $\bar{\psi}_3(z)$, and $\bar{T}(z)$ satisfy the equation,

$$(D^8 + PD^6 + QD^4 + RD^2 + S)(\bar{\phi}^*(z), \bar{\psi}_3(z), \bar{T}(z)) = B_{15}\bar{Q}_1 \tag{42}$$

which can be factorized as follows,

$$(D^2 - r_1^2)(D^2 - r_2^2)(D^2 - r_3^2)(D^2 - r_4^2)\bar{\psi}_1(z) = B_{15}\bar{Q}_1 \tag{43}$$

where r_n^2; (n = 1, 2, 3, 4) are the roots of Equation 42.

and $\bar{\psi}_4(z)$ and $\bar{\varphi}_2(z)$ satisfy the equation,

$$(D^6 + ED^4 + FD^2 + G)(\bar{\psi}_4(z), \bar{\varphi}_2(z)) = 0 \tag{44}$$

which can be factorized as follows,

$$(D^2 - h_1^2)(D^2 - h_2^2)(D^2 - h_3^2)\bar{\psi}_2(z) = 0 \tag{45}$$

where h_n^2; (n = 1, 2, 3) are the roots of Equation 44.

The series solution of Equation 42 has the form,

$$\bar{\psi}_1(z) = \sum_{n=1}^{4}[M_n(a,\omega)e^{-r_nz}] + N \tag{46}$$

$$\bar{\phi}^*(z) = \sum_{n=1}^{4}[M'_n(a,\omega)e^{-r_nz}] + N_1 \tag{47}$$

$$\bar{T}(z) = \sum_{n=1}^{4}[M''_n(a,\omega)e^{-r_nz}] + N_2 \tag{48}$$

$$\bar{\psi}_3(z) = \sum_{n=1}^{4}[M'''_n(a,\omega)e^{-r_nz}] + N_3 \tag{49}$$

The series solution of Equation 44 has the form,

$$\bar{\psi}_2(z) = \sum_{n=1}^{3} [L_n(a,\omega)e^{-h_n z}] \tag{50}$$

$$\bar{\varphi}_2(z) = \sum_{n=1}^{3} [L'_n(a,\omega)e^{-h_n z}] \tag{51}$$

$$\bar{\psi}_4(z) = \sum_{n=1}^{3} [L''_n(a,\omega)e^{-h_n z}] \tag{52}$$

where $M_n(a,\omega)$, $M'_n(a,\omega)$, $M''_n(a,\omega)$, $M'''_n(a,\omega)$, and $L_n(a,\omega)$, $L'_n(a,\omega)$, $L''_n(a,\omega)$ are the specific function depending upon a and ω.

Using Equations 46–49 in Equations 25, 28–30, we get the following relations,

$$M'_n(a,\omega) = H_{1n}M_n(a,\omega) \tag{53}$$

$$M''_n(a,\omega) = H_{2n}M_n(a,\omega) \tag{54}$$

$$M'''_n(a,\omega) = H_{3n}M_n(a,\omega) \tag{55}$$

similarly, using Equations 50–52 in Equations 26–27 and 31, we get the following relations,

$$L'_n(a,\omega) = R_{1n}L_n(a,\omega) \tag{56}$$

$$L''_n(a,\omega) = R_{2n}L_n(a,\omega) \tag{57}$$

Thus we have,

$$\bar{\phi}^*(z) = \sum_{n=1}^{4} [H_{1n}M_n(a,\omega)e^{-r_n z}] + N_1 \tag{58}$$

$$\bar{T}(z) = \sum_{n=1}^{4} [H_{2n}M_n(a,\omega)e^{-r_n z}] + N_2 \tag{59}$$

$$\bar{\psi}_3(z) = \sum_{n=1}^{4} [H_{3n}M_n(a,\omega)e^{-r_n z}] + N_3 \tag{60}$$

$$\bar{\varphi}_2(z) = \sum_{n=1}^{3} [R_{1n}L_n(a,\omega)e^{-h_n z}] \tag{61}$$

$$\bar{\psi}_4(z) = \sum_{n=1}^{3} [R_{2n}L_n(a,\omega)e^{-h_n z}] \tag{62}$$

$$\bar{t}_{xx}(z) = \sum_{n=1}^{4} [H_{4n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3} [R_{3n}L_n(a,\omega)e^{-h_n z}] - N_4 \tag{63}$$

$$\bar{t}_{xz}(z) = \sum_{n=1}^{4}[H_{5n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{4n}L_n(a,\omega)e^{-h_n z}] \tag{64}$$

$$\bar{t}_{zz}(z) = \sum_{n=1}^{4}[H_{6n}M_n(a,\omega)e^{-r_n z}] - \sum_{n=1}^{3}[R_{3n}L_n(a,\omega)e^{-h_n z}] - N_5 \tag{65}$$

$$\bar{q}_{xx}(z) = \sum_{n=1}^{4}[H_{7n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{5n}L_n(a,\omega)e^{-h_n z}] + N_6 \tag{66}$$

$$\bar{q}_{xz}(z) = \sum_{n=1}^{4}[H_{8n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{6n}L_n(a,\omega)e^{-h_n z}] \tag{67}$$

$$\bar{q}_{zz}(z) = \sum_{n=1}^{4}[H_{9n}M_n(a,\omega)e^{-r_n z}] - \sum_{n=1}^{3}[R_{5n}L_n(a,\omega)e^{-h_n z}] + N_7 \tag{68}$$

$$\bar{m}_{yz}(z) = \sum_{n=1}^{4}[H_{10n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{7n}L_n(a,\omega)e^{-h_n z}] + N_8 \tag{69}$$

$$\bar{\lambda}_3^*(z) = \sum_{n=1}^{4}[H_{11n}M_n(a,\omega)e^{-r_n z}] + \sum_{n=1}^{3}[R_{8n}L_n(a,\omega)e^{-h_n z}] \tag{70}$$

where

$$N = \frac{B_{15}}{S}\bar{Q}, N_1 = \frac{A_{12}YB_3\bar{Q}_1 - [(A_{12}B_{12} - A_{13}A_{17})B_8 + a^2B_3(A_{11}A_{17} + A_{12}A_{16})]N}{B_2(A_{17}A_{13} - A_{12}B_{12}) - B_3(A_{17}B_{11} + A_{12}A_{15}\omega)},$$

$$N_2 = \frac{-B_8N + B_2N_1}{B_3}, N_3 = \frac{-(B_6N_1 + B_7N_2)}{B_{13}}, N_4 = (A_{24}a^2N + N_2 - A_{26}N_1),$$

$$N_5 = (A_{25}a^2N + N_2 - A_{26}N_1), N_6 = (A_{24}a^2N_3), N_7 = (A_{30}a^2N_3), N_8 = (-iaA_{34}N_1),$$

$$H_{1n} = -\frac{[A_{12}r_n^4 - B_{16}r_n^2 + B_{17}]}{[(A_{12}B_2 + A_{17}B_3)r_n^2 - A_{12}B_{12}B_2 + A_{13}A_{17}B_2 - A_{17}B_3B_{11} - A_{12}A_{15}B_3]},$$

$$H_{2n} = \frac{(r_n^2 - B_8 + B_2H_{1n})}{B_3},\ H_{3n} = \frac{(B_6H_{1n} + B_7H_{2n})}{(r_n^2 - B_{13})},\ H_{4n} = (A_{25}r_n^2 - a^2A_{24}) - H_{2n} + A_{26}H_{1n},$$

$$H_{5n} = -iar_n(A_{27} + A_{28}), H_{6n} = (A_{24}r_n^2 - a^2A_{25}) - H_{2n} + A_{26}H_{1n}, H_{7n} = (a^2A_{29} - A_{30}r_n^2)H_{3n},$$

$$H_{8n} = ia(A_{31} + A_{32})r_nH_{3n}, H_{9n} = (a^2A_{30} - A_{29}r_n^2)H_{3n}, H_{10n} = -iaA_{34}H_{1n}, H_{11n} = -A_{35}r_nH_{1n},$$

$$R_{1n} = \frac{(B_9 - h_n^2)}{(A_5)}, R_{2n} = -\frac{(A_{23}\omega R_{1n})}{(h_n^2 - B_{14})}, R_{3n} = ia(A_{24} - A_{25})h_n, R_{4n} = -(A_{27}h_n^2 + a^2A_{28} + KR_{1n}),$$

$$R_{5n} = -ia(A_{29} - A_{30})h_nR_{2n}, R_{6n} = (h_n^2A_{31} + A_{32}a^2)R_{2n}, R_{7n} = -A_{33}h_nR_{1n}, R_{8n} = -iaA_{34}R_{1n},$$

$$B_{16} = (A_{12}\{B_{12} + B_8\} + A_{13}A_{17} - \{A_{11}A_{17} + A_{12}A_{16}\}B_3),$$

$$B_{17} = \{A_{12}B_{12} - A_{13}A_{17}\}B_8 + a^2B_3(A_{11}A_{17} + A_{12}A_{16}).$$

similarly for medium II (i.e. fluid half space), the solutions are of the form,

$$\bar{u}_1^f(z) = M_5(b,\omega)e^{-r_5 z} \tag{71}$$

$$\bar{u}_3^f(z) = M_5'(b,\omega)e^{-r_5 z} \tag{72}$$

where $M_5(a,\omega)$ and $M_5'(a,\omega)$ are the specific functions depending upon a and ω and r_5 is the root of characteristic equation,

$$(D^2 - a^2 + l\omega^2)\bar{u}_1^f(z) = 0 \tag{73}$$

where $l = \frac{\rho^f c_1^2}{\lambda^f}$ and $r_5 = \sqrt{a^2 - l\omega^2}$

Thus we have,

$$\bar{u}_3^f(z) = HM_5(b,\omega)e^{-r_5 z} \tag{74}$$

$$\bar{t}_{zz}^f(z) = IM_5(b,\omega)e^{-r_5 z} \tag{75}$$

$$\bar{t}_{xz}^f(z) = 0 \tag{76}$$

where $H = \frac{r_5^2 - l\omega^2}{iar_5}$ and $I = \frac{(\lambda^f)(iaH - r_5)}{\rho c_1^2}$.

3. Applications

In this section, we determine the parameters M_n; (n = 1, 2, 3, 4, 5) and L_n;(n = 1, 2, 3). In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. Constants M_1, M_2, M_3, M_4, M_5 and L_1, L_2, L_3 have to be selected such that boundary conditions at the surface $z = 0$ take the form,

$$t_{zz} = t_{zz}^f - F_1 e^{\omega t + iax},\ t_{xz} = t_{xz}^f,\ m_{yz} = 0,\ \lambda_3^* = 0,\ q_{zz} = 0,\ q_{xz} = 0,\ \frac{\partial T}{\partial z} = 0,\ \frac{\partial u_3}{\partial t} = \frac{\partial u_3^f}{\partial t} \tag{77}$$

where F_1 is the magnitude of mechanical force.

Using the expressions of $t_{zz}, t_{xz}, t_{zz}^f, t_{zx}^f, m_{yz}, \lambda_3^*, q_{zz}, q_{xz}, T, u_3, u_3^f$ into above boundary conditions (77), give the following equations satisfied by the parameters,

$$\sum_{n=1}^{4}[H_{6n}M_n] - \sum_{n=1}^{3}[R_{3n}L_n] - IM_5 = N_5 - F_1$$

$$\sum_{n=1}^{4}[H_{5n}M_n] + \sum_{n=1}^{3}[R_{4n}L_n] = 0$$

$$\sum_{n=1}^{4}[H_{10n}M_n] + \sum_{n=1}^{3}[R_{7n}L_n] = -N_8$$

$$\sum_{n=1}^{4}[H_{11n}M_n] + \sum_{n=1}^{3}[R_{8n}L_n] = 0$$

$$\sum_{n=1}^{4}[H_{9n}M_n] - \sum_{n=1}^{3}[R_{5n}L_n] = -N_7$$

$$\sum_{n=1}^{4}[H_{8n}M_n] + \sum_{n=1}^{3}[R_{6n}L_n] = 0$$

$$\sum_{n=1}^{4}[H_{2n}r_nM_n] = 0$$

$$\sum_{n=1}^{4}[-r_nM_n] + ia\sum_{n=1}^{3}L_n - HM_5 = 0$$

After solving these non-homogeneous system of equations, we get the values of constants M_1, M_2, M_3, M_4, M_5, L_1, L_2, L_3 and hence obtain the component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress at the interface of fluid half space and MTSM.

4. Special case

(1) If we neglect micropolarity effect i.e. $\alpha = \beta = \gamma = b_0 = \mu = K = J = 0$, we obtain the results for microstretch thermoelastic solid with microtemperatures without microrotational effect (TSMWM).

(2) If we neglect microstretch effect i.e. $\alpha_0 = \lambda_0 = \lambda_1 = \nu_1 = b_0 = \mu_2 = J_0 = 0$, we obtain the results for thermoelastic solid with microtemperatures without microstretch effect (TSMWS).

(3) If we neglect both micropolarity effect and microstretch effect i.e. $\alpha = \beta = \gamma = 0$, $\mu = K = J = \alpha_0 = \lambda_0 = \lambda_1 = \nu_1 = b_0 = \mu_2 = J_0 = 0$, we obtain the results for thermoelastic solid with microtemperatures (TSM).

5. Numerical results and discussions

In order to illustrate the theoretical results obtained in the preceding section, we present some numerical results for the physical constants,

The values of micropolar constants are (Eringen, 1984):

$\lambda = 9.4 \times 10^{10}$ N/m^2, $\mu = 4.0 \times 10^{10}$ N/m^2, $\rho = 1.74 \times 10^3$ kg/m^3, $K = 10^{10}$ Nm^{-2}, $\gamma = 7.79 \times 10^{-10}$ N, $J = 0.0000002 \times 10^{-14}$ m^2, $\beta = 0.32 \times 10^{10}$ N/m^2 K, $b_0 = 0.0098 \times 10^{10}$ N.

The values of thermal parameters are (Dhaliwal & Singh, 1980):

$c^* = 0.104 \times 10^4$ Nm/kg/K, $T_0 = 298$ K, $K^* = 1.7 \times 10^2$ Ns^{-1} K^{-1}, $\alpha_{t_1} = 0.05\text{K}^{-1}$, $\alpha_{t_2} = 0.05$ K^{-1}, $\tau_1 = 0.613 \times 10^3$ s.

The values of microstretch parameters are (Kumar & Kaur, 2014):

$j_0 = 0.000019 \times 10^{-13}$ m^2, $\lambda_0 = 0.21 \times 10^{11}\text{N/m}^2$, $\lambda_1 = 0.007 \times 10^{12}\text{N/m}^2$, $\alpha_0 = 0.008 \times 10^{-7}$ N, $b = 0.15 \times 10^{-10}$ N.

The values of microtemperature parameters are (Kumar & Kaur, 2014):

$k_1 = 0.0035$ Ns^{-1}, $k_2 = 0.045$ Ns^{-1}, $k_3 = 0.055$ NK^{-1}s^{-1}, $k_4 = 0.065 Ns^{-1}$ m^2, $k_5 = 0.076$ Ns^{-1} m^2, $k_6 = 0.096$ Ns^{-1} m^2, $\mu_1 = 0.0085$ N, $\mu_2 = 0.0095$ N.

The physical constants for water are given by Ewing et al. (1957):

$\lambda^f = 2.14 \times 10^9 \text{N/m}^2$, $\rho^f = 10^3$ kg/m^3.

The computations are carried out for the value of non-dimensional time $t = 0.2$ in the range $0 \le x \le 10$ and on the surface $z = 1.3$. The numerical values for normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress are shown in Figures 2–7 for mechanical force with magnitude.

$F_1 = 1.0$, $Q_0 = 1$, $\omega = \omega_0 + \iota\xi$, $\omega_0 = -0.3$, $\xi = 0.1$, $Q_1 = 10$ and $a = 0.9$ for

(a) MTSM by solid line with centered symbol ◆.

(b) TSMWM by solid line with centered symbol ■.

(c) TSMWS by dashed line with centered symbol ▲.

(d) TSM by dashed line with centered symbol ×.

6. Discussion

The variation of normal displacement for MTSM, TSMWM, and TSMWS is similar in nature. These values decrease sharply in the entire range. The values of normal displacement for TSM increase in the range $0 \le x \le 2.3$ and then the values approach zero with a straight curve. The variations of microtemperature for MTSM and TSMWS are opposite in nature which shows that microstructure has significant effect on microtemperature. The values of microtemperature for TSM are very less and lie in a very short range. These variations of normal displacement and microtemperature are shown in Figures 2 and 3, respectively.

Figure 4 depicts that the variations of normal force stress are opposite in nature for both MTSM and TSMWM. This concludes that micropolarity effect is more prominent in the study of normal force stress. The variations of normal force stress for TSMWS and TSM are similar in nature. The values are

Figure 2. Variation of normal displacement with horizontal distance.

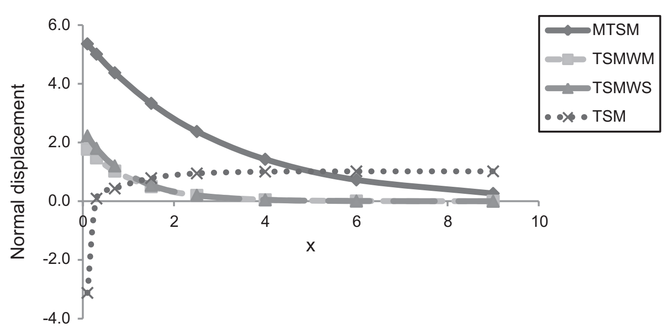
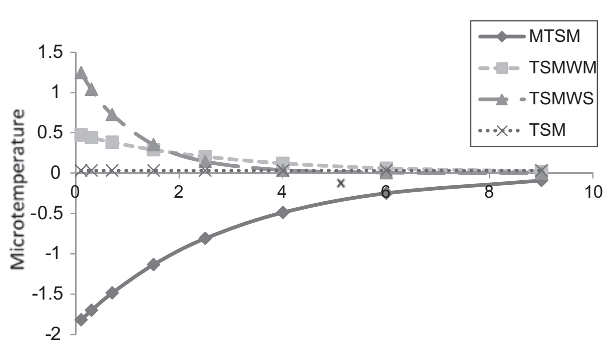


Figure 3. Variation of microtemperature with horizontal distance.



also quite close to each other. The values for these medium (TSMWS and TSM) decrease sharply and then follow a straight curve to converge. With difference in magnitude, the variation of microstress tensor for MTSM and TSMWM is similar in nature. These values decrease uniformly and then approach to zero with increase in horizontal distance. The variation of microstress tensor is shown in Figure 5.

Figure 6 shows that the variations of heat flux moment tensor are similar in nature for all mediums. There is difference in magnitude among all the solids which proves the effect of micropolarity and microstress in the medium. It is again observed that the values of heat flux moment tensor for TSM are very less and hence as compared to other medium, the variation lies in a very short range.

In the absence of stretch effect, the variation of couple stress is effected to a great extent as visible in Figure 7. The values increase in the range $0 \le x \le 4.0$ and then show a constant behavior. The variations are sharper for TSMWS in comparison to MTSM.

Figure 4. Variation of normal force stress with horizontal distance.

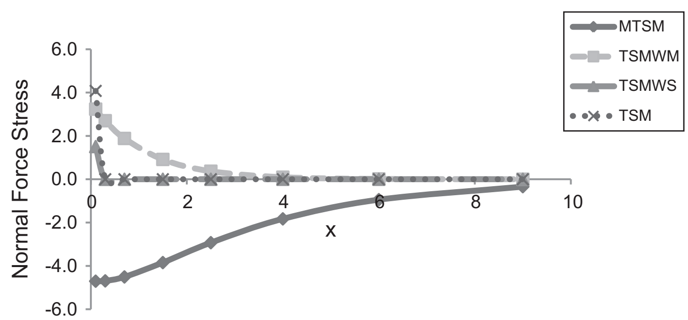


Figure 5. Variation of microstress tensor with horizontal distance.

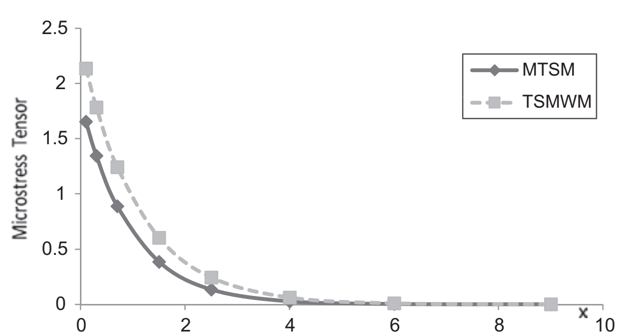
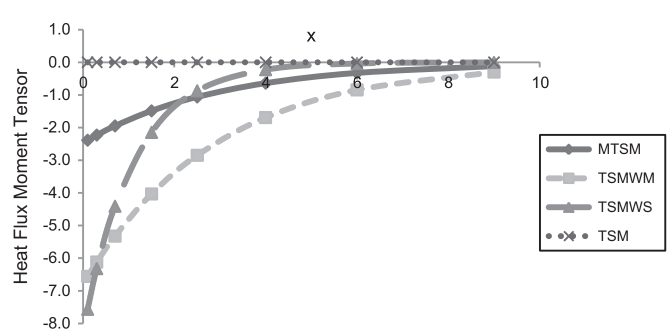


Figure 6. Variation of heat flux moment tensor with horizontal distance.



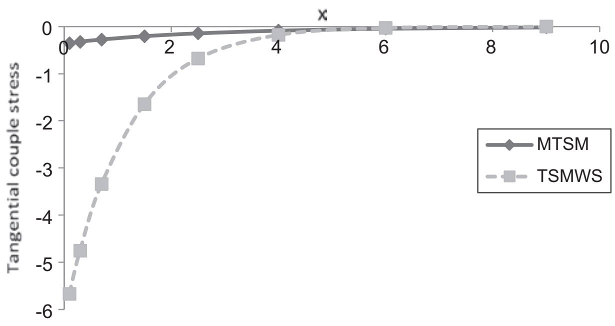


Figure 7. Variation of tangential coule stress with horizontal distance.

7. Conclusion

Both micropolarity and stretch effect have a significant effect on the normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and tangential couple stress. The values of all the quantities for a generalized TSM are less in magnitude as compared to the medium with micropolarity and stretch effect. Micropolarity does not show appreciable effect on microstress tensor but microstretch has a significant effect on couple stress. Such type of problems is very useful in the study of earthquake engineering, seismology, and volcanic eruptions. It helps us to study the effect of a heat source in the medium and the deformation caused in the medium due to the heat source.

Funding
The authors received no direct funding for this research.

Author details
Praveen Ailawalia[1]
E-mail: praveen_2117@rediffmail.com
Sunil Kumar Sachdeva[2,3]
E-mail: sunilsachdeva.daviet@gmail.com
Devinder Pathania[4]
E-mail: despathania@yahoo.com
ORCID ID: http://orcid.org/0000-0002-3324-9633
[1] Department of Applied Sciences and Humanities, M.M. University, Sadopur, Ambala City, Haryana, India.
[2] Department of Applied Sciences, D.A.V. Institute of Engineering and Technology, Kabir Nagar, Jalandhar, Punjab, India.
[3] Punjab Technical University, Jalandhar, Punjab, India.
[4] Department of Applied Sciences, Guru Nanak Engineering College, Ludhiana, Punjab, India.

Citation information
Cite this article as: Two dimensional deformation in microstretch thermoelastic half space with microtemperatures and internal heat source, Praveen Ailawalia, Sunil Kumar Sachdeva & Devinder Pathania, *Cogent Mathematics* (2015), 2: 1086293.

References

Abbas, I. A., & Othman, M. I. A. (2012). Plane waves in generalized thermo-microstretch elastic solid with thermal relaxation using finite element method. *International Journal of Thermophysics, 33*, 2407–2423. http://dx.doi.org/10.1007/s10765-012-1340-8

Aouadi, M. (2008). Some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 31*, 649–662. http://dx.doi.org/10.1080/01495730801981772

Casas, P. S., & Quintanilla, R. (2005). Exponential stability in thermoelasticity with microtemperatures. *International Journal of Engineering Science, 43*, 33–47. http://dx.doi.org/10.1016/j.ijengsci.2004.09.004

Chiriță, S., Ciarletta, M., & D'Apice, C. (2013). On the theory of thermoelasticity with microtemperatures. *Journal of Mathematical Analysis and Applications, 397*, 349–361. http://dx.doi.org/10.1016/j.jmaa.2012.07.061

Dhaliwal, R. S., & Singh, A. (1980). *Dynamic coupled thermoelasticity*. New Delhi: Hindustan Publication Corporation.

Eringen, A. C. (1965). *Linear theory of micropolar elasticity* (ONR Technical Report No. 29). School of Aeronautics, Aeronautics and Engineering Science, Purdue University, West Lafayette, IN.

Eringen, A. C. (1966). A unified theory of thermomechanical materials. *International Journal of Engineering Science, 4*, 179–202. http://dx.doi.org/10.1016/0020-7225(66)90022-X

Eringen, A. C. (1968). *Micropolar elastic solids with stretch*. In Mustafa Inan Anisina, Ari. Kitapevi Matbaassi, Istanbul (pp. 1–18).

Eringen, A. C. (1984). Plane waves in nonlocal micropolar elasticity. *International Journal of Engineering Science, 22*, 1113–1121. http://dx.doi.org/10.1016/0020-7225(84)90112-5

Eringen, A. C. (1990). Theory of thermo-microstretch elastic solids. *International Journal of Engineering Science, 28*, 1291–1301. http://dx.doi.org/10.1016/0020-7225(90)90076-U

Eringen, A. C. (1996). Linear theory of micropolar elasticity. *Journal of Mathematics and Mechanics, 15*, 909–923.

Eringen, A. C., & Suhubi, E. S. (1964). Nonlinear theory of simple micro-elastic solids I. *International Journal of Engineering Science, 2*, 189–203. http://dx.doi.org/10.1016/0020-7225(64)90004-7

Eringen, A. C., & Suhubi, E. S. (1964). Nonlinear theory of simple micro-elastic solids II. *International Journal of Engineering Science, 2*, 389–404.

Ewing, W. M., Jardetzky, W. S., & Press, F. (1957). *Elastic waves in layered media*. New York, NY: McGraw Hill.

Grot, R. A. (1969). Thermodynamics of a continuum with microstructure. *International Journal of Engineering Science, 7*, 801–814. http://dx.doi.org/10.1016/0020-7225(69)90062-7

Iesan, D. (2001). On a theory of micromorphic elastic solids with microtemperatures. *Journal of Thermal Stresses, 24*, 737–752.

Iesan, D., & Quintanilla, R. (2000). On a theory of thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 23*, 199–215.

Ieşan, D. (2007). Thermoelasticity of bodies with microstructure and microtemperatures. *International Journal of Solids and Structures, 44*, 8648–8662. http://dx.doi.org/10.1016/j.ijsolstr.2007.06.027

Ieşan, D., & Quintanilla, R. (2009). On thermoelastic bodies with inner structure and microtemperatures. *Journal of Mathematical Analysis and Applications, 354*, 12–23. http://dx.doi.org/10.1016/j.jmaa.2008.12.017

Kumar, R., & Kansal, T. (2011). Fundamental solution in the theory of thermomicrostretch elastic diffusive solids. *ISRN Applied Mathematics, 2011*, 1–15. http://dx.doi.org/10.5402/2011/764632

Kumar, R., & Kaur, M. (2014). Reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures. *Archive of Applied Mechanics, 84*, 571–590. http://dx.doi.org/10.1007/s00419-014-0818-1

Kumar, R., & Partap, G. (2009). Analysis of free vibrations for Rayleigh–Lamb waves in a microstretch thermoelastic plate with two relaxation times. *Journal of Engineering Physics and Thermophysics, 82*, 35–46.

Kumar, R., & Rupender, R. (2008). Reflection at free surface of magneto-thermo-microstretch elastic solid. *Bulletin of the Polish Academy of Sciences, 56*, 263–271.

Kumar, R., & Rupender, R. (2009). Propagation of plane waves at the imperfect boundary of elastic and electro-microstretch generalized thermoelastic solids. *Applied Mathematics and Mechanics, 30*, 1445–1454. http://dx.doi.org/10.1007/s10483-009-1110-6

Kumar, S., Sharma, J. N., & Sharma, Y. D. (2011). Generalized thermoelastic waves in microstretch plates loaded with fluid of varying temperature. *International Journal of Applied Mechanics, 3*, 563–586. http://dx.doi.org/10.1142/S1758825111001135

Magaña, A., & Quintanilla, R. (2006). On the time decay of solutions in one-dimensional theories of porous materials. *International Journal of Solids and Structures, 43*, 3414–3427. http://dx.doi.org/10.1016/j.ijsolstr.2005.06.077

Marin, M. (2010). Lagrange identity method for microstretch thermoelastic materials. *Journal of Mathematical Analysis and Applications, 363*, 275–286. http://dx.doi.org/10.1016/j.jmaa.2009.08.045

Othman, M. I. A., & Lotfy, Kh. (2010). On the plane waves of generalized thermo-microstretch elastic half space under three theories. *International Communications in Heat and Mass Transfer, 37*, 192–200. http://dx.doi.org/10.1016/j.icheatmasstransfer.2009.09.017

Othman, M. I. A., & Lotfy, Kh. (2011). Effect of rotation on plane waves in generalized thermo- microstretch elastic solid with one relaxation time. *Multidiscipline Modeling in Materials and Structures, 7*, 43–62. http://dx.doi.org/10.1108/15736101111141430

Othman, M. I. A., Lotfy, Kh., & Farouk, R. M. (2010). Generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. *Engineering Analysis with Boundary Elements, 34*, 229–237. http://dx.doi.org/10.1016/j.enganabound.2009.10.003

Quintanilla, R. (2011). On growth and continuous dependence in thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 34*, 911–922. http://dx.doi.org/10.1080/01495739.2011.586278

Řiha, P. (1976). On the microcontinuum model of heat conduction in materials with inner structure. *International Journal of Engineering Science, 14*, 529–535. http://dx.doi.org/10.1016/0020-7225(76)90017-3

Scalia, A., & Svanadze, M. (2006). On the representations of solutions of the theory of thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 29*, 849–863. http://dx.doi.org/10.1080/01495730600705448

Scalia, A., Svanadze, M., & Tracinà, R. (2010). Basic theorems in the equilibrium theory of thermoelasticity with microtemperatures. *Journal of Thermal Stresses*, 721–753. http://dx.doi.org/10.1080/01495739.2010.482348

Singh, B., & Kumar, R. (1998). Wave propagation in a generalized thermo-microstretch elastic solid. *International Journal of Engineering Science, 36*, 891–912. http://dx.doi.org/10.1016/S0020-7225(97)00099-2

Singh, D., Kumar, A., & Kumar, R. (2014). A problem in microstretch thermoelastic diffusive medium. *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, 8*, 24–27.

Steeb, H., Singh, J., & Tomar, S. K. (2013). Time harmonic waves in thermoelastic material with microtemperatures. *Mechanics Research Communications, 48*, 8–18. http://dx.doi.org/10.1016/j.mechrescom.2012.11.006

Tomar, S. K., & Khurana, A. (2009). Reflection and transmission of elastic waves from a plane interface between two thermo-microstretch solid half-spaces. *International Journal of Applied Mathematics and Mechanics, 5*, 48–68.