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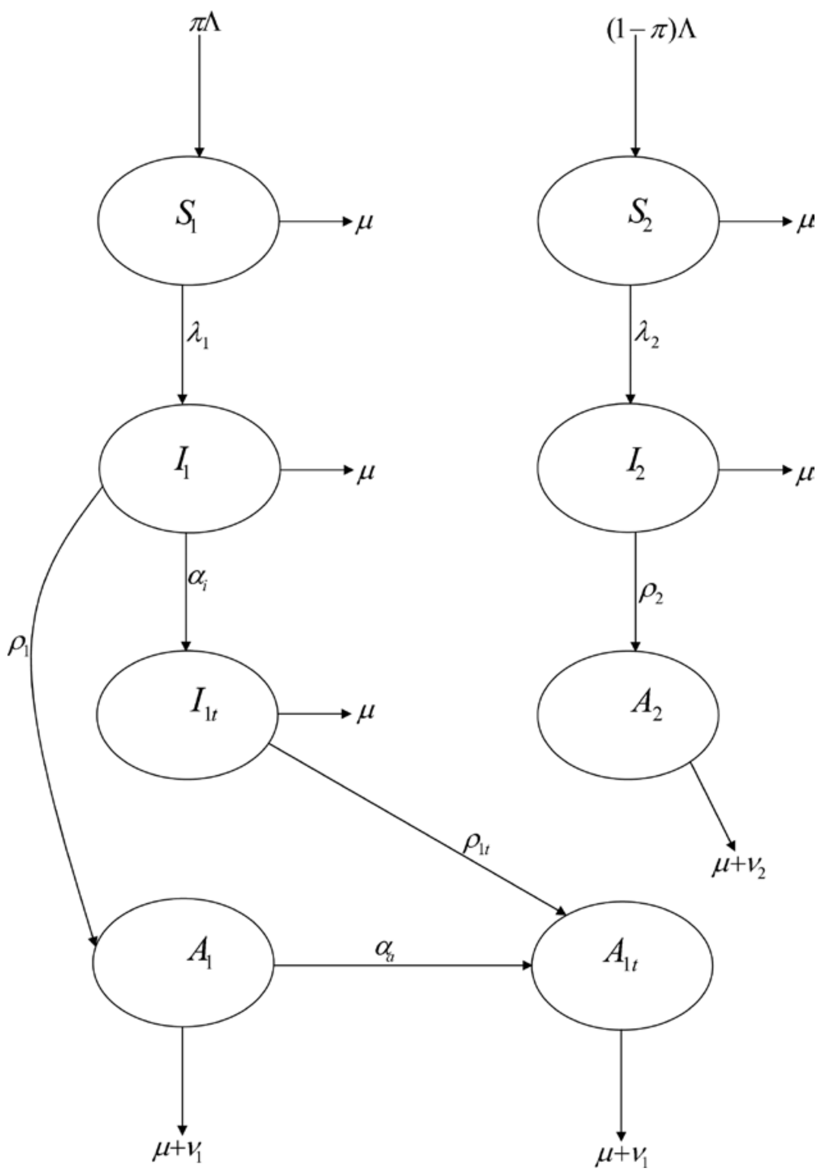

APPLIED & INTERDISCIPLINARY MATHEMATICS | RESEARCH ARTICLE

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Assessing the impact of homelessness on HIV/AIDS transmission dynamics

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Abstract: Care for the people living with HIV/AIDS is more than the provision of antiretroviral therapy. The effects of homelessness on HIV/AIDS transmission are captured through a mathematical model. The mathematical model is rigorously analyzed. The disease-free equilibrium is globally asymptotically stable when the reproduction number is less than unity. Results from the analysis of the reproduction number suggests that homelessness enhances both HIV transmission and progression to the AIDS stage. This is further supported by numerical simulations which show that some elements of homelessness (lack of entertainment) enhances HIV/AIDS transmission.

Subjects: Bioscience; Health and Social Care; Mathematics & Statistics; Medicine; Science

Keywords: mathematical model; HIV/AIDS and homelessness

1. Introduction

Homeless people are amongst the most vulnerable in the society and do not get the help they need to address their health, economic, and social issues. Homelessness and HIV/AIDS are intricately related (National Coalition for the Homeless (NCH), 2009) as homeless worsens HIV and the homeless are doubly affected by HIV. The pressure of daily needs, exposure to violence (including sexual exploitation), alcohol and drug-misuse to cope with stress or mental health issues and other conditions of the homelessness make homeless and unstably housed people extremely vulnerable to HIV infection (Aidala and Sumartjo, 2007). A 1995 survey of homeless adults found that 69% were at risk for HIV infection from unprotected sex with multiple partners, injection drug use (IDU), sex with IDU partners, or exchanging unprotected sex for money or drugs (Adams, 2003). People who

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C.P. Bhunu (BSc Hons, MSc, DPhil) is a professor in the Department of Mathematics, University of Zimbabwe. He currently serves as an external examiner for Chinhoyi University of Technology, Harare Institute of Technology and Zimbabwe Open University. He was visiting African scientist at Cambridge Infectious Disease Consortium (2010) and visiting professor at the University of Venda (2012), respectively. He is a life member of the Clare Hall College, University of Cambridge. He also serves as an editor and reviewer of several international journals in Applied Mathematics. His research interests lie in the field of mathematical modeling of issues affecting mankind ranging from social issues to biological issues as well as the theoretical analysis of the mathematical models that arise in all these applications.

PUBLIC INTEREST STATEMENT

A model is proposed to explore the impact homelessness has on HIV. Results from model analysis suggest that homeleness enhances HIV transmission and disease progression. Our results suggests that the fight against HIV is not won by the provision of ARVs alone.

are homeless or unstably housed have HIV infection rates as much as 16 times higher than people who have a stable place to live (Robertson et al., 2004). Homeless women and adolescents are particularly at risk (Adams, 2003). Stigma makes it difficult for those infected with HIV to access medical and mental health services (Tomaszewski, 2011). Homeless women have special barriers to health care. Homeless mothers, in particular, have been found to subordinate their own health care needs for the needs of their children (Song, 2003). Single homeless women are more likely to be victims of domestic violence and sexual abuse, both of which have been linked to HIV infection (Song, 2003).

Due to HIV infection, some individuals loose their homes due to the costs of medication and health care. At least half of all people living with HIV/AIDS experience homelessness or housing instability (Aidala, Lee, Abramson, Messeri' & Siegler, 2007). Thus, housing is the greatest unmet need of people living with HIV (Bekele et al., 2013; Shubert & Bernstine, 2007). In the United States of America about one-third to one-half of the HIV infected are either homeless, unable to afford their own housing or at imminent risk of homelessness (NCH, 2009; Song, 2003). The socially and culturally based stigma faced by people living with HIV/AIDS is exacerbated by co-factors of substance misuse, mental illness, and homelessness (Tomaszewski, 2011).

HIV/AIDS disease progression is affected by both medical and social factors which is a double blow for the HIV-infected homeless people. Homeless individuals lack basic needs such as food, clothing, and shelter which are necessary to care for the people living with HIV/AIDS (Tomaszewski, 2011). HIV-infected homeless are less likely to receive and adhere to antiretroviral therapy and are more likely to have higher death rate due to AIDS (Aidala et al., 2007; Kidder, Wolitski Campsmith' & Nakamura, 2007; Leaver, Bargh, Dunn, & Hwanget, 2007). It is against this background that we carry out this study. Mathematical models have been developed to understand the role of social and behavioral processes in HIV transmission (Ajay, Brendan, & David, 2009; Bhunu, Mhlanga, & Mushayabasa, 2014; Pedamallu, Ozdamar, Kropat, & Weber, 2012). However, none have looked into homelessness and HIV from the mathematical point of view. In our past work (Bhunu et al., 2014), we explored the impact of prostitution on HIV and now we model the effects of homelessness on the transmission dynamics of HIV/AIDS.

The paper is structured as follows. The model framework and its analysis are presented in Section 2. Numerical simulations are in Section 3 and the last Section concludes the paper.

2. Model description

The model subdivides the human population based on homelessness and HIV infection. The population is divided into the following sub-groups: non-homeless susceptibles $S_1(t)$, non-homeless HIV infected not yet showing AIDS symptoms $I_1(t)$, non-homeless HIV infected not yet showing AIDS symptoms and on treatment $I_{1_t}(t)$, non-homeless HIV infected displaying AIDS symptoms $A_1(t)$, non-homeless HIV infected displaying AIDS symptoms and on treatment $A_{1_t}(t)$, homeless susceptibles $S_2(t)$, homeless HIV infected not yet showing AIDS symptoms $I_2(t)$, homeless HIV infected showing AIDS symptoms $A_2(t)$. The total sub-populations for the non-homeless and the homeless are given by.

$$N_1(t) = S_1(t) + I_1(t) + I_{1_t}(t) + A_1(t) + A_{1_t}(t) \text{ and } N_2(t) = S_2(t) + I_2(t) + A_2(t), \text{ respectively.} \tag{1}$$

The total population size is given by $N(t) = N_1(t) + N_2(t)$. Individuals in different human sub-groups suffer from natural death at a constant rate μ, which is proportional to the number in each class. We assume that interaction is heterogeneous. The group j members make c_j ($j = 1, 2$) sexual contacts per unit time and a fraction of the contacts made by a member of group j with a member of group i is p_{ji} ($i = 1, 2$). Then $p_{11} + p_{12} = p_{22} + p_{21} = 1$. The total number of sexual contacts made in unit time by members of group '2' (homeless people) with members of group '1' (non-homeless people) is $c_2 p_{21} N_2$ and this must be equal to the total number of sexual contacts made by members of group '1' with members of group '2', we have a balance relation

$$\frac{p_{21}c_2}{N_1(t)} = \frac{p_{12}c_1}{N_2(t)} \tag{2}$$

The forces of HIV infection for the non-homeless and the homeless are given by λ_1 and λ_2, with

$$\begin{aligned}\lambda_1(t) &= \frac{p_{11}c_1\beta_1[A_1 + \phi_1 I_1 + \theta(A_{1_t} + \phi_1 I_{1_t})](t)}{N_1(t)} + \frac{p_{12}c_1\beta_2[A_2 + \phi_2 I_2](t)}{N_2(t)}\\ &\text{and}\\ \lambda_2(t) &= \frac{p_{22}c_2\beta_2[A_2 + \phi_2 I_2](t)}{N_2(t)} + \frac{p_{21}c_2\beta_1[A_1 + \phi_1 I_1 + \theta(A_{1_t} + \phi_1 I_{1_t})](t)}{N_1(t)}\end{aligned} \tag{3}$$

respectively. In Equation 3, β_i ($i = 1, 2$) is the probability of one individual being infected by one infectious individual from the 1 or 2 class [$\beta_2 = b_3\beta_1,\ b_3 \ge 1$, as a result of co-infections with other untreated STIs]; c_j ($j = 1, 2$) is the per capita effective sexual contact rate; $\phi_i \in (0, 1)$ [$i = 1, 2$] accounts for a reduction in infectiousness for those only infected with HIV not yet displaying AIDS symptoms since the viral load is correlated with infectiousness (WHO, 2005); $\theta \in (0, 1)$ accounts for a reduction in infectiousness for those on treatment when compared to those not yet on treatment. It is important to note that $c_2 = b_1c_1,\ b_1 \ge 1$ as homeless people lack other forms of entertainment, most of them will abuse alcohol/drugs (Didenko & Pankratz, 2007) and have many sexual partners.

Susceptible humans enter the population through sexual maturity at a rate Λ, a proportion π entering the non-homeless susceptibles and the complementary proportion $(1 - \pi)$ entering the homeless susceptibles. The non-homeless susceptibles $S_1(t)$ and homeless susceptibles $S_2(t)$ are infected with HIV at rates $\lambda_1(t)$ and $\lambda_2(t)$ to enter the $I_1(t)$ and $I_2(t)$-classes, respectively. Individuals in $I_1(t)$ and $I_2(t)$-classes progress to the AIDS stage ($A_1(t)$ and $A_2(t)$) at rates ρ_1 and ρ_2, respectively, with $\rho_2 = b_2\rho_1,\ b_2 \ge 1$ as homeless HIV positive individuals are more likely to progress to the AIDS stage of disease progression faster than their counterparts as they are more likely to be doubly infected with other infections and suffer from poor nutrition. Individuals in $I_1(t)$-class are put on antiretroviral therapy at a rate α_i to move into the $I_{1_t}(t)$-class. Individuals in $I_{1_t}(t)$-class progress to the AIDS stage $A_{1_t}(t)$ at a rate ρ_{1_t} with $\rho_{1_t} \le \rho_1$, as individuals on antiretroviral therapy are likely to progress the AIDS stage at a slower rate than those not yet on treatment. Those in $A_1(t)$-class are put on antiretroviral therapy at α_a to enter $A_{1_t}(t)$-class. AIDS-related deaths are experienced by individuals in the AIDS stage of disease progression at rates ν_1 and ν_2, for the homeless and non-homeless, respectively, with $\nu_2 = b_4\nu_1,\ b_4 \ge 1$ as homelessness experience higher AIDS related than the non-homeless due to failure to access medical care. Individuals who are homeless experience inadequate transportation, lack of comprehensive and/or culturally appropriate services, lack of awareness of services and resources, and poor provider attitudes (Tomaszewski, 2011). For that reason we assume there is no antiretroviral therapy for the HIV-infected homeless people.

We assume any transfer from non-homeless to homeless status or vice versa is negligible. The structure of the model is shown in Figure 1.

Based on these assumptions, the following system of differential equations describes the model.

$$\begin{aligned}
S_1'(t) &= \pi\Lambda - (\lambda_1(t) + \mu)S_1(t),\\
I_1'(t) &= \lambda_1(t)S_1(t) - (\rho_1 + \alpha_i + \mu)I_1(t),\\
I_{1_t}'(t) &= \alpha_i I_1(t) - (\rho_{1_t} + \mu)I_{1_t}(t),\\
A_1'(t) &= \rho_1 I_1(t) - (\alpha_a + \mu + \nu_1)A_1(t),\\
A_{1_t}'(t) &= \rho_{1_t} I_{1_t}(t) + \alpha_a A_1(t) - (\mu + \nu_1)A_{1_t}(t),\\
S_2'(t) &= (1 - \pi)\Lambda - (\lambda_2(t) + \mu)S_2(t),\\
I_2'(t) &= \lambda_2(t)S_2(t) - (\rho_2 + \mu)I_2(t),\\
A_2'(t) &= \rho_2 I_2(t) - (\mu + \nu_2)A_2(t).
\end{aligned} \tag{4}$$

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2.1. Invariant region

The model system 4 will be analyzed in a suitable region as follows. We first show that system 4 is dissipative. That is, all solutions are uniformly bounded in a proper subset $\Omega \subset \mathbb{R}^8_+$. Let $(S_1, I_1, I_{1_t}, A_1, A_{1_t}, S_2, I_2, A_2) \in \mathbb{R}^8_+$ be any solution with non-negative initial conditions. Adding all the equations in 4, we have

$$N'(t) = \Lambda - \mu N(t) - \nu_1(A_1 + A_{1_t} + b_4 A_2)(t). \tag{5}$$

Model system 4 has a varying population size ($N'(t) \neq 0$) and therefore a trivial equilibrium is not feasible. Then,

$$N'(t) \quad \leq \Lambda - \mu N(t). \tag{6}$$

So that (cf. Birkhoff & Rota, 1982)

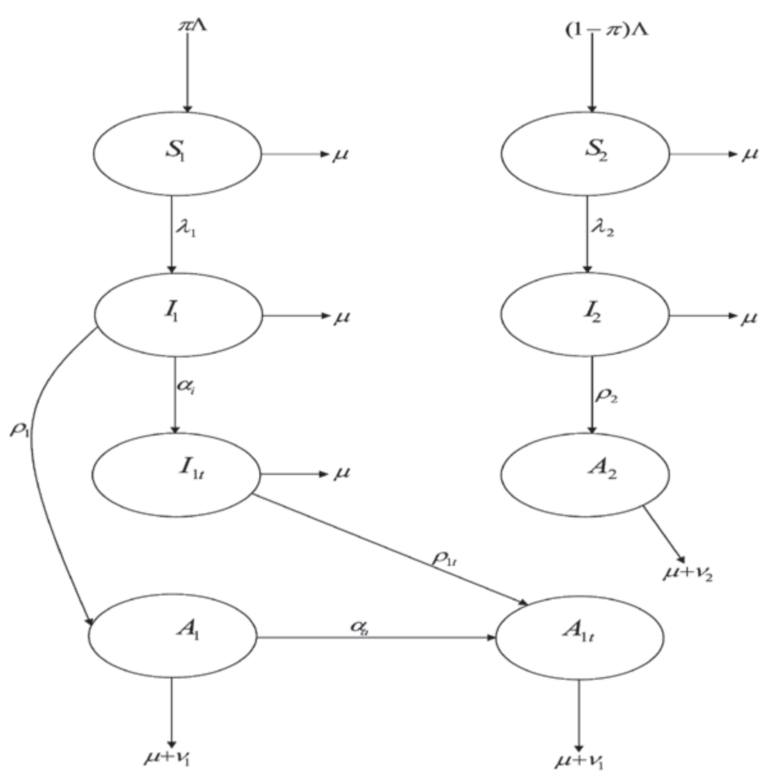
$$0 \leq N(t) \leq \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu} \right) e^{-\mu t}, \tag{7}$$

where $N(0)$ represents the value of 4 evaluated at the initial values of the respective variables. The lower limit comes naturally from the fact that the model variables and parameters are non-negative ($\forall\, t \geq 0$) since they monitor human populations. Thus, as $t \to \infty$, $0 \leq N(t) \leq \frac{\Lambda}{\mu}$. Therefore, all feasible solutions of system 4 enter the region

$$\Omega = \left\{ (S_1, I_1, I_{1_t}, A_1, A_{1_t}, S_2, I_2, A_2) \in \mathbb{R}^8_+ : N \leq \frac{\Lambda}{\mu} \right\} \tag{8}$$

Thus, Ω is positively invariant (it can also be shown that Ω is attracting) and it is sufficient to consider solutions in Ω. Existence, uniqueness, and continuation results for system 4 hold in this region. It can be shown that all solutions of system 4 starting in Ω remain in Ω for all $t \geq 0$. All parameters and state variables for model system 4 are assumed to be non-negative for $t \geq 0$.

Figure 1. Structure of the model.



2.2. Disease-free equilibrium and stability analysis

The disease-free equilibrium of model system 4 is given by

$$\begin{aligned}\mathcal{E}^0 &= (S_1^0, I_1^0, I_{1_t}^0, A_1^0, A_{1_t}^0, S_2^0, I_1^0, A_2^0) \\ &= \left(\frac{\Lambda\pi}{\mu}, 0, 0, 0, 0, \frac{\Lambda(1-\pi)}{\mu}, 0, 0\right)\end{aligned} \tag{9}$$

Following Van den Driessche and Watmough (2002), we have $\mathcal{R}_{H_H}$ as the reproduction number of the model system 4. $\mathcal{R}_{H_H}$ which is defined as the number of secondary HIV infections produced by one infected individual in the presence of antiretroviral therapy in a completely susceptible population with some homeless people is given by

$$\begin{aligned}\mathcal{R}_{H_H} &= \frac{H_1 + \sqrt{H_1^2 - H_2}}{2g_1g_2g_3g_4g_5g_6} \\ H_1 &= g_5g_6p_{11}c_1\beta_1[\theta g_3\alpha_i(\rho_{1_t} + \phi_1 g_4) + g_2(\theta\alpha_a\rho_1 + (\rho_1 + \phi_1 g_3)g_4)] + g_1g_2g_3g_4p_{22}c_2\beta_2(\rho_2 + \phi_2 g_6), \\ H_2 &= 4g_1g_2g_3g_4g_5g_6c_1c_2\mathcal{P}[\theta g_3\alpha_i(\rho_{1_t} + \phi_1 g_4) + g_2(\theta\alpha_a\rho_1 + (\rho_1 + \phi_1 g_3)g_4)](\rho_2 + \phi_2 g_4)\beta_1\beta_2\end{aligned} \tag{10}$$

with

$$\begin{aligned}g_1 &= \rho_1 + \alpha_i + \mu,\ g_2 = \rho_{1_t} + \mu,\ g_3 = \alpha_a + \mu + \nu_1,\ g_4 = \mu + \nu_1,\ g_5 = \rho_2 + \mu, \\ g_6 &= \mu + \nu_2,\ g_7 = \mu + \rho_1,\ \mathcal{P} = p_{11}p_{22} - p_{12}p_{21} \text{ throughout the manuscript.}\end{aligned} \tag{11}$$

Local stability of the disease-free equilibrium is assured by Theorem 2 (Van den Driessche & Watmough, 2002)

THEOREM 1 *The disease-free equilibrium $\mathcal{E}^0$ for model system 4 is locally asymptotically stable if $\mathcal{R}_{H_H} < 1$ and unstable otherwise.*

Using a theorem from Castillo-Chavez, Feng, and Huang (2002), we show global stability when the reproduction number is less than unity.

THEOREM 2 *The disease-free equilibrium $\mathcal{E}^0$ for model system (4) is globally asymptotically stable provided $\mathcal{R}_{T_p} < 1$.*

Proof Following the method by Castillo-Chavez et al. (2002), two conditions should be met to guarantee the global asymptotic stability of the disease-free equilibrium . We write system (4) as

$$\begin{aligned}\frac{dX}{dt} &= F(X(t), Z(t)), \\ \frac{dZ}{dt} &= G(X(t), Z(t)), \quad G(X(t), 0) = 0,\end{aligned} \tag{12}$$

where $X = (S_1, S_2)$ and $Z = (I_1, I_{1_t}, A_1, A_{1_t}, I_2, A_2)$ with $X \in \mathbb{R}_+^2$ representing the number of uninfected individuals and $Z \in \mathbb{R}_+^6$ representing the number of infected individuals. The disease-free equilibrium can now be written as $\mathcal{E}^0 = (\mathbf{X}_0, 0)$ where

$$\mathbf{X}_0 = \left(\frac{\Lambda\pi}{\mu}, \frac{\Lambda(1-\pi)}{\mu}\right) \tag{13}$$

Conditions H_1 and H_2 below must be met to guarantee global asymptotic stability.

$$\begin{aligned}&H_1: \quad \text{For } \frac{dX}{dt} = F(X, 0),\ X_0 \text{ is globally asymptotically stable,} \\ &H_2: \quad G(X, Z) = AZ - \hat{G}(X, Z), \hat{G}(X, Z) \geq 0 \text{ for } (X, Z) \in \mathcal{R}_+^6 \subset \Omega,\end{aligned} \tag{14}$$

where $A = D_Z G(X^*, 0)$ is an M-matrix (the off-diagonal elements of A are non-negative) and $\mathbb{R}^6_+$ is the region where the model makes biological sense.

In this case

$$F(X,0) = \begin{bmatrix} \pi\Lambda - \mu S_1 \\ (1-\pi)\Lambda - \mu S_2 \end{bmatrix}, \; G(X,Z) = AZ - \widehat{G}(X,Z),$$

$$A = \begin{bmatrix} -g_1 + p_{11}c_1\beta_1\phi_1 & p_{11}c_1\beta_1\theta\phi_1 & p_{11}c_1\beta_1 & p_{11}c_1\beta_1\theta & \dfrac{p_{12}c_1\beta_2\phi_2\pi}{1-\pi} & \dfrac{p_{12}c_1\beta_2\pi}{1-\pi} \\ \alpha_i & -g_2 & 0 & 0 & 0 & 0 \\ \rho_1 & 0 & -g_3 & 0 & 0 & 0 \\ 0 & \rho_{1_t} & \alpha_a & -g_4 & 0 & 0 \\ \dfrac{p_{21}c_2\beta_1\phi_1(1-\pi)}{\pi} & \dfrac{p_{21}c_2\beta_1\theta\phi_1(1-\pi)}{\pi} & \dfrac{p_{21}c_2\beta_1(1-\pi)}{\pi} & \dfrac{p_{21}c_2\beta_1\theta(1-\pi)}{\pi} & p_{22}c_2\beta_2\phi_2 - g_5 & p_{22}c_2\beta_2 \\ 0 & 0 & 0 & 0 & \rho_2 & -g_4 \end{bmatrix}. \tag{15}$$

It follows that

$$\widehat{G}(X,Z) = \begin{bmatrix} \widehat{G_1}(X,Z) \\ \widehat{G_2}(X,Z) \\ \widehat{G_3}(X,Z) \\ \widehat{G_4}(X,Z) \\ \widehat{G_5}(X,Z) \\ \widehat{G_6}(X,Z) \end{bmatrix} = \begin{bmatrix} p_{11}c_1\beta_1[A_1 + \phi_1 I_1 + \theta(A_{1_t} + \phi_1 I_{1_t})]\left(1 - \frac{S_1}{N_1}\right) + p_{12}c_1\beta_2[A_2 + \phi_2 I_2]\left(\frac{\pi}{1-\pi} - \frac{S_1}{N_2}\right) \\ 0 \\ 0 \\ 0 \\ p_{21}c_2\beta_1[A_1 + \phi_1 I_1 + \theta(A_{1_t} + \phi_1 I_{1_t})]\left(\frac{1-\pi}{\pi} - \frac{S_2}{N_1}\right) + p_{22}c_2\beta_2[A_2 + \phi_2 I_2]\left(1 - \frac{S_2}{N_2}\right) \\ 0 \end{bmatrix} \tag{16}$$

We need to show that $\widehat{G}_i(X,Z) \geq 0, \; (i = 1, 5)$. To do this, we prove by contradiction. Assume statements in 17 and 18 are true.

$$\frac{\pi}{1-\pi} < \frac{S_1}{N_2} \tag{17}$$

and

$$\frac{1-\pi}{\pi} < \frac{S_2}{N_1} \tag{18}$$

From 17 together we have

$$\frac{\pi}{1-\pi} < \frac{S_1}{N_2} \Rightarrow \frac{N_2}{S_1} < \frac{1-\pi}{\pi} \tag{19}$$

From 18 and 19 it follows that

$$\frac{N_2}{S_1} < \frac{1-\pi}{\pi} < \frac{S_2}{N_1} \Rightarrow N_1 N_2 < S_1 S_2 \tag{20}$$

a contradiction as statement 20 is not true. Thus, $N_1 N_2 \geq S_1 S_2$ implying that $\dfrac{S_1}{N_2} \leq \dfrac{\pi}{1-\pi}$ and $\dfrac{S_2}{N_1} \leq \dfrac{1-\pi}{\pi}$. Thus, $\widehat{G}(X,Z) \geq 0$. Therefore, the disease-free equilibrium is globally asymptotically stable. □

2.2.1. Analysis of the reproduction number $\mathcal{R}_{H_H}$

In the case that the like only have sexual contacts with the like, $p_{11} = p_{22} = 1,\ p_{12} = p_{21} = 0$, then $\mathcal{R}_{H_H} = \max\{\mathcal{R}_{H_1}, \mathcal{R}_{0_{H_2}}\}$ where

$$\begin{aligned}\mathcal{R}_{H_1} &= \frac{\beta_1 c_1[\theta g_3 \alpha_i(\rho_{1_t} + \phi_1 g_4) + (\theta \alpha_a \rho_1 + (\rho_1 + \phi_1 g_3) g_4) g_2]}{g_1 g_2 g_3 g_4} \\ \mathcal{R}_{0_{H_1}} &= \frac{\beta_1 c_1(\rho_1 + \phi_1 g_4)}{g_4 g_7},\ \mathcal{R}_{0_{H_2}} = \frac{\beta_2 c_2(\rho_2 + \phi_2 g_6)}{g_5 g_6}\end{aligned} \tag{21}$$

These are: (i) the antiretroviral-induced reproduction number for HIV transmission when non-homeless people have sexual contacts only with the non-homeless ($\mathcal{R}_{H_1}$) and (ii) the basic reproduction number for HIV transmission when the homeless only have sexual contacts with the homeless ($\mathcal{R}_{0_{H_2}}$). In the absence of any intervention strategy $\mathcal{R}_{H_1}$ becomes $\mathcal{R}_{0_{H_1}} = \frac{\beta_1 c_1(\rho_1 + \phi_1 g_4)}{g_4 g_7}$, $\mathcal{R}_{0_{H_2}} = \frac{\beta_2 c_2(\rho_2 + \phi_2 g_6)}{g_5 g_6}$. This allows us to compare the various components of the two basic reproduction numbers $\mathcal{R}_{0_{H_2}}$ and $\mathcal{R}_{0_{H_1}}$ for different scenarios such as lack of entertainment, poor nutrition, co-infections with other STIs, and reduced socio-economic status.

In Table 1, the various components of homelessness are singly assessed: Case 1 suggests that homeless people are at a comparative disadvantage when it comes to entertainment, as lack of entertainment leaves sexual intercourse as the only form of entertainment, making the homeless people more prone to HIV infections than their non-homeless counterparts. Case 2 attempts to describe the effect of poor nutrition by capturing the increased probability of progressing to the AIDS stage among the homeless than among the non-homeless. Poor nutrition tends to compromise one's immunity, thus contributing to an increase in the progression to the AIDS stage of disease progression for the HIV infected. Due to the lack of entertainment, proper medical advice and treatment, co-infections with other STIs are common among the homeless, making them more prone to HIV infections than the non-homeless as noted in Case 3. Generally homeless people in the AIDS stage of disease progression experience higher AIDS-induced death rates than their counterparts. Results from Table 1 suggest that homeless people are at an increased disadvantage when it comes to HIV infection. All the signifiers of homelessness serve to exasperate the risk of HIV transmission between the homeless and the non-homeless. These results suggest that fighting homelessness should be addressed to alleviate the plight of the homeless HIV-infected people who find it impossible to access medical care. This is in total agreement with Wolitski et al. (2010) and Buchanan, Kee, Sadowski, and Garcia (2009) who showed a positive linkage between housing assistance for low-income people living with HIV/AIDS and better access to health care services. Results from Table 1 are further illustrated in Figure 2.

Table 1. Effects of lack of entertainment, poor nutrition, co-infections with other STIs, and reduced socio-economic status on HIV/ AIDS transmission dynamics. In each case, all parameters are equalized between the homeless and non-homeless, except those under the heading "Conditions"

Case	Description	Conditions	$\mathcal{R}_{0_{H_2}} - \mathcal{R}_{0_{H_1}}$
1	Lack of entertainment	$c_2 = b_1 c_1,\ b_1 > 1$	$\frac{(b_1 - 1)c_1\beta_1(\rho_1 + \phi_1 g_4)}{g_4 g_7} > 0$
2	Poor nutrition	$\rho_2 = b_2\rho_1, b_2 > 1$	$\frac{c_1\beta_1(b_2 - 1)(\mu - \phi_1 g_4)}{g_4 g_5 g_7} > 0,\ 0 < \phi_1 < \frac{\mu}{g_4}$
3	Co-infections with other STIs	$\beta_2 = b_3\beta_1, b_3 \geq 1$	$\frac{(b_3 - 1)c_1\beta_1(\rho_1 + \phi_1 g_4)}{g_4 g_7} > 0$
4	Reduced socio-economic status	$v_2 = b_4 v_1, b_4 \geq 1$	$\frac{(b_4 - 1)c_1\beta_1 v_1(\phi_1 v_1 - \rho_1)}{g_4 g_6 g_7} > 0,\ \frac{\rho_1}{v_1} \leq \phi_1 < 1$

Figure 2. Effects of lack of entertainment or co-infections with other STIs (solid line), poor nutrition (dashed line), and reduced socio-economic status (dotted line) noting that $c_2 = b_1 c_1$, $\rho_2 = b_2\rho_1$, $\beta_2 = b_3\beta_1$, and $\nu_2 = b_4\nu_1$. Parameter values are as in Table 2.

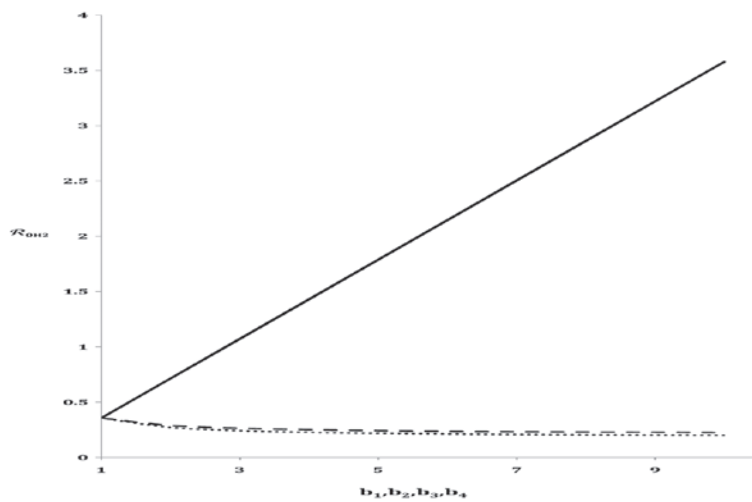


Figure 2 shows the effects of homelessness on HIV acquistion. It shows that lack of entertainment or co-infection with other STIs enhance the growth of the homeless-induced reproduction number. This suggests that lack of entertainment and/or co-infection with other STIs enhance the transmission of HIV.

2.3. Endemic equilibria

Model system 4 has three possible equilibria states: the homeless-only, the non-homeless-only, and the co-existence equilibrium. It is worth mentioning that the homeless and non-homeless-only equilibria states are simple HIV/AIDS endemic equilibria not worth discussing as so many researchers have analyzed them (see Bhunu, Garira, & Magombedze 2009 and references cited there in). The co-existence equilibrium occurs when there is sexual interaction between the homeless and non-homeless and is given by $\mathcal{E}^*$ where

$$\begin{aligned}
\mathcal{E}^* &= \left(S_1^*, I_1^*, I_{1_t}^*, A_1^*, A_{1_t}^*, S_2^*, I_2^*, A_2^*\right)\\
&\text{where}\\
S_1^* &= \frac{\pi\Lambda}{\lambda_1^*+\mu},\ I_1^* = \frac{\pi\Lambda\lambda_1^*}{g_1(\lambda_1^*+\mu)},\ I_{1_t}^* = \frac{\pi\alpha_i\Lambda\lambda_1^*}{g_2 g_1(\lambda_1^*+\mu)},\ A_1^* = \frac{\pi\rho_1\Lambda\lambda_1^*}{g_3 g_1(\lambda_1^*+\mu)}, \qquad (22)\\
A_{1_t}^* &= \frac{\pi\Lambda\lambda_1^*[\rho_{1_t}\alpha_i g_3 + \rho_1\alpha_a g_2]}{g_1 g_2 g_3 g_4(\lambda_1^*+\mu)},\ S_2^* = \frac{(1-\pi)\Lambda}{\lambda_2^*+\mu},\ I_2^* = \frac{(1-\pi)\Lambda\lambda_2^*}{g_5(\lambda_2^*+\mu)},\ A_2^* = \frac{(1-\pi)\rho_2\Lambda\lambda_2^*}{g_6 g_5(\lambda_2^*+\mu)}.
\end{aligned}$$

Substituting equation 22 into the equation for the forces of infection λ_1^* in equation 3 we obtain

$$\begin{aligned}
&\lambda_1^* = \frac{p_{11}c_1\beta_1\lambda_1^* D_1}{D_5+\lambda_1^* D_2} + \frac{p_{12}c_1\beta_2 D_3\lambda_2^*}{D_6+\lambda_2^* D_4},\\
&D_1 = \theta g_3\alpha_i(\rho_{1_t}+\phi_1 g_4) + g_2(\theta\alpha_a\rho_1 + (\rho_1+\phi_1 g_3)g_4),\ D_3 = \rho_2+\phi_2 g_6,\ D_4 = g_6+\rho_2 \qquad (23)\\
&D_2 = g_2 g_3 g_4 + \alpha_i g_3 g_4 + \rho_1 g_2 g_4 + \rho_{1_t}\alpha_i g_3 + \rho_1\alpha_a g_2,\ D_5 = g_1 g_2 g_3 g_4,\ A_6 = g_5 g_6
\end{aligned}$$

Substituting equation 22 into the equation for the forces of infection λ_1^* in equation 3 we obtain

$$\lambda_2^* = \frac{p_{22}c_2\beta_2 D_3\lambda_2^*}{D_6+\lambda_2^* D_4} + \frac{p_{21}c_2\beta_1\lambda_1^* D_1}{D_5+\lambda_1^* D_2} \qquad (24)$$

Expressing λ_1^* as the subject of the formula in equations 23 and 24 and equating the two forms of λ_1^* to obtain an equation in λ_2^* which upon being solved we obtain $\lambda_2^* = 0$ corresponding to the disease-free equilibrium, $\lambda_2^* = \dfrac{B_1(\mathcal{R}_{H_H} - 1)}{B_2}$, $[B_1, B_2$ are positive and in terms of D_k's, p_{ij}, c_i, i, j=(1,2), k=(1, 2, 3, 4, 5, 6)] being the endemic equilibrium which clearly exists when $\mathcal{R}_{H_H} > 1$ and the other two complex roots which are going to be discarded since we are dealing with real populations. This result is summarized in Lemma 1.

Lemma 1 *The endemic equilibrium $\mathcal{E}^*$ exists whenever $\mathcal{R}_{H_H} > 1$.*

3. Numerical simulations

Unless otherwise stated, values used in the analysis and simulations are given in Table 2

The fourth-order Runge–Kutta numerical scheme coded in C + + programming language is used to graphically depict disease progression over time. Numerical simulations using a set of reasonable parameter values in Table 2 are carried out for illustrative purpose and to support the analytical results.

Figure 3 shows antiretroviral therapy for the non-homeless also has a beneficial effect on the homeless as noted by a decline in the number of new HIV cases among the homeless whenever levels of antiretroviral therapy are increased. This result suggests effective control of HIV lie in antiretroviral therapy for the non-homeless. This result further suggests control of HIV require strategies that remove people from the streets (homeless) into decent homes where they can be tracked and put on therapy. This result further suggests that care and control of HIV goes beyond provision of drugs. This is all in support of Wolitski et al. (2010) and Buchanan et al. (2009) who found positive linkage between provision of housing for the people living with HIV/AIDS and improved access to health care services.

Figure 4 is a graphical representation showing the effects of lack of entertainment with regard to cumulative new HIV cases in the homeless and non-homeless communities, respectively. It shows that lack of entertainment increases the rate of acquiring HIV as noted by increase in new HIV cases

Table 2. Default (baseline) model parameters used in the analysis and simulations. Here, ZIMSTAT stands for Zimbabwe National Statistics Agency (2012)

Definition	Symbol	Value (Range)	Source
Recruitment rate	Λ	0.029 yr^{-1}×4.2×10^6	ZIMSTAT
Natural mortality rate	μ	0.02 yr^{-1}	ZIMSTAT
Modification parameters	ϕ_1, ϕ_2, θ	0.125 (0.1–1)	Bhunu and Mushayabasa (2012)
Rate of being put on treatment	α_i, α_a	0.33(0.01–1) yr^{-1}	Bhunu et al., (2009)
Product of effective contact rate			
for HIV transmission and probability	$\beta_i c_j$ $(i, j = 1, 2)$	0.125(0.011–0.95) yr^{-1}	Hyman, Li, and Stanley (1999)
of HIV transmission per sexual contact			
Rate of progression to AIDS	ρ_1, ρ_{1_t}	0.1(0.075–0.95) yr^{-1}	Bhunu et al. (2009)
AIDS-related death (non-homeless)	ν_1	0.333 (0.3–0.75) yr^{-1}	Bhunu et al. (2009)
Homegeneous mixing	p_{11}, p_{22}	0.67	Assume
Heterogeneous mixing	p_{12}, p_{21}	0.33	Assume
Modification parameter	b_k $(k = 1, \cdots, 4)$	1.125(≥1)	Assume

Figure 3. Simulations showing the possible effects of antiretroviral therapy using parameter values in Table 2. The direction of the arrow shows an increase in the levels of treatment.

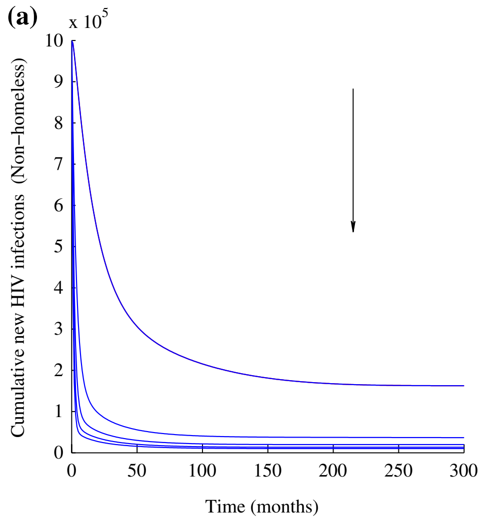
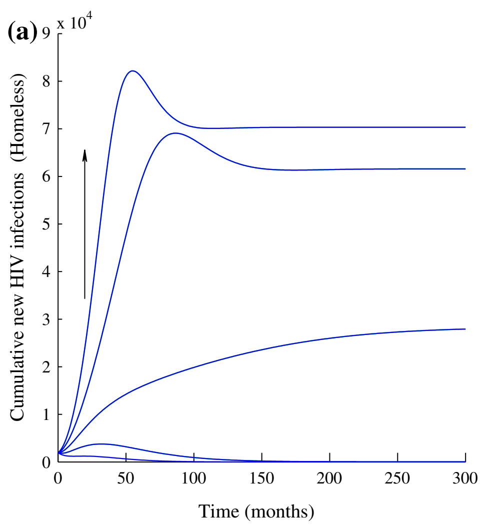


Figure 4. Simulations showing the possible effects of lacking entertainment using parameter values in Table 2 in the presence of antiretroviral therapy. The direction of the arrow shows a decrease in the levels of entertainment.



with decrease in entertainment levels. This greatly affects the homeless more than the non-homeless as noted in Figure 4(a). This suggest that provision of entertainment facilities which cuts across the homeless and non-homeless boundaries will play a crucial role in reducing the spread of HIV as some people resort to sexual intercourse due to the lack of entertainment.

4. Discussion

Homelessness expose an individual to a number of social, economic, and health risks and challenges. A mathematical model to explore the potential effects of homelessness on HIV/AIDS transmission dynamics is presented as a system of non-linear differential equations. The reproduction number of the model is computed and analyzed. The disease-free equilibrium is shown to be globally asymptotically stable whenever the reproduction number is less than unity. Numerical simulations show that antiretroviral therapy for the non-homeless also has a beneficial effect on the homeless as noted by a decline in the number of cumulative new HIV cases among the homeless with increase in antiretroviral therapy among the non-homeless. This is in total support of Wolitski et al. (2010) and Buchanan et al. (2009) who showed a positive linkage between provision of housing for those living with HIV/ AIDS and improved access to health care services which in turn improves the quality of their lives (living longer healthy productive lives). This result suggests the effect of creation of homes for the homeless where antiretroviral therapy and monitoring can be easily administered will have a more beneficial effect on the general population. Furthermore, numerical simulations show that the lack of entertainment also play a significant role in the spread of HIV and mostly so for the homeless. Thus, there is a need for policy-makers to provide recreational facilities which cuts across the homeless and non-homeless boundaries as some people resort to sex due to lack of other forms of entertainment. This on its own creates more social, economic, and health problems: unwanted pregnancies and increased risk of contracting HIV among others. Results from this theoretical study (comparison of homeless- and non-homeless-induced reproduction numbers) show that lack of entertainment, poor nutrition, and co-infections with other STIs worsen HIV/AIDS (HIV transmission and AIDS-related deaths are higher among the homeless than their counterparts). There are a number of limitations to our study, which should be acknowledged. We assumed that "homeless" versus "non-homeless" is a state assigned at birth, with no possibility of transfer later in life. While true to some degree in large parts of the world, this is obviously not always the case.

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References

Adams, M. (2003). HIV and Homeless shelters: Policy and practice, American civil liberties union AIDS project. Retrieved June 20, 2003, from archive.aclu.org/issues/gay/hiv_homeless.html

Aidala, A. A., Lee, G., Abramson, D. M., Messeri, P., & Siegler, A. (2007). Housing need, housing assistance, and connection to HIV medical care. *AIDS & Behavior, 11*, 101–115.

Aidala, A., & Sumartojo, E. (2007). Why housing? *AIDS & Behavior, 11*(6, Suppl. 2), S1–S6.

Ajay, M., Brendan, O., & David, E. B. (2009). *Needle sharing and HIV transmission: A model with markets and purposive behavior*. NBER Working Papers. Retrieved from http://ideas.repec.org/p/nbr/nberwo/14823.html

Bekele, T., Rourke, S. B., Tucker, R., Greene, S., Sobota, M., Koornstra, J., ... Guenter, D. (2013). Direct and indirect effects of perceived social support on health-related quality of life in persons living with HIV/AIDS. *AIDS Care, 25*, 337–346.

Bhunu, C. P., Garira, W., & Magombedze, G. (2009). Mathematical analysis of a two strain HIV/AIDS model with antiretroviral treatment. *Acta Biotheoretica, 57*, 361–381.

Bhunu, C. P., & Mushayabasa, S. (2012). Assessing the impact of using antiretroviral drugs as pre-exposure vaccines. *HIV & AIDS Review, 11*, 42–48.

Bhunu, C. P., Mhlanga, A. N., & Mushayabasa, S. (2014). Exploring the impact of prostitution on HIV/ AIDS transmission. *International Scholarly Research Network, 2014,* 10.

Buchanan, D., Kee, R., Sadowski, L. S., & Garcia, D. (2009). The health impact of supportive housing for HIV-Positive homeless patients: A randomized controlled trial. *American Journal of Public Health, 99* (Suppl. 3), S675–S680.

Birkhoff, G., & Rota, G. C. (1982). *Ordinary differential equations*. Boston, MA: Ginn.

Castillo-Chavez, C., Feng, Z., & Huang, W. (2002). On the computation of R_0 and its role on global stability. In C. Castillo Chavez, S. Blower, P. Driessche, D. Kirschner, & A. Yakubu (Eds.), *Mathematical approaches for emerging and reemerging infectious diseases: Models, methods, and theory*. IMA, (Vol. 126, pp. 215–230) New York, NY: Springer.

Didenko, E., & Pankratz, N. (2007). Substance use: Pathways to homelessness? Or a way of adapting to street life? *Visions: BC's Mental Health and Addictions Journal, 4*, 9–10.

Hyman, J. M., Li, J., & Stanley, E. A. (1999). The differential infectivity and staged progression models for the transmission of HIV. *Mathematical Biosciences, 155*, 77–109.

Kidder, D., Wolitski, R., Campsmith, M., & Nakamura, G. (2007). Health status, health care use, medication use, and medication adherence in homeless and housed people living with HIV/AIDS. *American Journal of Public Health, 97*, 2238–2245.

Leaver, C. A., Bargh, G., Dunn, J. R., & Hwang, S. W. (2007). The effects of housing status on health-related outcomes in people living with HIV: A systematic review of the literature. *AIDS & Behavior, 11* (Suppl. 2), S85–S100.

National Coalition for the Homeless. (2009). HIV/AIDS and homelessness. Retrieved from www.nationalhomeless.org/factsheets/hiv.html

Robertson, M. J., Clark, R. A., Charlebois, E. D., Tulsky, J., Long, H. L., Bangsberg, D. R., & Moss, A. R. (2004). HIV seroprevalence among homeless and marginally housed adults in San Francisco. *American Journal of Public Health., 94*, 1207–1217.

Pedamallu, C. S., Ozdamar, L., Kropat, E., & Weber, G. W. (2012). A system dynamics model for intentional transmission of HIV/AIDS using cross impact analysis. *CEJOR, 20*, 319–336.

Shubert, V., & Bernstine, N. (2007). Moving from fact to policy: Housing is HIV prevention and health care. *AIDS & Behavior, 11* (Suppl. 2), S167–S171.

Song, J. (2003). AIDS housing of Washington. *AIDS housing Survey* (p. 1). Washington, DC: Springer.

Tomaszewski, E. P. (2011). HIV/ AIDS and homeless. National Association of Social Workers. Retrieved from www.socialworkers.org/practice/hiv_aids/spectrum

Van den Driesche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for the compartmental models of disease transmission. *Mathematical Biosciences, 180*, 29–48.

Wolitski, R. J., Kidder, D. P., Pals, S. L., Royal, S., Aidala, A., Stall, R., & Courtenay-Quirk, C. (2010). Randomized trial of the effects of housing assistance on the health and risk behaviors of homeless and unstably housed people living with HIV. *AIDS & Behavior, 14*, 493–503.

WHO. (2005). Guidelines for HIV diagnosis and monitoring of antiretroviral therapy. Geneva: WHO