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Strong (skew) ξ-Lie commutativity preserving maps on algebras

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Abstract: Let $\mathcal{A}$ be any unital *-algebra over the real or complex field $\mathbb{F}$, and let $\xi \in \mathbb{F}$ with $\xi \neq 1$. Assume that $\Phi: \mathcal{A} \to \mathcal{A}$ is a map. It is shown that, Φ satisfies $\Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A) = AB - \xi BA$ for all $A, B \in \mathcal{A}$ if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, the center of $\mathcal{A}$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$; if $\Phi(I) = \Phi(I)^*$, then Φ satisfies $\Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A)^* = AB - \xi BA^*$ for all $A, B \in \mathcal{A}$ if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$; if $|\xi| = 1$ and Φ is surjective, then Φ satisfies $\Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A)^* = AB - \xi BA^*$ for all $A, B \in \mathcal{A}$ if and only if $\Phi(I) = \Phi(I)^* \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$, and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$.

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1. Introduction

Let $\mathcal{R}$ be a ring. Then $\mathcal{R}$ is a Lie ring under the Lie product $[A, B] = AB - BA$. Recall that a map $\Phi: \mathcal{R} \to \mathcal{R}$ preserves commutativity if $[\Phi(A), \Phi(B)] = 0$ whenever $[A, B] = 0$ for all $A, B \in \mathcal{R}$. The problem of characterizing linear (additive) maps preserving commutativity had been studied intensively on various rings and algebras (see [Brešar, 1993; Brešar & Šemrl, 2005; Choi, Jafarian, & Radjavi, 1987] and the references therein).

In Bell and Daif (1994), the authors gave the conception of strong commutativity preserving maps. Let $\mathcal{S}$ be a subset of $\mathcal{R}$. A map $\Phi: \mathcal{S} \to \mathcal{R}$ is called strong commutativity preserving if $[\Phi(T), \Phi(S)] = [T, S]$ for all $T, S \in \mathcal{S}$. Note that a strong commutativity preserving map must be commutativity preserving, but the inverse is not true generally. Bell and Daif (1994) proved that $\mathcal{R}$ must be commutative, if $\mathcal{R}$ is a prime ring and $\mathcal{R}$ admits a derivation or a non-identity endomorphism which is strong commutativity preserving on a right ideal of $\mathcal{R}$. Brešar and Miers (1994) proved that every strong commutativity preserving additive map Φ on a semiprime ring $\mathcal{R}$ is of the form $\Phi(A) = \lambda A + \mu(A)$, where $\lambda \in \mathcal{C}$, the extended centroid of $\mathcal{R}$, $\lambda^2 = 1$, and $\mu: \mathcal{R} \to \mathcal{C}$ is an additive map. Recently, Lin and Liu (2008) obtained the similar result on a noncentral Lie ideal of a prime ring. Qi and Hou (2010; 2012) gave a complete characterization of strong commutativity preserving surjective maps (without the assumption of additivity) on prime rings and triangular algebras, respectively.

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PUBLIC INTEREST STATEMENT

Preserver problem had attracted many mathematicians' attentions for many years. In this paper, the authors discuss the general maps preserving strong (skew) ξ-Lie commutativity on any algebras and give a complete characterization for such maps.

Let $\mathcal{R}$ be a *-ring. For any $A, B \in \mathcal{R}$, $[A, B]_* = AB - BA^*$ denotes the skew Lie product of A and B. This kind of product is found playing a more and more important role in some research topics such as representing quadratic functionals with sesquilinear functionals, and its study has attracted many authors' attention (see [Brešar & Fosňer, 2000; Chebotar, Fong, & Lee, 2005; Cui & Hou, 2006] and the reference therein). Molnár (1996) initiated the systematic study of this skew Lie product, and studied the relation between subspaces and ideals of $\mathcal{B}(H)$, the algebra of all bounded linear operators acting on a Hilbert space H.

Recall that a map $\Phi: \mathcal{R} \to \mathcal{R}$ is called zero skew Lie product preserving, if $\Phi(A)\Phi(B) - \Phi(B)\Phi(A)^*$ =0 whenever $AB - BA^* = 0$ for any $A, B \in \mathcal{R}$. Additive or linear maps preserving zero skew Lie products on various rings and algebras had been studied by many authors (see, Bell & Daif, 1994 and the references therein). More specially, Φ is strong skew commutativity preserving, if $[\Phi(A), \Phi(B)]_* = [A, B]_*$ for all $A, B \in \mathcal{R}$. It is obvious that strong skew commutativity preserving maps must be zero skew Lie product preserving. However, the inverse is not true generally. In Cui and Park (2012), they proved that, if $\mathcal{R}$ is a factor von Neumann algebra, then every strong skew commutativity preserving map Φ on $\mathcal{R}$ has the form $\Phi(A) = \Psi(A) + h(A)I$ for all $A \in \mathcal{R}$, where $\Psi: \mathcal{R} \to \mathcal{R}$ is a linear bijective map satisfying $\Psi(A)\Psi(B) - \Psi(B)\Psi(A)^* = AB - BA^*$ for all $A, B \in \mathcal{R}$ and h is a real linear functional of $\mathcal{R}$ with $h(0) = 0$; particularly, if $\mathcal{R}$ is of type I, then $\Phi(A) = cA + h(A)I$ for each $A \in \mathcal{R}$, where $c \in \{-1, 1\}$. Recently, Qi and Hou (2013) generalized the above result to von Neumann algebras without central summand of type I_1.

Recall that A commutes with B up to a factor $\xi \in \mathbb{F}$ if $AB = \xi BA$. Note that the concept of commutativity up to a factor for pairs of operators is important and has been studied in the context of operator algebras and quantum groups (see Brooke, Busch, & Pearson, 2002 and Kassel, 1995). Motivated by this, a binary operation $[A, B]_\xi = AB - \xi BA$, called ξ-Lie product of A and B, was introduced in Qi and Hou (2009). Thus, we also can define the skew ξ-Lie product of A and B. Let $\mathcal{A}$ be a *-algebra over $\mathbb{F}$, where $\mathbb{F}$ is a field with an involution $*$. For $A, B \in \mathcal{A}$ and $\xi \in \mathbb{F}$, we call $AB - \xi BA^*$ the skew ξ-Lie product of A and B. It is obvious that the skew ξ-Lie product is the skew Lie product if $\xi = 1$. Now, based on these concepts, we say that a map $\Phi: \mathcal{A} \to \mathcal{A}$ is preserving strong ξ-Lie commutativity if $[\Phi(T), \Phi(S)]_\xi = [T, S]_\xi$ for all $T, S \in \mathcal{A}$; is preserving strong skew ξ-Lie commutativity if $\Phi(T)\Phi(S) - \xi\Phi(S)\Phi(T)^* = TS - \xi ST^*$ for all $T, S \in \mathcal{A}$.

The purpose of this paper is to consider nonlinear strong (skew) ξ-Lie commutativity preserving maps on general algebras with $\xi \neq 1$. Let $\mathcal{A}$ be any unital algebra over any field $\mathbb{F}$ and $\xi \in \mathbb{F}$ with $\xi \neq 1$. Denote by $\mathcal{Z}(\mathcal{A})$ the center of $\mathcal{A}$. Assume that $\Phi: \mathcal{A} \to \mathcal{A}$ is a map. In Section 2, we prove that Φ preserves strong ξ-Lie commutativity if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$, and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$ (Theorem 2.1). In Section 3, we furthermore assume that $\mathcal{A}$ is a *-algebra. It is shown that, if $\Phi(I) = \Phi(I)^*$, then Φ preserves strong skew ξ-Lie commutativity if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$ (Theorem 3.1); if $|\xi| = 1$ and Φ is surjective, then Φ preserves strong skew ξ-Lie commutativity if and only if $\Phi(I) = \Phi(I)^* \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$ (Theorem 3.2).

2. Maps preserving strong ξ-Lie commutativity

In this section, we will give a characterization of nonlinear strong ξ-Lie commutativity preserving maps on general algebras. The following is our main result.

THEOREM 2.1 *Let $\mathcal{A}$ be any algebra with unit I over a field $\mathbb{F}$, and let $\xi \in \mathbb{F}$ with $\xi \neq 1$. Assume that $\Phi: \mathcal{A} \to \mathcal{A}$ is a map. Then Φ preserves strong ξ-Lie commutativity, that is, Φ satisfies $[\Phi(A), \Phi(B)]_\xi = [A, B]_\xi$ for all $A, B \in \mathcal{A}$, if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$.*

Proof The "if" part is obvious. For the "only if" part, since $[\Phi(I), \Phi(I)]_\xi = [I, I]_\xi$, we have $(1 - \xi)\Phi(I)^2 = (1 - \xi)I$. It follows that $\Phi(I)^2 = I$ as $\xi \neq 1$.

In the sequel, we will complete the proof by considering two cases.

Case 1 $\xi = -1$.

Take any $A \in \mathcal{A}$. Then

$$2A = AI + IA = \Phi(A)\Phi(I) + \Phi(I)\Phi(A) \tag{2.1}$$

Multiplying $\Phi(I)$ from the left- and the right-hand side in Equation 2.1, respectively, one gets

$$2\Phi(I)A = \Phi(I)\Phi(A)\Phi(I) + \Phi(I)^2\Phi(A) = \Phi(I)\Phi(A)\Phi(I) + \Phi(A)$$

and

$$2A\Phi(I) = \Phi(A)\Phi(I)^2 + \Phi(I)\Phi(A)\Phi(I) = \Phi(A) + \Phi(I)\Phi(A)\Phi(I)$$

Comparing the above two equations, we obtain $\Phi(I)A = A\Phi(I)$ for each $A \in \mathcal{A}$. It follows from the arbitrariness of $A \in \mathcal{A}$ that $\Phi(I) \in \mathcal{Z}(\mathcal{A})$. This and Equation 2.1 imply $\Phi(I)\Phi(A) = A$. Note that $\Phi(I)^2 = I$. So $\Phi(A) = \Phi(I)A$ holds for all $A \in \mathcal{A}$, completing the proof of the theorem.

Case 2 $\xi \neq -1$.

Take any $A, B \in \mathcal{A}$ and note that $(1-\xi)[A,B]_{-1} = [A,B]_\xi + [B,A]_\xi$. Since Φ preserves strong ξ-Lie commutativity, we have

$$\begin{aligned}(1-\xi)[A,B]_{-1} &= [\Phi(A),\Phi(B)]_\xi + [\Phi(B),\Phi(A)]_\xi \\ &= (1-\xi)(\Phi(A)\Phi(B) + \Phi(B)\Phi(A)).\end{aligned}$$

That is,

$$AB + BA = \Phi(A)\Phi(B) + \Phi(B)\Phi(A)$$

holds for all $A, B \in \mathcal{A}$. Now, by Case 1, the theorem is true.

Combining Case 1 and Case 2, the proof of the theorem is complete.

3. Maps preserving strong skew ξ-Lie commutativity

In this section, we will discuss the maps preserving strong skew ξ-Lie commutativity on general algebras.

THEOREM 3.1 Let $\mathcal{A}$ be any *-algebra with unit I over the real or complex field $\mathbb{F}$ and let $\xi \in \mathbb{F}$ with $\xi \neq 1$. Assume that $\Phi : \mathcal{A} \to \mathcal{A}$ is a map. If $\Phi(I) = \Phi(I)^*$, then Φ preserves strong skew ξ-Lie commutativity, that is, Φ satisfies $\Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A)^* = AB - \xi BA^*$ for all $A, B \in \mathcal{A}$, if and only if $\Phi(I) \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$.

Proof Still, one only needs to prove the "only if" part.

By the assumption, for any $A, B \in \mathcal{A}$, we have

$$\Phi(A)\Phi(B) - \xi\Phi(B)\Phi(A)^* = AB - \xi BA^* \tag{3.1}$$

Taking $A = B = I$ in Equation (3.1), one gets $\Phi(I)^2 - \xi\Phi(I)\Phi(I)^* = (1-\xi)I$. Note that $\Phi(I) = \Phi(I)^*$ and $\xi \neq 1$. We obtain

$$\Phi(I)^2 = I \tag{3.2}$$

Taking $A = I$ in Equation 3.1, one gets $\Phi(I)\Phi(B) - \xi\Phi(B)\Phi(I)^* = (1-\xi)B$, that is,

$$\Phi(I)\Phi(A) - \xi\Phi(A)\Phi(I) = (1-\xi)A \text{ for all } A \in \mathcal{A} \tag{3.3}$$

Taking $B = I$ in Equation 3.1, one has

$$\Phi(A)\Phi(I) - \xi\Phi(I)\Phi(A)^* = A - \xi A^* \tag{3.4}$$

This implies

$$\Phi(I)^*\Phi(A)^* - \bar{\xi}\Phi(A)\Phi(I)^* = A^* - \bar{\xi}A \text{ for all } A \in \mathcal{A}$$

Multiplying ξ from both sides in the above equation, we get

$$\xi\Phi(I)^*\Phi(A)^* - |\xi|^2\Phi(A)\Phi(I)^* = \xi A^* - |\xi|^2 A \text{ for all } A \in \mathcal{A} \tag{3.5}$$

Combining Equations 3.4 and 3.5, we have

$$\Phi(A)(\Phi(I) - |\xi|^2\Phi(I)^*) - \xi(\Phi(I) - \Phi(I)^*)\Phi(A)^* = (1-|\xi|^2)A \text{ for all } A \in \mathcal{A} \tag{3.6}$$

Note that $\Phi(I) = \Phi(I)^*$. Equation 3.6 implies

$$(1-|\xi|^2)\Phi(A)\Phi(I) = (1-|\xi|^2)A \text{ for all } A \in \mathcal{A} \tag{3.7}$$

In the following, we will prove the theorem by two cases.

Case 1 $|\xi| \neq 1$.

In this case, Equation 3.7 implies

$$\Phi(A)\Phi(I) = A \text{ for all } A \in \mathcal{A} \tag{3.8}$$

Multiplying $\Phi(I)$ from the right-hand side in Equation 3.8, by Equation (3.2), one gets

$$\Phi(A) = A\Phi(I) \text{ for all } A \in \mathcal{A} \tag{3.9}$$

On the other hand, combining Equations 3.3 and 3.8, one has $\Phi(I)\Phi(A) = A$. Multiplying $\Phi(I)$ from the left-hand side in this equation and noting that Equation (3.2), one gets

$$\Phi(A) = \Phi(I)A \text{ for all } A \in \mathcal{A} \tag{3.10}$$

It follows from Equations 3.9 to 3.10 that $\Phi(I) \in \mathcal{Z}(\mathcal{A})$. The proof is finished.

Case 2 $|\xi| = 1$.

Multiplying $\Phi(I)$ from the left- and the right-hand side in Equation 3.3, respectively, by Equation 3.2 again, one can obtain

$$\Phi(A) - \xi\Phi(I)\Phi(A)\Phi(I) = (1-\xi)\Phi(I)A$$

and

$$\Phi(I)\Phi(A)\Phi(I) - \xi\Phi(A) = (1-\xi)A\Phi(I)$$

Comparing the above two equations gets $(1-\xi^2)\Phi(A) - \xi(1-\xi)A\Phi(I) = (1-\xi)\Phi(I)A$, that is,

$$(1+\xi)\Phi(A) = \xi A\Phi(I) + \Phi(I)A \text{ holds for all } A \in \mathcal{A} \tag{3.11}$$

We claim $A\Phi(I) = \Phi(I)A$, and so $\Phi(I) \in \mathcal{Z}(\mathcal{A})$. In fact, if $\xi = -1$, Equation (3.11) implies $A\Phi(I) = \Phi(I)A$; if $\xi \neq -1$, multiplying $\bar{\xi}$ from both sides in Equation (3.11), one has

$$(1+\bar{\xi})\Phi(A) = A\Phi(I) + \bar{\xi}\Phi(I)A \text{ for all } A \in \mathcal{A} \tag{3.12}$$

as $|\xi| = 1$. On the other hand, Equation 3.4 implies $(1+\xi)\Phi(A)\Phi(I) = \xi(1+\xi)\Phi(I)\Phi(A)^* + (1+\xi)A - \xi(1+\xi)A^*$. This and Equations 3.11–3.12 yield

$$(\xi A\Phi(I) + \Phi(I)A)\Phi(I) = \xi\Phi(I)(A\Phi(I) + \bar{\xi}\Phi(I)A)^* + (1+\xi)A - \xi(1+\xi)A^*$$

Note that $\Phi(I) = \Phi(I)^*$ and Equation 3.2. The above equation can be reduced to

$$\Phi(I)A\Phi(I) = \xi^2\Phi(I)A^*\Phi(I) + A - \xi^2A^*$$

Multiplying $\Phi(I)$ from the right side in the above equation yields

$$\Phi(I)A - A\Phi(I) = \xi^2(\Phi(I)A^* - A^*\Phi(I)) \text{ for all } A \in \mathcal{A} \tag{3.13}$$

Replacing A by iA in Equation (3.13), one can get $\Phi(I)(iA) - (iA)\Phi(I) = \xi^2(\Phi(I)(iA)^* - (iA)^*\Phi(I))$, that is,

$$\Phi(I)A) - A\Phi(I) = \xi^2(-\Phi(I)A^* + A^*\Phi(I)) \text{ for all } A \in \mathcal{A} \tag{3.14}$$

Combining Equations 3.13 and 3.14, one achieves $\Phi(I)A = A\Phi(I)$ for all $A \in \mathcal{A}$. The claim holds.

Now, it follows from Equation 3.3 that $\Phi(I)\Phi(A) = \Phi(A)\Phi(I) = A$, and so $\Phi(A) = \Phi(I)A = A\Phi(I)$ holds for all $A \in \mathcal{A}$. The theorem holds.

We complete the proof of the theorem.

If Φ is surjective and $|\xi| = 1$, then the condition $\Phi(I) = \Phi(I)^*$ in Theorem 3.1 can be deleted.

THEOREM 3.2 Let $\mathcal{A}$ be any *-algebra with unit I over the real or complex field $\mathbb{F}$, and let $\xi \in \mathbb{F}$ with $\xi \neq 1$ and $|\xi| = 1$. Assume that $\Phi:\mathcal{A} \to \mathcal{A}$ is a surjective map. Then Φ preserves strong skew ξ-Lie commutativity if and only if $\Phi(I) = \Phi(I)^* \in \mathcal{Z}(\mathcal{A})$, $\Phi(I)^2 = I$ and $\Phi(A) = \Phi(I)A$ for all $A \in \mathcal{A}$.

Proof By Theorem 3.1, to complete the proof of the theorem, one only needs to prove $\Phi(I) = \Phi(I)^*$.

Indeed, by checking the proof of Theorem 3.1, Equation 3.6 still holds, that is,

$$\Phi(A)(\Phi(I) - |\xi|^2\Phi(I)^*) - \xi(\Phi(I) - \Phi(I)^*)\Phi(A)^* = (1-|\xi|^2)A \text{ holds for all } A \in \mathcal{A}$$

Since $|\xi| = 1$, the above equation reduces to $\Phi(A)(\Phi(I) - \Phi(I)^*) - \xi(\Phi(I) - \Phi(I)^*)\Phi(A)^* = 0$ for all $A \in \mathcal{A}$. As Φ is surjective, there exists $A \in \mathcal{A}$ such that $\Phi(A) = I$. So $\Phi(I) - \Phi(I)^* = \xi(\Phi(I) - \Phi(I)^*)$. It follows from the fact $\xi \neq 1$ that $\Phi(I) = \Phi(I)^*$.

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